JOURNAL OF NUMERICAL ANALYSIS AND APPROXIMATION THEORY
J. Numer. Anal. Approx. Theory, vol. 48 (2019) no. 2, pp. 112–121
ictp.acad.ro/jnaat

ON THE UNIQUE SOLVABILITY AND NUMERICAL STUDY OF ABSOLUTE VALUE EQUATIONS

MOHAMED ACHACHE*

Abstract. The aim of this paper is twofold. Firstly, we consider the unique solvability of absolute value equations (AVE), $Ax - B|x| = b$, when the condition $\|A^{-1}\| < \frac{1}{\|B\|}$ holds. This is a generalization of an earlier result by Mangasarian and Meyer for the special case where $B = I$. Secondly, a generalized Newton method for solving the AVE is proposed. We show under the condition $\|A^{-1}\| < \frac{1}{4\|B\|}$, that the algorithm converges linearly global to the unique solution of the AVE. Numerical results are reported to show the efficiency of the proposed method and to compare with an available method.

MSC 2010. 90C33, 19K56.
Keywords. Absolute value equations, Complementarity, Generalized Newton method, Global convergence.

1. INTRODUCTION

We consider the following absolute value equations (AVE) of the type:

$$Ax - B\,|x| = b, \tag{1}$$

where $A, B \in \mathbb{R}^{n\times n}$ ($B \neq 0$), $b \in \mathbb{R}^n$ and $|x|$ denotes the vector with absolute values of each component of $x \in \mathbb{R}^n$. When $B = I$ where I is the identity matrix, the AVE (1) reduces to the absolute value equations of the type:

$$Ax - |x| = b. \tag{2}$$

In [6], [8], [9], [14], some frequently optimization problems such as linear programming, convex quadratic programming, bimatrix games [4] and the linear complementarity problem (LCP) can be equivalently reformulated as the AVE (1)–(2). Up to now several authors interested to study this topic, cf. *e.g.* Mangasarian [6], Mangasarian and Meyer [9], Prokopyev [14], Rohn [16], [18] and Lotfi and Veiseh [12]. Besides, various numerical methods for solving the AVE (1)–(2) have been proposed, cf. *e.g.* Cacetta [3], Haghani *et al.* [5], Noor *et al.* [10], Jiang and Zhang [15], Rhon [17], Abdallah *et al.* [2] and recently, Achache and Hazzam [1]. Generally, the AVE problem is an NP-hard in its general form since the LCP can be formulated as the AVE (see [8]). In [7], a

* Department of Mathematics, University of Abbas Ferhat Sétif 1, Sétif, Algeria, e-mail: achache_m@univ-setif.dz.

generalized Newton algorithm for the AVE (2) was investigated, in which it was proved that this method converges globally linear from any starting point to the unique solution of the AVE (2) under the condition that $\|A^{-1}\| < \frac{1}{4}$. Later on, Li [11], developed a modified generalized Newton for solving the AVE (2). His approach is based on introducing an identity matrix such that a modified generalized Newton method for solving the AVE (2) can be established. He showed under certain conditions that this method converges linearly global to a solution of AVE (2).

In this paper we are interested in a first part of it in the unique solvability of the AVE (1). We show that the condition $\|A^{-1}\| < \frac{1}{\|B\|}$ is sufficient to guarantee the unique solvability of the AVE (1) for each $b \in \mathbb{R}^n$. This generalizes an earlier result by Mangasarian and Meyer who showed in [9, Proposition 4], that the absolute value equations of the type (2) is uniquely solvable for each $b \in \mathbb{R}^n$ if $\|A^{-1}\| < 1$. To do so, we transform the AVE (1) into a standard linear complementarity problem (LCP) and show under the above mentioned condition that the LCP has a unique solution, so is the AVE (1) for every $b \in \mathbb{R}^n$.

In the second part, inspired by the work of Mangasarian [7], we propose a generalized Newton method to solve the AVE (1). When $\|A^{-1}\| < \frac{1}{4\|B\|}$, we show that the method is globally convergent to the unique solution of AVE (1) for every $b \in \mathbb{R}^n$. Finally, some numerical results are presented to show the efficiency of this approach. In addition, a numerical comparison is made with an available method.

