Chapter 7

Granular flows

In [16] it was shown that with the introduction of a very small dissipation in the microscopic dynamics, the usual hydrodynamics approach for granular flows fails to give a correct picture for a many-particle system. In this chapter we analyze the causes of this failure. The method presented in the previous chapter gives the possibility to prove the validity of the balance equations in this case as well. The conclusion is that the constitutive relations included into the granular flows equations are not adjusted to the particular corpuscular system from [16]. The results presented in the following can be found in [67].

7.1 Balance equations

Consider N identical sizeless particles confined within the space interval $[0\,,\,1]$ bounded by two walls of infinite mass. When two particles collide, their velocities after the collision are v_1' and v_2' are expressed in terms of their velocities before the collision v_1 and v_2 by the relation

$$\begin{aligned} v_1' &= \epsilon v_1 + (1-\epsilon) v_2 \\ v_2' &= (1-\epsilon) v_1 + \epsilon v_2 \end{aligned} \tag{7.1}$$

where $\epsilon = (1-r)/\,2$ and r is the restitution coefficient defined by $v_1^{'} - v_2^{'} = -\,r\,(v_1 - v_2)$. If $r = 1$ ($\epsilon = 0$), the collision is perfectly elastic and if $r = 0$ ($\epsilon = 1$) the collision is perfectly plastic. If the system is isolated, all the particles will be at rest when the initial kinetic energy is dissipated through collisions. In our case the energy is continuously pumped in the system through the left side wall. When the particle situated near the left wall collides this wall it is reversed with a random velocity with a Gaussian distribution proportional to $\exp(-v_o^2/2T_o)$. The collision with the right side

wall of the particle situated to the extreme right is perfectly elastic, without the change of the kinetic energy. These boundary conditions correspond to a left side wall at the constant temperature T_o and to a right side wall thermically isolated.

In [16] numerical simulations of the evolution of this corpuscular system have been performed by computation of the motion of each particle of the system. Excepting the particle that collides with the left wall, the other $N-1$ particles squeeze in a small clump. The first particle moves with a relative large velocity between the left wall and the clump of the particles which have a much smaller velocity. When ϵ decreases, the particles squeeze together in a smaller region and they move slower. This result is independent of how the energy is pumped in at the boundaries. A similar phenomenon has been reported in the cooling of inelastic particle system [23].

The usual hydrodynamic approach to such a corpuscular system is based on the equations of granular flow derived in [24] and [28]

$$\begin{aligned} \partial_t c &= -\partial_x(cu)\,, \\ c\partial_t u &= -cu\partial_x u - \partial_x(C_1 c\theta)\,, \\ c\partial_t \theta &= -cu\partial_x\theta - C_1 c\theta\partial_x u + \partial_x^2(C_2\theta^{3/2}) - C_3\epsilon\theta^{3/2}\,, \end{aligned} \tag{7.2}$$

where c is the volume concentration of the particle number, u is the average velocity, θ is the granular temperature (all these quantities will be rigorously defined in the following) and C_1, C_2, and C_3 are numerical constants. One can see that these equations are identical to the balance equations (1.2-1.4) for the one-dimensional case if the density ρ is replaced with the concentration c. This replacement is possible because all the particles have the same mass m and then $\rho = m\,c$.

The equations (7.2) include the constitutive relations for relatively dense granular materials in high shear flow. For these granular flows the flux of the kinetic energy is proportional to the gradient of the kinetic temperature (the Fourier law). But the Fourier law is not valid for the considered corpuscular system. Indeed, the fast moving particle running between the wall and the clump of the other particles transports the energy gained from the collision with the wall and loses it during the inelastic collisions with the neighboring particle. This nonvanishing energy flux near the wall corresponds to a vanishing gradient of the kinetic temperature, because in this region only the fast particle can move and its average state does not depend on position.

This contradiction leads to erroneous results in the hydrodynamic simulation. For example, the stationary and without mean motion solution of the equations (7.2) contradict the numerical solution. For the boundary conditions $\theta(0,t) = T_o$, $\partial_x\theta(1,t) = 0$ and $u(0,t) = u(1,t) = 0$ corresponding

to the corpuscular system previously described, for $u \equiv 0$ and all the time derivatives equal to zero, the equations (7.2) become

$$\begin{aligned} \partial_x(C_1 c\theta) &= 0\,, \\ \partial_x^2(C_2\theta^{3/2}) - C_3\epsilon\theta^{3/2} &= 0\,. \end{aligned}$$

One can see that there is a nontrivial solution $c(x)$ and $\theta(x)$. For $\epsilon \longrightarrow 0$, $c(x)$ and $\theta(x)$ become constant functions contradicting the numerical simulation in [16] for which the cluster of the $N-1$ particles does not disappear when the dissipation diminishes. In fact the clustering particles group together in a smaller space and move with smaller velocity.

