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A NOTE ON THE FIXED POINT METHOD AND THE LINEAR COMPLEMENTARITY PROBLEM*

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Abstract. In this paper, we propose an extended form of a fixed point method for processing the large and sparse linear complementarity problem (LCP). We obtain an equivalent form of LCP by using two positive diagonal matrices and prove the equivalence. For the proposed method, we provide some convergence conditions when the system matrix is a P-matrix or an H_+-matrix or a symmetric positive definite matrix.

MSC. 65F10, 65F50.

Keywords. Linear complementarity problem, P-matrix, H_+-matrix, symmetric positive definite matrix, matrix splitting, convergence.

1. INTRODUCTION

In the literature on mathematical programming, the linear complementarity problem has received considerable attention. It also appears in a number of applications in operations research applications, control theory, mathematical economics, optimization theory, stochastic optimal control, the American option pricing problem, economics, elasticity theory, the free boundary problem, and the Nash equilibrium point of the bimatrix game. For details, see [18], [19]. For recent works on this problem, see [14], [15] and references therein.

Consider $A_1 \in \mathbb{R}^{n\times n}$ and a vector $q \in \mathbb{R}^n$. The linear complementarity problem denoted as $\mathrm{LCP}(q, A_1)$ is to find the solution $z \in \mathbb{R}^n$ to the following system

$$z \geq 0,\ A_1 z + q \geq 0,\ z^T(A_1 z + q) = 0. \tag{1}$$

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There are various methods of solving the LCP using an iterative process, namely the projected methods [4], [9], [12], [16], [20], [21], the modulus algorithm [2], [3], [5], [10] and the modulus based matrix splitting iterative methods [8], [11], [22].

A general fixed point method (GFP) is proposed by Fang [6] assuming the case where $\phi = \omega A_{1D}^{-1}$ with $\omega > 0$ and A_{1D} is a diagonal matrix of A_1. The GFP approach takes less iterations than the modulus-based successive overrelaxation (MSOR) iteration method [2]. In this paper we present an extended form of the fixed point method [6] that incorporates projected type iteration techniques by including two positive diagonal parameter matrices ϕ_1 and ϕ_2. We also show that the fixed point equation and the linear complementarity problem are equivalent and discuss the convergence conditions as well as provide some convergence domains for the proposed method.

The rest of this paper is organized as follows: In Section 2, we present some notation, definitions and lemmas in order to establish our key findings. Section 3 discusses an extended form of a fixed point method for solving $\text{LCP}(q, A_1)$ with convergence analysis. In the last section, we give the conclusion.

2. PRELIMINARIES

Some notations, introductory definitions and required lemmas are introduced. For details, see [2], [6].

Let $A_1 = (a_{ij}) \in \mathbb{R}^{n\times n}$ and $B_1 = (b_{ij}) \in \mathbb{R}^{n\times n}$. We use $A_1 \geq (>)\ B_1$ to denote $a_{ij} \geq (>)\ b_{ij}$ for all i, j. The comparison matrix $\langle A_1\rangle = (\langle a_{ij}\rangle)$ of A_1 is defined by $\langle a_{ij}\rangle = |a_{ij}|$ with $i = j$ and $\langle a_{ij}\rangle = -|a_{ij}|$ with $i \neq j$ for $i, j = 1, 2, \ldots, n$. The matrix A_1 is called a Z-matrix if all of its non-diagonal elements are less than or equal to zero; an Z-matrix is called an M-matrix if $A_1^{-1} \geq 0$; an H-matrix if $\langle A_1\rangle$ is an M-matrix. The splitting $A_1 = M_1 - N_1$ is called an M-splitting if $\det(M_1) \neq 0$ and $N_1 \geq 0$, an H-splitting if $\langle M_1\rangle - |N_1|$ is an M-matrix [7]. An H-matrix is called an H_+-matrix [1] if $a_{ii} > 0$ for $i = 1, 2, \ldots, n$. Let $A_1 \in \mathbb{R}^{n\times n}$, then A_1 is said to be a P-matrix if all its principle minors are positive that is $\det(A_{1\alpha\alpha}) > 0$ for all $\alpha \subseteq \{1, 2, \ldots, n\}$. Suppose $A_1 = (a_{ij}) \in \mathbb{R}^{n\times n}$ be a square matrix, then $|A_1| = (c_{ij})$ is defined by $c_{ij} = |a_{ij}|\ \forall\ i, j$ and A_1^T denotes the transpose of A_1.

