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BOOK REVIEWS

Martin J. Gander, Felix Kwok, *Numerical Analysis of Partial Differential Equations Using Maple and MATLAB*, SIAM, Philadelphia, USA, 2018, X + 153 pp., ISBN 978-1611975307.

This book represents an excellent introduction to numerical methods for elliptic partial differential equations (PDEs). It contains detailed descriptions for four major classes of discretization methods for PDEs: *the Finite Difference (FD) Method*, *the Finite Volume (FV) Method*, *The Spectral Method* and *the Finite Element Method (FEM)*. The structure of the book contains a Preface, an Introduction, a chapter for each of these methods, and ends with a Bibliography consisting of 63 entries and an Index. The chapters are written in a similar manner: each chapter begins with an interesting historical description (including quotes for main contributions in the field) and ends with some concluding remarks and a set of useful problems. Besides their didactic purpose the problems have a practical significance.

The first chapter represents the introduction. Besides the notation some classical ODEs and PDEs are provided. For example, *the pendulum equation*, *the Lotka-Volterra model*, *the heat equation*, *the wave equation*, *the advection-reaction-diffusion equation*, *the Maxwell's equations* and *the Navier-Stokes equations* are introduced in a didactic manner.

In the second chapter the first major class of discretization methods for PDEs, namely the FD method, is introduced. To this end, the two-dimensional Poisson equation with Dirichlet boundary condition on unit square domain is considered. Moreover, using the truncated Taylor series in each variable the associated discrete Poisson equation is derived. Theorem 2.6 represents the main result of the chapter and it shows the convergence of finite difference approximation. The proof is based on a truncation error estimate, a discrete maximum principle and a Poincaré-type estimate. The chapter is completed with relevant results related to general boundary conditions, general differential operators and nonrectangular domains.

Chapter 3 is dedicated to finite volume method. A general two-dimensional diffusion equation is considered to describe the method. Relation between FV and FD and consistency of FV method are provided. Also, a convergence result for one-dimensional example is proved and an error estimate is established.

In the next chapter a different approach is presented, namely the spectral methods. The method originates from the series based solutions of a partial differential equation, and for illustration the Fourier series solution for the one-dimensional Poisson equation is presented. Next the Fourier spectral method is introduced and more specifically the Fourier collocation spectral method, the differentiation matrices are defined. For the introduced Fourier method the convergence analysis is carried out, the spectral convergence is defined. The well known and widely used Chebyshev spectral collocation method is introduced. Finally the two most important conclusions are presented: the spectral method works extremely well for smooth solutions and spectral methods are difficult to use on non rectangular geometries.

In the final chapter the finite element method is introduced. The material can be divided into three parts. The first part contains a clear analysis of Poisson equation in one spatial domain with homogeneous Dirichlet boundary conditions. More exactly, the *strong form* is defined and the associated *variational formulation* and *minimization problem* are derived. The equivalence between them is provided. The corresponding Galerkin and Ritz approximations are introduced. Additional results concerning more general boundary conditions or stiffness and mass matrices are discussed. In the second part, a convergence analysis of the FEM is presented. To this end, the authors recall some Sobolev space arguments and other outstanding mathematical results as Cèa lemma, Poincaré inequality or Lax-Milgram theorem. In the last part of the chapter results related to two-dimensional problem are provided.

The book offers well-organized MATLAB and Maple routines, which may turn out to be very useful for graduate students as well as for researchers interested in solving PDEs.

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