However, the failure of the equations (7.2) can not be attributed entirely to the constitutive relations. It is possible that there exists no description by continuous fields of a system formed only by 100 particles like the system used in the numerical simulation in [16]. The number of particles is too small to consider that the local equilibrium principle is satisfied or to apply the statistical mechanics methods. Thus, in this case it is questionable the existence itself of the balance equations. This problem can be solved by the approach described in Chapter 3 since the coarse-grained averages exist for an arbitrary number of particles, even for a single one. For the simple corpuscular system considered in [16] we can give a shorter derivation of the relation (4.24). The particles interact only when they collide and no exterior forces act on them, except that due to the collisions with the walls. Between two collisions, the particles move uniformly and the velocity, momentum, and kinetic energy of the particles have constant values and their time derivatives are zero. At the moment of collision these quantities have a discontinuous jump. Hence, according to the discrete description presented in chapter 2, each collision should mean the disappearance of the particles with the old velocities and the appearance of new particles with new velocities. To make the computation easier we will use another approach, i.e., we consider that the N particles are permanent and the quantities attached to them may have discontinuous variations. Thus, we obtain a verification of the results in chapter 4.

When the particle i collides, its coordinate x_i varies continuously, but its velocity $\xi_i = \dot{x}_i$ undergoes a jump. In order to obtain the discrete analogue of the equations (7.2), the microscopic physical quantity φ_i will be equal only to the mass (m), the momentum $(m\,\xi_i)$ and the kinetic energy $(m\,\xi_i^2/2)$. Since the only variations of the velocity ξ_i are jumps from one constant value to another, then the functions $\varphi_i(t)$ have the same behavior. Hence the functions $\varphi_i(t)$ are no more analytical, but only piece-wise analytical.

In the one-dimensional case the sphere $S(\mathbf{r}, a)$ is replaced by the interval

$(x-a, x+a)$ of length $2a$ and the coarse-grained average (4.5) becomes

$$\langle\varphi\rangle(x,t) \;=\; \frac{1}{4\,\tau\,a}\,\sum_{i=1}^{N}\,\int_{t-\tau}^{t+\tau}\,G_i(x,t')\;dt', \tag{7.3}$$

where the integrand is identical with (4.12) but written in a slightly different form

$$G_i(x,t) \;=\varphi_i(t)\;H^+(a-|x_i(t)-x|)\,. \tag{7.4}$$

In the same way as for the three-dimensional case, for a given x, the integrand (7.4) is a continuous function (constant) with respect to t, except at a finite number of points where it has jumps. Hence G_i is Riemann integrable and its partial derivative with respect to t is of the form (4.19)

$$\partial_t\langle\varphi\rangle(x,t) \;=\; \frac{1}{4\tau a}\sum_{i=1}^{N}[G_i(x,t+\tau)\;-\;G_i(x,t-\tau)]\,. \tag{7.5}$$

The function (7.3) depends on x through the moments u when the particle i enters or leaves the interval $(x-a, x+a)$. Instead of the equation (4.6) these moments are given by the zeros of the equation

$$x_i(u)\;-\;x\;\pm\;a\;=\;0\,,$$

and its derivative (4.8) is $du\,/\,dx\;=\;1/\,\xi_i(u)$. If $u\in(t-\tau,t+\tau)$, then u appears as the integration limit in (7.3) and the derivative of $\langle\varphi\rangle$ with respect to x is

$$\partial_x\,\langle\varphi\rangle \;=\; \frac{1}{4\,\tau\,a}\,\sum_{i=1}^{N}\,\left[\,\sum_{u\in U_i'}\frac{\varphi_i(u)}{\xi_i(u)}\;-\;\sum_{u\in U_i''}\frac{\varphi_i(u)}{\xi_i(u)}\right]\,, \tag{7.6}$$

where analogously to (4.10) U_i' (respective U_i'') is the set containing the moments when the ith particle leaves (enters) the interval $(x-a, x+a)$ during the temporal interval $(t-\tau, t+\tau)$. The proof of the a.e. continuity of the derivatives (7.5) and (7.6) is not repeated.