LEMMA 1 ([13]). *The LCP(q, A_1) has a unique solution for any $q \in \mathbb{R}^n$ if $A_1 \in \mathbb{R}^{n\times n}$ is a P-matrix.*

LEMMA 2 ([6]). *Let $A_1 \in \mathbb{R}^{n\times n}$ and $A_1 = M_1 - N_1$ be an M-splitting with M_1 an M-matrix and $N_1 \geq 0$. Then $\rho(M_1^{-1}N_1) < 1$.*

3. MAIN RESULTS

For a given vector $\xi \in \mathbb{R}^n$, we indicate the vectors $\xi_+ = \max\{0, \xi\}$, $\xi_- = \max\{0, -\xi\}$. Since $\xi_+ \geq 0$, $\xi_- \geq 0$, $\xi = \xi_+ - \xi_-$, $\xi_+^T\xi_- = 0$. Let $z = \phi_1\xi_+$ and

$w = \phi_2 \xi_-$, where ϕ_1 and ϕ_2 are positive diagonal matrices of order n. Now we convert the LCP into a fixed point formulation that is

$$\xi = (I_1 - \phi_2^{-1} A_1 \phi_1)\xi_+ - \phi_2^{-1} q, \tag{2}$$

where I_1 is an identity matrix of order n.

In the following result, we provide an equivalent formulation of the LCP(q, A_1) for solving LCP(q, A_1).

THEOREM 1. *Suppose $A_1 \in \mathbb{R}^{n\times n}$ and $q \in \mathbb{R}^n$. Then ξ^* is a solution of (2) if and only if $z^* = \phi_1 \xi_+^*$ is a solution of LCP(q, A_1).*

Proof. Let ξ^* be a solution of (2). Then

$$\xi^* = (I_1 - \phi_2^{-1} A_1 \phi_1)\xi_+^* - \phi_2^{-1} q,$$
$$\phi_2 \xi_-^* = A_1 \phi_1 \xi_+^* + q.$$

Since $\phi_2 \xi_-^* \geq 0$,

$$A_1 \phi_1 \xi_+^* + q \geq 0.$$

Moreover,

$$(\phi_1 \xi_+^*)^T (A_1 \phi_1 \xi_+^* + q) = (\phi_1 \xi_+^*)^T (\phi_2 \xi_-^*) = 0,$$

and $\phi_1 \xi_+^* \geq 0$. Therefore $z^* = \phi_1 \xi_+^*$ is a solution of LCP(q, A_1).

Let $z^* = \phi_1 \xi_+^*$ and $w^* = \phi_2 \xi_-^*$, and $\xi^* = \xi_+^* - \xi_-^*$. Now from LCP$(q, A_1)$

$$\phi_2 \xi_-^* = A_1 \phi_1 \xi_+^* + q,$$
$$\xi^* = \xi_+^* - \phi_2^{-1}(A_1 \phi_1 \xi_+^* + q),$$
$$\xi^* = (I_1 - \phi_2^{-1} A_1 \phi_1)\xi_+^* - \phi_2^{-1} q.$$

Thus, ξ^* is a solution of (2). □

Now we show that the solution of (2) is unique when the matrix A_1 is a P-matrix.

THEOREM 2. *For any positive diagonal matrices ϕ_1 and ϕ_2, (2) has a unique solution if $A_1 \in \mathbb{R}^{n\times n}$ is a P-matrix.*

Proof. Since A_1 is a P-matrix, for any $q \in \mathbb{R}^n$ LCP(q, A_1) has a unique solution. Let y^* and u^* be the solutions of (2). Then

$$y^* = y_+^* - \phi_2^{-1}(A_1 \phi_1 y_+^* + q).$$

and

$$u^* = u_+^* - \phi_2^{-1}(A_1 \phi_1 u_+^* + q).$$

As $y_+^* = u_+^*$,

$$y^* + \phi_2^{-1}(A_1 \phi_1 y_+^* + q) = u^* + \phi_2^{-1}(A_1 \phi_1 u_+^* + q).$$

Hence

$$y^* = u^*.$$

□

Based on (2), we obtain an extended form of the fixed point method which is referred to as Method 1.