The paper is built as follows. In section 2, the main result of the unique solvability of the AVE (1) is stated. In section 3, a generalized Newton method is proposed for solving the AVE (1). The linearly global convergence of the method is established. In section 4, numerical results are reported to validate the efficiency of our approach. In section 5, some concluding remarks are given.

The notation used throughout the paper is as follows. The scalar product of two vectors x and y in $\mathbb{R}^n$ will be denoted by $\langle x, y\rangle = x^T y$. For $x \in \mathbb{R}^n$, the norm $\|x\|$ will denote the Euclidean norm $(x^T x)^{\frac{1}{2}}$. For a matrix $A \in \mathbb{R}^{n\times n}$, the transpose of A is denoted by A^T. The induced spectral norm of a matrix A is denoted by $\|A\| = \max_{\|x\|=1} \|Ax\|$. This definition implies that $\|Ax\| \le \|A\|$ $\|x\|$ and $\|AB\| \le \|A\|\|B\|$ for any matrices $A, B \in \mathbb{R}^{n\times n}$. Let $\{\alpha_k\}$ be a non negative sequence of real numbers converging to zero, then the convergence of $\{\alpha_k\}$ is said to be linear if there exists $\rho \in (0, 1)$ such that $\alpha_{k+1} \le \rho\alpha_k$ for $k \ge k_0$. For a matrix $A \in \mathbb{R}^{n\times n}$, its eigenvalues are denoted by $\lambda(A)$ with $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are called the smallest and the greatest eigenvalue of A, respectively. Finally, the notation $x \ge 0$, means that the components of x are greater or equal to zero.

2. THE UNIQUE SOLVABILITY OF THE AVE

Throughout the paper, we make the following assumption on the AVE (1).

ASSUMPTION 1. *The matrix A is nonsingular and the pair of matrices $[A, B]$, satisfies the condition*

$$\|A^{-1}\| < \tfrac{1}{\|B\|}.$$

In the proof of the main result, we shall reformulate the AVE (1) as an equivalent standard LCP. Then, one may investigate the AVE (1) across the LCP by making use of the theory of LCP where we invoke under our assumption the class of $\mathcal{P}$-matrix, to conclude the unique solvability of the AVE (1) for each $b \in \mathbb{R}^n$.

Now, we recall some necessary definitions and auxiliary results that will be used later to prove the unique solvability of the AVE (1).

DEFINITION 1. *A matrix $M \in \mathbb{R}^{n\times n}$ is positive definite if $\langle Mx, x\rangle > 0$ for all non zero $x \in \mathbb{R}^n$.*

DEFINITION 2. *A matrix $M \in \mathbb{R}^{n\times n}$ is a $\mathcal{P}$-matrix if the determinants of all principal submatrices of M are positive. Consequently, any positive definite matrix is a $\mathcal{P}$-matrix.*

DEFINITION 3. *The standard LCP consists of finding $x, y \in \mathbb{R}^n$ such that*

$$x \geq 0,\ y \geq 0, y = Mx + q,\ \langle x, y\rangle = 0, \tag{3}$$

where $M \in \mathbb{R}^{n\times n}$ and $q \in \mathbb{R}^n$ are given. The standard LCP with a $\mathcal{P}$-matrix is denoted by $\mathcal{P}-LCP$.

As a consequence an important result has been proved by Cottle *et al.* [4], where any $\mathcal{P}$-LCP has a unique solution for every $q \in \mathbb{R}^n$.

THEOREM 4. [4, Theorem 3.3.7] *A matrix $M \in \mathbb{R}^{n\times n}$ is a $\mathcal{P}$-matrix if and only if the LCP has a unique solution for $q \in \mathbb{R}^n$.*

DEFINITION 5. *For $x \in \mathbb{R}^n$, we define the vectors x^+ and x^- by $x_i^+ = \max(x_i, 0)$ and $x_i^- = \max(-x_i, 0), 1 \leq i \leq n$, respectively. Then*

$$x^+ \geq 0,\ x^- \geq 0,\ x = x^+ - x^-,\ |x| = x^+ + x^-,\ \langle x^+, x^-\rangle = 0. \tag{4}$$