Relation (7.5) shows that $\partial_t\langle\varphi\rangle$ is related to the change of G_i from $t-\tau$ to $t+\tau$. Since the time derivatives of the two factors of G_i in (7.4) vanishes a.e. $\dot\varphi_i\equiv 0$ and $\dot H\equiv 0$, the changes of G_i are only jumps. The jumps of G_i related to $\dot H$ occur when the particle i enters (leaves) the interval $(x-a, x+a)$ and the jump of G_i is $+\varphi_i(u)$ (respectively $-\varphi_i(u)$). According to (7.6) the variation of $\partial_t\langle\varphi\rangle$ due to these processes is $-\partial_x\,\langle\varphi\xi\rangle$. The change of G_i due to $\dot\varphi$ occurs when the particles collide inside the interval $(x-a, x+a)$. Therefore the generation term (4.31) is replaced by a collision term

$$\delta_c\,\varphi \;=\; \frac{1}{4\,\tau\,a}\,\sum_{i=1}^{N}\,\sum_{s\in V_i}\,[\,\varphi_i(s+0)\;-\;\varphi_i(s-0)\,], \tag{7.7}$$

where V_i is the set containing the moments s when the particle i collides inside $(x-a, x+a)$ during $(t-\tau, t+\tau)$, $\varphi_i(s-0)$ is the limit to the left and $\varphi_i(s+0)$ is the limit to the right. Then from (7.5) it follows the analogue of the relation (4.24)

$$\partial_t \langle\varphi\rangle \; + \; \partial_x \langle\varphi\,\xi\rangle \; = \; \delta_c\,\varphi. \tag{7.8}$$

The particles have the same mass and then $\langle m\rangle = m\,\langle 1\rangle$, where the notation in Chapter 4 is used. For the mean baricentric velocity we have $\bar{p} = \langle m\xi\rangle \,/\, \langle m\rangle = \langle\xi\rangle \,/\, \langle 1\rangle$. In the following, this simplification with the common mass m is made implicitly. Then the discrete analogue of the balance equation for the number of particles and for the mass have the same form

$$\partial_t \langle 1\rangle \; + \; \partial_x \left(\langle 1\rangle\,\bar{p}\right) \; = \; 0, \tag{7.9}$$

where the term $\delta_c\,\varphi$ vanishes because the collisions do not imply the variation of the particle number.

Because the corpuscular system is one-dimensional, the stress tensor (4.39) has only a single component proportional with the kinetic temperature. We define the kinetic temperature $\theta = \langle \frac{1}{2}(\xi-\bar{p})^2\rangle \,/\langle 1\rangle$ (for $\langle 1\rangle \neq 0$) analogously to the definition (4.40). For the granular materials it is known also as the granular temperature.

For $\varphi_i = m\,\xi_i$ we obtain $\langle\varphi\rangle = m\,\langle 1\rangle\,\bar{p}$ and $\langle\varphi\,\xi\rangle = m\,\langle 1\rangle\bar{p}^2 + 2m\,\langle 1\rangle\theta$. The momentum is conserved when two particles collide such that this process does not contribute to the value of the term $\delta_c\,\xi$. The collision with the walls induces a variation of the particle momentum and $\delta_c\,\xi$ is nonvanishing if the spatial averaging interval $(x-a, x+a)$ contains one of the walls. Then (7.8) becomes

$$\partial_t \left(\langle 1\rangle\,\bar{p}\right) \; + \; \partial_x \left(\langle 1\rangle\,\bar{p}^2\right) \; + \; \partial_x \left(2\langle 1\rangle\,\theta\right) \; = \; \delta_c\,\xi, \tag{7.10}$$

i.e., the discrete analogue of the momentum balance equation.

In a similar manner, for $\varphi_i = m\,\xi_i^2\,/\,2$ we obtain from (7.8) the discrete analogue of the balance equation of the kinetic energy

$$\partial_t[\langle 1\rangle\,(\frac{1}{2}\bar{p}^2+\theta)] \,+\, \partial_x[\langle 1\rangle\,(\frac{1}{2}\bar{p}^2+\theta)\bar{p}] \,+\, \partial_x\,(\,\langle 1\rangle\,\theta\,\bar{p}) \,+\, \partial_x\,q \;=\; \frac{1}{2}\delta_c\,\xi^2\;. \tag{7.11}$$

Here $\langle 1\rangle\,(\frac{1}{2}\bar{p}^2+\theta)$ is proportional to the density of the kinetic energy and $q = \langle(\xi-\bar{p})^3\rangle$ is the microscopic flux of the granular temperature. It corresponds to the quantity (4.41) but it does not contain a dynamical part.