METHOD 1. *Let* $A_1 \in \mathbb{R}^{n\times n}$ *and* $q \in \mathbb{R}^n$. *Suppose* $\xi^{(0)} \in \mathbb{R}^n$ *an initial vector and the sequence* $\{z^{(k)}\}_{k=1}^{+\infty} \subset \mathbb{R}^n$. *Let Residue be an Euclidean norm of the error vector and define the Residue as*

$$\mathrm{Res}(z^{(k)}) = \|\min(z^{(k)}, A_1 z^{(k)} + q)\|_2,$$

where $z^{(k)}$ *is the* k^{th} *approximate solution of the LCP*(q, A_1). *The iteration process stop if* $(z^{(k)}) < 10^{-5}$ *or the number of iteration reached 900. For computing* $\xi^{(k+1)} \in \mathbb{R}^n$ *is as follows:*

(1) *Given an initial vector* $\xi^{(0)} \in \mathbb{R}^n$, *error* $\epsilon > 0$ *and set* $k = 0$.

(2) *Using the following scheme, create the sequence* $\xi^{(k)}$*:*

$$\xi^{(k+1)} = (I_1 - \phi_2^{-1} A_1 \phi_1)\xi_+^{(k)} - \phi_2^{-1} q \tag{3}$$

and set $z^{(k+1)} = \phi_1 \xi_+^{(k+1)}$.

(3) *If* $(z^{(k)}) < \epsilon$ *then stop; otherwise, set* $k = k + 1$ *and return to step 2.*

REMARK 3. Fang [6] introduced a fixed point method, which is a special case of (3) with $\phi_2 = \phi^{-1}$ and $\phi_1 = I_1$, where ϕ is a positive diagonal matrix. □

In the following result, we discuss the convergence condition when the system matrix A_1 is a P-matrix.

THEOREM 4. *Let* $A_1 \in \mathbb{R}^{n\times n}$ *be a* P*-matrix. Let* $\rho(|I_1 - \phi_2^{-1} A_1 \phi_1|) < 1$ *and* ξ^* *be the solution of* (2). *Then the sequence* $\{z^{(k)}\}_{k=1}^{+\infty}$ *generated by Method 1 converges to* z^* *for any initial vector* $\xi^{(0)} \in \mathbb{R}^n$.

Proof. Suppose A_1 is a P-matrix. Then ξ^* is a unique solution of (2). Thus

$$\xi^* = (I_1 - \phi_2^{-1} A_1 \phi_1)\xi_+^* - \phi_2^{-1} q.$$

From (3), it results

$$\xi^{(k+1)} - \xi^* = (I_1 - \phi_2^{-1} A_1 \phi_1)(\xi_+^{(k)} - \xi_+^*).$$

It follows that

$$\begin{aligned} |\xi^{(k+1)} - \xi^*| &= |(I_1 - \phi_2^{-1} A_1 \phi_1)| \cdot |\xi_+^{(k)} - \xi_+^*| \\ &\le |(I_1 - \phi_2^{-1} A_1 \phi_1)| \cdot |\xi^{(k)} - \xi^*|. \end{aligned}$$

Since $\rho(|I_1 - \phi_2^{-1} A_1 \phi_1|) < 1$. Hence, for any initial vector $\xi^{(0)} \in \mathbb{R}^n$ the sequence $\{z^{(k)}\}_{k=1}^{+\infty}$ converges to the z^*. □

In the following result, we provide the convergence conditions for *Method* 1 when the system matrix is an H_+-matrix.

Theorem 5. *Assume $A_1 \in \mathbb{R}^{n\times n}$ is an H_+-matrix, $A_{1D} = diag(A_1)$ and $B = A_{1D} - A_1 \in \mathbb{R}^{n\times n}$. Let $\phi_1 = \alpha_1 D_1$, $\phi_2 = \omega^{-1}A_{1D}$ and*

$$\rho\Big(A_{1D}^{-1}|B|D_1\Big) \le \rho\Big(A_{1D}^{-1}|B|\Big)\rho(D_1),$$

where α_1, ω are positive constants and D_1 is a positive diagonal matrix. Let $\omega\alpha_1 = \beta$ and ξ^* be the solution of (2). Then the sequence $\{z^{(k)}\}_{k=1}^{+\infty}$ generated by Method 1 converges to z^* for any initial vector $\xi^{(0)} \in \mathbb{R}^n$ if