LEMMA 6. *If Assumption (1) holds, then $(A - B)$ is nonsingular.*

Proof. Indeed $(A-B)$ is nonsingular. For if not then for some $x \neq 0$, we have $(A-B)x = 0$, which will derive a contradiction. This implies that $x = A^{-1}Bx$. Hence by Assumption 1 it follows that $\|x\| \leq \|A^{-1}\|\|B\|\|x\| < \|x\|$ which is a contradiction. Therefore $(A - B)$ is nonsingular. This achieves the proof. $\square$

2.1. Reformulation of the *AVE* as a standard *LCP*.

Proposition 7. *The AVE is as a standard LCP.*

Proof. According to the decomposition of x and $|x|$ in (4), the *AVE* (1) can be reformulated to the following standard *LCP*: find $x^+ \geq 0, x^- \geq 0$, such that

$$x^+ = Mx^- + q, \ \langle x^+, x^- \rangle = 0, \tag{5}$$

where $M = (A-B)^{-1}(A+B)$ and $q = (A-B)^{-1}b$. This achieves the proof. □

We are now ready to state our main result concerning the unique solvability of the *AVE*s.

Theorem 8. *Under Assumption 1, the AVE (1) is uniquely solvable for each $b \in \mathbb{R}^n$.*

Proof. We shall prove that the *LCP* in (5) has a unique solution *i.e.*, we prove $M = (A-B)^{-1}(A+B)$ is positive definite. We have for all $x \neq 0$,

$$\langle (A-B)^{-1}(A+B)x, x \rangle = \langle (A+B)x, (A^T - B^T)^{-1}x \rangle.$$

Letting $(A^T - B^T)^{-1}x = z$, hence we get,

$$\begin{aligned} \langle (A-B)^{-1}(A+B)x, x \rangle &= \langle (A+B)(A^T - B^T)z, z \rangle \\ &= \langle (AA^T - BB^T)z, z \rangle + \langle (BA^T - AB^T)z, z \rangle \\ &= \langle (AA^T - BB^T)z, z \rangle, \end{aligned}$$

since $\langle (BA^T - AB^T)z, z \rangle = 0$, for all $z \in \mathbb{R}^n$. We have by Cauchy-Schwartz inequality and Assumption 1 that

$$\begin{aligned} \langle (AA^T - BB^T)z, z \rangle &= \langle AA^T z, z \rangle - \langle BB^T z, z \rangle \\ &= \|Az\|^2 - \|Bz\|^2 \\ &= \|y\|^2 - \|BA^{-1}y\|^2 \\ &\geq (1 - \|A^{-1}\|\|B\|)^2 \|y\|^2 > 0, \text{ for all } y \neq 0, \end{aligned}$$

where $y = Az$. Therefore $(A-B)^{-1}(A+B)$ is positive definite and the *LCP* in (5) is a $\mathcal{P}-LCP$. By Theorem 4, the *LCP* has a unique solution for each b and so the *AVE* (1). This achieves the proof. □

In the next section, we propose a generalized Newton method for solving the *AVE* (1). In the sequel of the analysis of the proposed method, we will guard the Assumption 1, where we prove under this latter that the generalized Newton method is well-defined *i.e.*, the generalized Jacobian matrix is nonsingular.

3. A GENERALIZED NEWTON METHOD FOR THE *AVE*

By defining the function $f : \mathbb{R}^n \to \mathbb{R}^n$ by

$$f(x) = Ax - B|x| - b, \tag{6}$$

it is clear that x is a solution of the AVE (1) if and only if

$$f(x) = 0. \tag{7}$$

Notice that $|x|$ is not differentiable. A generalized Jacobian $\partial|x|$ of $|x|$ based on a subgradient [13] of its components is given by the diagonal matrix $D(x)$: $\partial|x| = D(x) = \text{diag}(\text{sign}(x))$, with

$$\text{sign}(x) = \begin{cases} 1, & \text{if } x_i > 0, \\ 0, & \text{if } x_i = 0, \\ -1, & \text{if } x_i < 0. \end{cases}$$

Noting that $D(x)x = |x|$ and $\|D(x)\| \leq 1$ for all x.