Relations (7.9)-(7.11) are of the same form as the usual balance equations (3.14), (3.18) and (3.25), but they have a wider validity. They also hold for simple mechanical systems not possessing statistical properties characteristic to thermodynamic systems. So, even for the corpuscular system of 100

particles for which the numerical simulations in [16] have been performed, the hydrodynamics equations (7.2) have not given the correct results, not because of the balance equations, but because of the constitutive relations inappropriate to the corpuscular system analyzed.

7.2 Constitutive relations

The relations (7.9)-(7.11) are either identities or equations, according to the available information on the microscopic structure. If the motion of each particle is explicitly known, then (7.9)-(7.11) are simple identities containing only known functions. Otherwise, they become balance equations for the coarse-grained averages which are unknown functions. As their number is greater than the number of the balance equations, in order to obtain a solvable problem, we need constitutive relations.

The first order partial derivatives (7.5) and (7.6) have discontinuous variations when a collision occurs or when a particle enters or leaves the spatial interval $(x-a, x+a)$. For certain corpuscular systems we can choose a and τ such that these discontinuous variations vanish or are negligible and the smoothness specific to the continuous fields is obtained. In this section we consider a very simple case of the corpuscular system presented in the previous section for which we may obtain the smoothness without using the local equilibrium hypothesis. The simplicity of the system allows the complete knowledge of the particles motion and consequently to verify if the constitutive relations from (7.2) are correct.

The corpuscular system which we analyze is identical to the system in the previous section, but it contains only two particles in periodic motion. In figure 7.1 we present the motion of the two particles in a period. We impose the additional conditions that the left side wall return the first particle with constant velocity $v_0 = 1$ and $0 < \epsilon < 1/3$. We consider the first particle is initially in the origin $x_1(0) = 0$ and its velocity due to the collision to the wall is $\xi_1(0) = v_1 = 1$. The initial position of the second particle is $x_2(0) = (1-3\epsilon)\,/\,(1-2\epsilon-\epsilon^2)$ and has the velocity $\xi_2(0) = v_2 = -(1-\epsilon)/(1+\epsilon) < 0$. The two particles collide at the moment $t_c = \frac{1}{2} - \epsilon^2/(1-2\epsilon-\epsilon^2)$ at the coordinate point $x_c = v_1\, t_c = t_c$. According to (7.1) after the collision the velocity of the first particle becomes $v_1^{'} = -(1-3\,\epsilon)/(1+\epsilon) < 0$ and the velocity of the second particle $v_2' = (1-\epsilon)/(1+\epsilon) = -v_2$, i.e., the absolute value of the velocity remains the same. The time the first particle needs to reach again the origin is $x_c\,/\,|v_1'| = \frac{1}{2}(1+\epsilon)^2/(1-2\epsilon-\epsilon^2)$. During this time the second particle covers a distance equal to $\frac{1}{2}(1-\epsilon^2)/(1-2\epsilon-\epsilon^2) = 2-x_c-x_2(0)$, i.e., after the elastic collision to the right side wall the particle

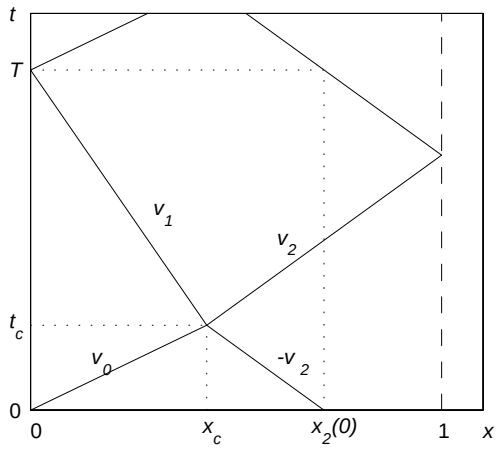

Figure 7.1: The periodic motion of two particles which collide inelastically between them ($x = x_c$) and elastically with the right wall ($x = 1$) maintained in motion by the collision with the left wall ($x = 0$).

reaches again $x_2(0)$. So the motion of the particle is periodic with the period $T = t_c + x_c \,/\, |v_1'| = (1 - \epsilon^2)/(1 - 2\,\epsilon - \epsilon^2)$.

