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BOOK REVIEWS

JAN S. HESTHAVEN, *Numerical Methods for Conservation Laws. From Analysis to Algorithms*, SIAM (Computational Science and Engineering Series), Philadelphia, 2018, xi + 570 pp., ISBN 978-1-611-975-09-3.

This impressive monograph is divided in three parts, namely: I Conservative Laws, II Monotone Schemes and III High-Order Schemes. The work also contains an Introduction and an Index. Each chapter ends with its own bibliography.

The first part contains only two chapters and it is centered on the concept of weak and entropy solutions for scalar and systems of conservation laws. They provide a proper understanding of conservation laws as well as of the nature of their solutions.

The second part is subdivide in four chapters. The author makes the transition from continuous to discrete systems using mainly finite difference and finite volume schemes. He treats both linear and nonlinear problems as well as extensions to two-dimensional spatial problems. The central idea is to design numerical schemes that inherit the properties of conservation laws. In spite of being powerful, flexible and robust these methods have only first order of accuracy.

The third part is by far the most consistent of the work. It contains six broad and dense chapters. In the first chapter of this part the author discusses the good and the bad of high-order accuracy schemes. In the next two chapters he addresses respectively the issues of strong stability of time integration schemes and the accuracy of the spatial discretization. Various flux limited schemes are analyzed. Essentially nonoscillatory schemes (ENO) and weighted essentially nonoscillatory schemes (WENO) have a devoted chapter. They respond to the desire to find schemes of an arbitrary order of accuracy for general conservation schemes. As an alternative to very computational expensive (ENO) and (WENO) schemes, the author develops Galerkin's discontinuous methods in a separate chapter. These are examined in the context of multidimensional problems. The final chapter is devoted to the Fourier spectral method which implies periodic boundary conditions. Both Galerkin and collocation variants of this method are considered. Filtering and postprocessing aspects of these methods are briefly reviewed.

The style of the book is almost conversational, but that does not mean the material is elementary. The author follows various methods once presented "in action" and carries out a lot of relevant observations about their capabilities.

The programming environment is MATLAB and the work contains a large list of MATLAB scripts.

Graduate students and researchers in applied mathematics and engineering working in fluid dynamics, scientific computing and numerical analysis will find this book of real interest.

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