To solve the equation (7), we use the Newton method with a generalized Jacobian $\partial f(x)$ of $f(x)$ given by:

$$\partial f(x) = A - BD(x). \tag{8}$$

The generalized Newton method for finding a zero of the equation $f(x) = 0$ consists of the following iteration:

$$f(x^k) + \partial f(x^k)(x^{k+1} - x^k) = 0. \tag{9}$$

Based on the values of $f(x)$ and $\partial f(x)$ in (6) and (8), the generalized Newton iteration in (9), for solving the AVE (1), simplifies to

$$(A - BD(x^k))x^{k+1} = b. \tag{10}$$

The next lemma establishes under Assumption 1 that the generalized Newton iteration in (10), is well-defined.

Lemma 9. *If Assumption 1 holds, then $(A - BD)$ is nonsingular for every diagonal matrix D whose diagonal elements are 1, -1 or 0.*

Proof. If $(A-BD)$ is singular then for some $x \neq 0$, we have $(A-BD)x = 0$. It follows that $x = A^{-1}BDx$. By Assumption 1, it follows that

$$\|x\| \leq \|A^{-1}\|\|B\|\|D\|\|x\| < \|D\|\|x\| \leq \|x\|.$$

Then we have a contradiction. Hence $(A-BD)$ is nonsingular. This completes the proof. $\square$

Now, we can formally describe the corresponding generalized Newton algorithm for solving the AVE (1) as follows.

3.1. Algorithm.

Input:
An accuracy $\epsilon > 0$;
an initial starting point $x^0 \in \mathbb{R}^n$;
two matrices A and B and a vector b;
set $k := 0$;
while $\|f(x^k)\| > \epsilon$ **do**
begin
compute x^k from the linear system $(A - BD(x^{k-1}))x^k = b$;
update $k := k + 1$;
end.

A generalized Newton algorithm for the *AVE*.

Following Mangasarian [7], we establish the boundedness of the iterates of (10) and hence the existence of an accumulation point for them.

PROPOSITION 10. *If Assumption 1 holds, then the iterates in (10) of the generalized Newton method are well-defined and bounded. Consequently, there exists an accumulation point* $\bar{x}$ *such that* $(A - BD)\bar{x} = b$ *for some diagonal* D *whose diagonal elements equal* ± 1 *or* 0.

Proof. By Lemma 6, the matrix $(A - BD)^{-1}$ exists and therefore the sequence of the generalized Newton iterations in (10), is well-defined. Suppose now on the contrary that the sequence $\{x^k\}$ is unbounded. Then there exists a subsequence $\{x^{k_i+1}\} \to \infty$ with nonzero x^{k_i+1} such that $D(\, x^{k_i}) = \tilde{D}$ where $\tilde{D}$ is a fixed diagonal matrix whose diagonal elements equal ± 1 or 0 extracted from the finite number of possible configurations for $D(x^k)$ in the sequence $\{D(x^k)\}$ and such that the bounded subsequence $\{\frac{x^{k_i+1}}{\|x^{k_i+1}\|}\}$ converges to $\tilde{x}$. Hence,

$$(A - B\tilde{D})\frac{x^{k_i+1}}{\|x^{k_i+1}\|} = \frac{b}{\|x^{k_i+1}\|}.$$

Letting $i \to \infty$, gives $(A - B\tilde{D})\tilde{x} = 0, \|\tilde{x}\| = 1$. In summary, there exists a $\tilde{x} \neq 0$ such that $(A - B\tilde{D})\tilde{x} = 0$, which is a contradiction to Lemma 9. Consequently, the sequence $\{x^k\}$ is bounded and there exists an accumulation point $\bar{x}$, of $\{x^k\}$ such that $f(\bar{x}) = 0$. This completes the proof. □

We are now ready to study the global convergence of the generalized Newton method. First we give the following lemma.

LEMMA 11. *Let* x *and* y *be points in* $\mathbb{R}^n$. *Then,* $\|\,|x| - |y|\,\| \leq 2\,\|x - y\|$.