Usually, in continuous mechanics, the spatial average is taken over a volume microscopic in comparison to the macroscopic scale of the continuous phenomenon. For our approach this corresponds to $a \to 0$. First we take the coarse-grained average (7.3) over a spatial interval much smaller than the system length, i.e., $a \ll 1$, and then we study the limit case. For a more condensed presentation of the results we distinguish five spatial regions defined by the points $-a$, a, $x_c - a$, $x_c + a$, $1 - a$, and $1 + a$ (fig. 7.2). For $x \in (x_c - a, x_c + a)$ the averaging interval $(x - a, x + a)$ contains the collision point of the two particles and one particle moves at the left of this point while the other particle moves at the right. Therefore only in this transition region $(x_c - a, x_c + a)$ both particles contribute to the coarse-grained average. There are two boundary regions $(-a, a)$ and $(1 - a, 1 + a)$ each containing one wall and if x lies within one of these intervals then the particle can move only within a part of the averaging interval $(x - a, x + a)$. The transition region and the boundary regions bound two regions $(a, x_c - a)$ and $(x_c + a, 1 - a)$ where only one particle contribute to the coarse-grained average, the particle moving in the entire averaging interval $(x - a, x + a)$.

Since the considered system is a simple mechanical one, the coarse-grained averages $\langle \varphi \rangle$ have discontinuous variations. They can be completely eliminated because the particles motion is periodical, although the local equilib-

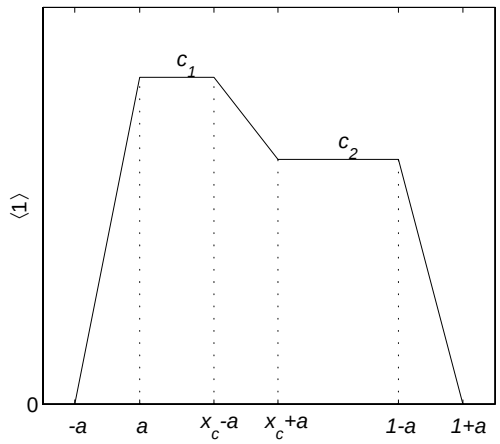

Figure 7.2: The discrete analogue of the concentration for the periodic motion of the two particles.

rium principle is not valid. Consider $\tau = K\,T + \delta$, where K is a natural number and δ a positive real number smaller than T. Then the time averaging is performed on $2K$ complete periods and two intervals of δ length, smaller than a period and (7.3) is written as

$$\langle\varphi\rangle(x,t) \;=\; \frac{1}{4\,(KT+\delta)\,a}\sum_{i=1}^{2}\left[2K\int_0^T G_i(x,t')\;dt' + \int_{-\delta}^{0} G_i(x,t')\;dt' + \int_0^{\delta} G_i(x,t')\;dt'\right]$$

If $K \to \infty$, i.e., $\tau \to \infty$, for $\langle\varphi\rangle$ one obtains a value independent of time with the time averaging performed on a period of the particles motion

$$\langle\varphi\rangle(x) \;=\frac{1}{2\,T\,a}\sum_{i=1}^{2}\int_0^T G_i(x,t')\;dt'\,. \tag{7.12}$$

Then all the coarse-grained averages have null time derivatives, so that the system remains in average in a "stationary" state.

First we compute the quantities in the discrete analogue of the continuity equation (7.9). For $\varphi_i(t) = 1$, we obtain from (7.12)

$$\langle 1\rangle(x) \;=\; \frac{1}{2\,T\,a}\;\sum_{i=1}^{2}\;\int_0^T H^{+}(a - |x_i(t') - x|)\;dt'. \tag{7.13}$$

Consider $x \in (a, x_c - a)$. Then the space averaging interval $(x-a, x+a)$ is entirely contained into the interval $(0, x_c)$ in which the first particle moves and only the first particle contributes to the sum in (7.13). The integral in (7.13) is equal with the time interval in which the first particle lies within the space averaging interval, i.e., $2a/v_1 + 2a/|v_1'| = 4a(1-\epsilon)/(1-3\epsilon)$. In this case we obtain for $\langle 1 \rangle$ a constant value

$$c_1 = \frac{2(1-\epsilon)}{T(1-3\,\epsilon)}\,. \tag{7.14}$$

In the same way, for $x \in (x_c + a, 1-a)$ only the second particle contributes to the sum in (7.13) and the integral is equal with $2a/|v_2| + 2a/v_2' = 4a(1+\epsilon)/(1-\epsilon)$ and $\langle 1 \rangle$ has a constant value

$$c_2 = \frac{2(1+\epsilon)}{T(1-\epsilon)}\,. \tag{7.15}$$

It is easy to verify that for $0 < \epsilon < 1/3$, we have $c_1 > c_2$. For $x \in (-a, a)$ the first particle moves over the space averaging interval only over the interval $(0, x+a)$, so that the integral is equal with $(x+a)/v_1 + (x+a)/|v_1'| = 2(x+a)(1-\epsilon)/(1-3\epsilon)$. One can see that in this boundary region $\langle 1 \rangle$ varies linearly from the value zero for $x = -a$ to the value c_1 given by (7.14) for $x = a$

$$\langle 1 \rangle(x) = \frac{c_1}{2a}(x+a)\,. \tag{7.16}$$

The same linear variation takes place in the other boundary region $(1-a, 1+a)$. In the transition region $(x_c - a, x_c + a)$ the function $\langle 1 \rangle$ varies linearly between the values c_1 and c_2

$$\langle 1 \rangle(x) = -\frac{c_1 - c_2}{2a}(x - x_c) + \frac{c_1 + c_2}{2}\,. \tag{7.17}$$

Thus we obtain for $\langle 1 \rangle$ the graph in fig. 7.2.

In the continuity equation (7.9) we also have the mean velocity $\bar{p}$. To compute it we need the coarse-grained average

$$\langle \xi \rangle(x) \;=\; \frac{1}{2\,T\,a} \sum_{i=1}^{2} \int_0^T \xi_i(t')\, H^+(a - |x_i(t') - x|)\; dt'\,.$$

We notice that in this integral, $\xi_i(t')\,dt'$ is the distance covered by the i particle in the dt' interval taken with the sign of the particle velocity. Then for i fixed, the integral is equal with the distance covered by the i particle in the space averaging interval $(x_c - a, x_c + a)$ in the positive sense of the axis

minus the distance covered in the space averaging interval $(x_c - a, x_c + a)$ in the negative sense of the axis. Since the motion is periodic and the integral is performed over a period it follows that the integral vanishes and $\bar{p} = 0$. This result can also be obtained by direct computation. Taking into account that $\langle 1 \rangle$ does not vary in time, it follows that all the therms of the equation (7.9) vanish, so the relation is satisfied.

In order to compute the kinetic temperature, first the coarse-grained average need to be computed

$$\langle \xi^2 \rangle(x) \;=\; \frac{1}{2\,T\,a} \sum_{i=1}^{2} \int_0^T \xi_i^2(t')\, H^+(a - |x_i(t') - x|)\; dt' \,. \tag{7.18}$$

We proceed similarly to the computation of the integral in (7.13). For $x \in (a, x_c - a)$ the integral in (7.18) is equal with $2av_1 + 2a|v_1'|$, i.e., it has a constant value

$$e_1 = 4a\,\frac{1-\epsilon}{1+\epsilon}\,. \tag{7.19}$$

For $x \in (x_c + a, 1 - a)$ the integral in (7.18) is equal with $2a|v_2| + 2av_2'$ and has the same value as it has in the previous case since from the momentum conservation at the particles collision we have $v_1 + v_2 = v_1' + v_2'$ where $v_2 < 0$ and $v_1' < 0$. Also due to the momentum conservation, the integral has the same value in the transition region $(x_c - a, x_c + a)$ too. So $\langle \xi^2 \rangle$ is constant for $x \in (a, 1 - a)$ and in the boundary regions has a linear variation of the form (7.16). Applying the formula $\theta = \langle \frac{1}{2}(\xi - \bar{p})^2 \rangle \,/ \langle 1 \rangle$ with $\bar{p} = 0$, from the values obtained by calculating (7.18) and (7.13) we obtain for the kinematic temperature two constant values

$$\theta_1 = \frac{1 - 3\,\epsilon}{2(1+\epsilon)} \tag{7.20}$$

for $x \in (-a, x_c - a)$ and

$$\theta_2 = \frac{1}{2}\left(\frac{1-\epsilon}{1+\epsilon}\right)^2 \tag{7.21}$$

for $x \in (x_c + a, 1 + a)$. It is easy to verify that for $0 < \epsilon < 1/3$ we have $\theta_1 < \theta_2$. In the transition region $(x_c - a, x_c + a)$ the variation is given by the inverse of the function (7.17). We obtain the graph in figure 7.3.