Proof. For a detailed proof, see Lemma 5 in [7]. □

LEMMA 12. *Under the assumption* $\|(A - BD)^{-1}\| < \frac{1}{3\|B\|}$, *the generalized Newton iteration converges linearly from any starting point to a solution* x^* *for any solvable AVE (1).*

Proof. Let x^* be a solution of the AVE (1), then $(A-BD(x^*))x^* = b$. Noting that $|x^*| = D(x^*)x^*$ and $|x^k| = D(x^k)x^k$. Now, subtracting $(A-BD(x^*))x^* = b$ from $(A - BD(x^k))x^{k+1} = b$, we obtain

$$\begin{aligned} A(x^{k+1} - x^*) &= BD(x^k)x^{k+1} - BD(x^*)x^* \\ &= BD(x^k)(x^{k+1} + x^k - x^k) - BD(x^*)x^* \\ &= B(|x^k| - |x^*|) + BD(x^k)(x^{k+1} - x^* + x^* - x^k) \\ &= B(|x^k| - |x^*|) - BD(x^k)(x^k - x^*) + BD(x^k)(x^{k+1} - x^*). \end{aligned}$$

Hence

$$(A - BD(x^k))(x^{k+1} - x^*) = B(|x^k| - |x^*|) - BD(x^k)(x^k - x^*).$$

Consequently,

$$(x^{k+1} - x^*) = (A - BD(x^k))^{-1}B(|x^k| - |x^*|) - BD(x^k)(x^k - x^*)).$$

By Lemma 7, we have,

$$\left\|x^{k+1} - x^*\right\| \leq \left\|(A - BD(x^k))^{-1}\right\|(2\left\|B\right\|\left\|x^k - x^*\right\|) + \left\|B\right\|\left\|x^k - x^*\right\|).$$

Hence, $\left\|x^{k+1}-x^*\right\| \leq 3\left\|(A-BD(x^k))^{-1}\right\|\left\|B\right\|(\left\|x^k-x^*\right\|)$. So by the condition $\left\|(A - BD)^{-1}\right\| < \frac{1}{3\|B\|}$, it follows that $\left\|x^{k+1} - x^*\right\| < \left\|x^k - x^*\right\|$. Hence the sequence $\{x^k\}$ converges linearly to x^*. This completes the proof. ☐

We are now ready to prove our main result of the global convergence. We quote first the following lemma.

Lemma 13. *If Assumption 1 holds, then* $(A - BD)$ *is nonsingular and*

$$\left\|(A - BD)^{-1}\right\| \leq \frac{\left\|A^{-1}\right\|}{1-\left\|A^{-1}B\right\|}.$$

Proof. The first part follows directly from Lemma 9. For the proof of the second part, we have $(A - BD)^{-1}$ can be written in the form

$$(A - BD)^{-1} = (I - A^{-1}BD)^{-1}A^{-1}.$$

But since $(I - A^{-1}BD)^{-1}(I - A^{-1}BD) = I$, it follows that

$$(I - A^{-1}BD)^{-1} = I + (I - A^{-1}BD)^{-1}A^{-1}BD.$$

By introducing an induced matrix norm, we get

$$\left\|(I - A^{-1}BD)^{-1}\right\| \leq \frac{1}{1-\left\|A^{-1}B\right\|\left\|D\right\|} \leq \frac{1}{1-\left\|A^{-1}B\right\|}.$$

Because,

$$\left\|(A - BD)^{-1}\right\| = \left\|(I - A^{-1}BD)^{-1}A^{-1}\right\|,$$

it implies that

$$\left\|(A - BD)^{-1}\right\| \leq \frac{\left\|A^{-1}\right\|}{1-\left\|A^{-1}B\right\|}.$$

This completes the proof. ☐

Proposition 14. *Let $\|A^{-1}\| < \frac{1}{4\|B\|}$ and $D(x^k) \neq 0$ for all k. Then the AVE (1) is uniquely solvable for each $b \in \mathbb{R}^n$ and the generalized Newton iteration is well-defined and converges globally from any starting point to a solution x^* for any solvable AVE.*