Since the mean velocity $\bar{p}$ vanishes, in the equation (7.10) only the last two terms are nonzero. According to (7.7) the collision term is equal to

$$\delta_c\,\xi \;=\; \frac{1}{2T\,a} \sum_{i=1}^{2} \sum_{s \in V_i} [\xi_i(s+0) \;-\; \xi_i(s-0)\,], \tag{7.22}$$

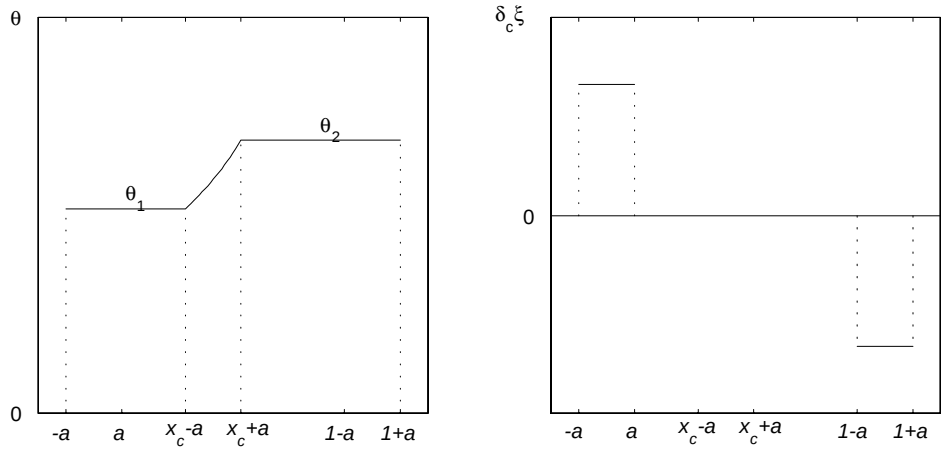

Figure 7.3: The discrete analogue of the kinetic temperature and the collision term for the periodic motion of the two particles.

and is proportional to the average variation of the particles momentum. It is nonzero if within the space averaging interval $(x-a, x+a)$ a collision takes place. For $x \in (-a, a)$ the variation of the first particle velocity at the collision with the left wall is equal with $v_1 - v_1' = 2(1-\epsilon)/(1+\epsilon)$. For $x \in (1-a, 1+a)$ the variation of the second particle velocity at the collision with the right wall is equal to $v_2' - v_2 = -2(1-\epsilon)/(1+\epsilon)$. For $x \in (x_c - a, x_c + a)$, the both particles velocities vary and $\delta_c \xi$ vanishes since $v_1' - v_1 + v_2 - v_2' = 0$ from the momentum conservation. In figure 7.3 we have represented the collision term (7.22) which is nonzero only in the boundary regions where it has the absolute value equal to $(1-\epsilon)/\,a\,T\,(1+\epsilon)$. Let us verify that the relation (7.10) is identically satisfied. We have shown that the coarse-grained average $\langle \xi^2 \rangle = \langle 1 \rangle\, \theta$ has a linear variation in the boundary regions and it is constant in the rest. For $x \in (-a, a)$, the function $\langle \xi^2 \rangle$ linearly increases from zero up to the value (7.19), so that $\partial_x \langle \xi^2 \rangle = e_1/2a$, i.e., it is equal to $\delta_c \xi$. In the other boundary region $\langle \xi^2 \rangle$ decreases with the same slope and (7.10) is satisfied.

In relation (7.11) the first three terms vanish because $\bar{p} = 0$ and θ does not depend on time. In order to compute the kinetic temperature flux we have to compute first the coarse-grained average

$$\langle \xi^3 \rangle(x) \;=\; \frac{1}{2\,T\,a} \sum_{i=1}^{2} \int_0^T \xi_i^3(t')\, H^+(a - |x_i(t') - x|)\; dt' \,. \tag{7.23}$$