Proof. The uniquely solvable of AVE (1) for any b follows from Theorem 8 which requires $\|A^{-1}\| < \frac{1}{\|B\|}$. Now by Lemma 13, we have,

$$\|(A - BD)^{-1}\| \leq \frac{\|A^{-1}\|}{1-\|A^{-1}B\|} < \frac{\frac{1}{4\|B\|}}{1-\frac{1}{4}} = \frac{1}{3\|B\|}.$$

Hence by Lemma 12, the sequence generated by the generalized Newton method converges to the unique solution of the AVE (1) from any starting point x^0. This completes the proof. □

4. NUMERICAL RESULTS

In this section, we report some numerical results of the algorithm on two examples of absolute value equations. The experiments are done by the software MATLAB on a PC. In the implementation, our tolerance is $\epsilon = 10^{-6}$ and the number of iterations and the time produced by the algorithm are denoted by **It** and **CPU**, respectively. To this end and in order to demonstrate the performance of the algorithm, we compare the obtained numerical results with those obtained by the modified generalized method in [11]. For these examples we will ensure that the sufficient condition of solvability of the AVE problem is guaranteed *i.e.*, the AVE (1) has a unique solution for each b.

Problem 15. *The data (A, B, b) of the AVE problem is taken as*

$$A = (a_{ij}) = \begin{cases} 10 & i=j, \\ -1 & |i-j|=1, \\ 0 & \text{otherwise} \end{cases} \qquad B = (b_{ij}) = \begin{cases} 5, & i=j, \\ -1, & |i-j|=1, \\ 0, & \text{otherwise.} \end{cases}$$

Observe first that the two matrices A and B can be written as follows:

$$A = 10I - \bar{A},\ B = 5I - \bar{A}$$

where

$$\bar{A} = (\bar{a}_{ij}) = \begin{cases} 0, & i=j, \\ -1, & |i-j|=1, \\ 0, & \text{otherwise.} \end{cases}$$

Since A and B are symmetric real matrices, then their eigenvalues are reals. Let us denote by $\lambda(A)$ and $\lambda(B)$ the eigenvalues of A and B, respectively. It follows from the matrix calculus that their eigenvalues are given by

$$\lambda(A) = 10 - \lambda(\bar{A}) \text{ and } \lambda(B) = 5 - \lambda(\bar{A}),$$

where $\lambda(\bar{A})$ are the eigenvalues of $\bar{A}$. Substituting $\lambda(\bar{A})$ from the equation $\lambda(B) = 5 - \lambda(\bar{A})$ into $\lambda(A) = 10 - \lambda(\bar{A})$, it follows that $\lambda(A) - \lambda(B) = 5 > 0$, i.e.,$\lambda(A) > \lambda(B)$. Consequently, $\lambda_{\min}(A) > \lambda_{\max}(B)$ which is equivalent to say that $\|A^{-1}\| < \frac{1}{\|B\|}$. Therefore, the Problem 15 is uniquely solvable for each $b \in \mathbb{R}^n$. For $b = (A - B)e$, the exact unique solution is given by $x^* = e$. The *obtained numerical results for different values of size* n *and with the initial point* $x^0 = (1, 2, \ldots, n)^T$ *are summarized in Table 1.*

Methods $\rightarrow$	**Gen. Newt**		**M. Gen. Newt**	
size $n \downarrow$	**It**	**CPU**	**It**	**CPU**
20	2	0.00058	12	0.00239
500	2	0.43522	15	2.75882
1000	2	2.73065	16	23.39024
1500	2	8.72754	16	69.01293

Table 1. Numerical results for Problem 15.

PROBLEM 16. *This example is an AVE problem of type* (2) *and it is taken as:*

$$A = (a_{ij}) = \begin{cases} 100 & i=j \\ 10 & |i-j|=1, \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j = 1, .., n,$$

and $B = I$*. Also with the same reasoning as in the first example we can checked that Assumption 1 in this example is satisfied and so the Problem 16 has a unique solution for each* $b \in \mathbb{R}^n$*. For* $b = (A - I)e \in \mathbb{R}^n$*, the exact solution is* $x^* = e$*. The obtained numerical results for different values of size* n *and with the initial point* $x^0 = (1, 3, \ldots, 2n+1)^T$*, are summarized in Table 2.*

Methods $\rightarrow$	***Gen. Newt***		***M. Gen. Newt***	
size $n \downarrow$	***It***	***CPU***	***It***	***CPU***
20	2	0.0006	6	0.00164
500	2	0.9280	7	1.38453
1000	2	7.4371	8	12.43491
1500	2	29.632463	8	37.71990

Table 2. Numerical results for Problem 16.