We proceed similarly as for the computation of the integral in (7.13) and

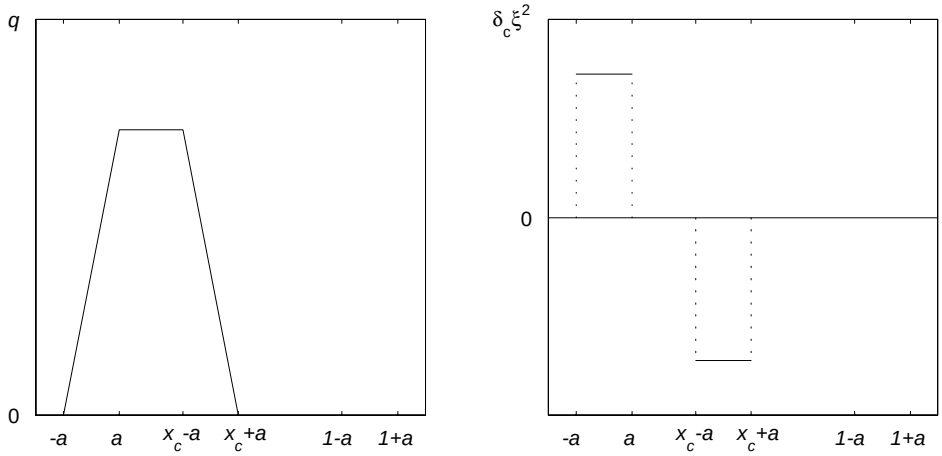

Figure 7.4: The discrete analogue of the heat flux and the collision term for the periodic motion of the two particles.

(7.18). For $x \in (a, x_c - a)$ the integral in (7.23) is equal with

$$2av_1^2 - 2av_1'^2 = 8a\epsilon(1-\epsilon)/(1+\epsilon)^2.$$

For $x \in (x_c + a, 1 - a)$ the integral in (7.23) is equal with $-2av_2^2 + 2av_2'^2$ i.e., it vanishes because $|v_2|{=}v_2'$. As represented in figure 7.4 the flux q has a constant value

$$q_1 = \frac{4\,\epsilon(1-\epsilon)}{T\,(1+\epsilon)^2} \tag{7.24}$$

for $x \in (a, x_c - a)$ and linear variations for $x \in (-a, a) \cap (x_c - a, x_c + a)$. To the collision term

$$\delta_c\,\xi^2 = \frac{1}{2T\,a}\,\sum_{i=1}^{2}\,\sum_{s\in V_i}\,[\xi_i^2(s+0)\;-\;\xi_i^2(s-0)\,],$$

only the first particle contributes since the absolute value of the second particle velocity does not vary due to its elastic collisions. Similarly to the computation of (7.22) we obtain that $\delta_c\,\xi^2 = 4\,\epsilon\,(1-\epsilon)/\,a\,T\,(1+\epsilon)^2$ if $x \in (-a, a)$ and $\delta_c\,\xi^2 = -4\,\epsilon\,(1-\epsilon)/\,a\,T\,(1+\epsilon)^2$ if $x \in (x_c - a, x_c + a)$ (figure 7.4). Taking into account (7.24) it follows that the relation (7.11) is identically satisfied.

In order to obtain a description comparable with the hydrodynamic one given by (7.2), we have to take the limit $a \to 0$. The boundary regions disappear and the domain of definition of the continuous fields becomes $(0, 1)$. The transition region is reduced to the collision point x_c separating the two

regions with uniform motion where all the coarse-grained averages do not vary in terms of x. In these regions all the terms of the balance equations (7.9)-(7.11) are zero. However the values of the continuous fields correspond to a state that is analogous to that described in [16]. Indeed, there are two distinct regions with different concentration and temperature. Because $c_1 > c_2$ and $\theta_1 < \theta_2$, the denser and colder clump is composed by the first particle near the left wall.

Now we can discuss the validity of the constitutive relations used in (7.2)

$$\delta_c \xi = 0\,,\ q = -\partial_x \left(C_2\, \theta^{3/2}\right),\ \delta_c \xi^2 = -C_3\, \epsilon\, \langle 1 \rangle^2\, \theta^{3/2}.$$

The first, $\delta_c \xi = 0$, is identically satisfied. But the other two constitutive relations containing the granular temperature θ are inappropriate for this corpuscular system. In particular the proportionality of the granular temperature flux q to the gradient of the temperature, i.e., the Fourier law, is not appropriate in this case. Indeed, although θ is constant for $x \in (0\,, x_c)$, $q = q_1$ is nonvanishing. This flux really exists because the first particle transports the energy gained from the collision with the left wall and loses it during the inelastic collision with the second particle. The energy dissipation is described by the term $\delta_c \xi^2$ which for $a \to 0$ is proportional with the Dirac function $-\delta(x - x_c)$. The derivation of the Fourier law even for the simple materials (ideal gas) remains a problem without solution [8].