In these tables "Gen. Newt" and "M. Gen. Newt" are denoted for the Generalized Newton method and the Modified Generalized Newton method [11]*, respectively. From Tables 1 and 2, we see that the number of iterations and the CPU times of the generalized Newton method are always less than those obtained by the modified generalized Newton method which indicates the superiority of the generalized Newton method.*

5. CONCLUDING REMARKS

In this paper, we have studied the unique solvability of the *AVE* under a suitable sufficient condition. Moreover, we have extended a generalized Newton method for solving the *AVE* (1) which globally and finitely converges to the unique solution of the *AVE* (1) when $\|A^{-1}\| < \frac{1}{4\|B\|}$. We have also reported some numerical results which indicate the effectiveness of the proposed method compared with an available generalized modified Newton method. Possible future work may consist to consider Traub's Newton method for solving the *AVE* (1) which is known as the two-step Newton method.

REFERENCES

[1] M. Achache, N. Hazzam, *Solving absolute value equations via complementarity and interior point methods*, J. Nonl. Funct. Anal., (2018), pp. 1–10

[2] L. Abdellah, M. Haddou, T. Migot, *Solving absolute value equation using complementarity and smoothing functions*, J. Comput. Appl. Math., **327** (2018), pp. 196–207.

[3] L. Caccetta, B. Qu, G. Zhou, *A globally and quadratically convergent method for absolute value equations*, Comput. Optim. Appl., **48** (2011), pp. 45–58

[4] R.W. Cottle, J.S. Pang, R.E. Stone, *The Linear Complementarity Problem*, Academic Press, New-York, 1992.

[5] F.K. Haghani, *On generalized Traub's method for absolute value equations*, J. Optim. Theory Appl., 166 (2015), pp. 619–625.

[6] S.L. Hu, Z.H. Huang, *A note on absolute value equations*, Optimization Letters, **4** (2010), pp. 417–424.

[7] O.L. Mangasarian, *A generalized Newton method for absolute value equations*, Optimization Letters, **3** (2009), pp. 101–108.

[8] O.L. Mangasarian, *Linear complementarity as absolute value equation solution*, Optimization Letters, **8** (2014), pp. 1529–1534.

[9] O.L. Mangasarian, R.R. Meyer, *Absolute value equations*, Linear Algebra and its Applications, **419** (2006), pp. 359–367.

[10] M.A. Noor, J. Iqbal, K.I. Noor, A. Al-Said, *On a iterative method for solving absolute value equations*, Optimization Letters, **6** (2012), pp. 1027–1033.

[11] C.X. Li, *A modified generalized Newton method for absolute value equations*, J. Optim. Theory Appl., **170** (2016), pp. 1055–1059.

[12] T. Lotfi, H. Veiseh, *A note on unique solvability of the absolute value equation*, Journal of Linear and Topological Algebra, **2** (2013), pp. 77–81.

[13] B.T. Polyak, *Introduction to Optimization*, Optimization Software Inc., Publications Division, New York, 1987.

[14] O. Prokopyev, *On equivalent reformulations for absolute value equations*, Computational Optimization and Applications, **44** (2009), pp. 363–372.

[15] X. Jiang, Y. Zhang, *A smoothing-type algorithm for absolute value equation*, Journal of Industrial and Management Optimization, **9** (2013), pp. 789–798.

[16] J. Rohn, *On unique solvability of the absolute value equations*, Optimization Letters, **3** (2009), pp. 603–606.

[17] J. Rohn, *An algorithm for computing all solutions of an absolute value equation*, Optimization Letters, **6** (2012), pp. 851–856.

[18] J. Rohn, *A theorem of the alternatives for the equation $Ax + B|x| = b$. Linear and Multilinear Algebra*, **52** (2004), pp. 421–426.

Received by the editors: March 21, 2019; accepted: June 17, 2019; published online: January 29, 2020.