$$0 < \beta < \frac{2}{\big(1+\rho(A_{1D}^{-1}|B|)\big)\rho(D_1)}.$$

Proof. We have A_1 is an H_+-matrix, $A_{1D} = diag(A_1)$, $B = A_{1D} - A_1$ and $\rho(A_{1D}^{-1}|B|) < 1$. For $\phi_1 = \alpha_1 D_1$ and $\phi_2 = \omega^{-1}A_{1D}$, we obtain

$$\begin{aligned}
|I_1 - \phi_2^{-1}A_1\phi_1| &= |I_1 - (\omega^{-1}A_{1D})^{-1}A_1\alpha_1 D_1| \\
&= |I_1 - (\omega^{-1}A_{1D})^{-1}(A_{1D} - B)\alpha_1 D_1| \\
&= |I_1 - \omega\alpha_1 D_1 + \omega A_{1D}^{-1}B\alpha_1 D_1| \\
&\le |I_1 - \omega\alpha_1 D_1| + |\omega A_{1D}^{-1}B\alpha_1 D_1| \\
&\le |I_1 - \beta D_1| + \beta A_{1D}^{-1}|B|D_1.
\end{aligned}$$

It follows that

$$|I_1 - \beta D_1| + \beta A_{1D}^{-1}|B|D_1 = \begin{cases} (I_1 - \beta D_1) + \beta A_{1D}^{-1}|B|D_1, & \text{if } 0<\beta D_1 \le I_1, \\ (\beta D_1 - I_1) + \beta A_{1D}^{-1}|B|D_1, & \text{if } \beta D_1 > I_1. \end{cases}$$

Now we write

$$\rho(|I_1 - \phi_2^{-1}A_1\phi_1|) \le \begin{cases} 1 - (1 - \rho(A_{1D}^{-1}|B|))\beta\rho(D_1), & \text{if } 0<\beta\rho(D_1) \le 1, \\ (1 + \rho(A_{1D}^{-1}|B|))\beta\rho(D_1) - 1, & \text{if } \beta\rho(D_1)>1. \end{cases} \tag{4}$$

From (4) we can seen that $\rho(|I_1 - \phi_2^{-1}A_1\phi_1|) < 1$ for $\beta\rho(D_1) \in (0,1]$ and for $\beta\rho(D_1) > 1$, $\rho(|I_1 - \phi_2^{-1}A_1\phi_1|) < 1$ if and only if

$$(1 + \rho(A_{1D}^{-1}|B|))\beta\rho(D_1) - 1 < 1$$

such that $\beta < \frac{2}{\big(1+\rho(A_{1D}^{-1}|B|)\big)\rho(D_1)}$. Therefore, if

$$0 < \beta < \frac{2}{\big(1+\rho(A_{1D}^{-1}|B|)\big)\rho(D_1)},$$

for any initial vector $\xi^{(0)} \in \mathbb{R}^n$, the sequence $\{z^{(k)}\}_{k=1}^{+\infty}$ converges to z^*. □

In the following result, we provide the convergence conditions for *Method* 1 when the system matrix is a symmetric positive definite (SPD) matrix.

Theorem 6. *Let $A_1 \in \mathbb{R}^{n\times n}$ be the SPD matrix. Let $\phi_2 = \omega^{-1}I_1$ and $\phi_1 = \alpha_1 D_1$, where D_1 is a scalar matrix and denote the minimum and the maximum eigenvalues of $A_1 D_1$ by ν_{min} and ν_{max} respectively. Let ξ^* be the solution of (2). Then the sequence $\{z^{(k)}\}_{k=1}^{+\infty}$ generated by Method 1 converges to z^* for any initial vector $\xi^{(0)} \in \mathbb{R}^n$ if $0 < \beta < \frac{2}{\nu_{max}}$.*

Proof. From Theorem 4, we have

$$\xi^{(k+1)} - \xi^* = (I_1 - \phi_2^{-1} A_1 \phi_1)(\xi_+^{(k)} - \xi_+^*).$$

This implies that

$$\|\xi^{(k+1)} - \xi^*\|_2 = \|(I_1 - \phi_2^{-1} A_1 \phi_1)(\xi_+^{(k)} - \xi_+^*)\|_2.$$

Since $\|(\xi_+^{(k)} - \xi_+^*)\|_2 \leq \|(\xi^{(k)} - \xi^*)\|_2$,

$$\begin{aligned} \|\xi^{(k+1)} - \xi^*\|_2 &\leq \|(I_1 - \phi_2^{-1} A_1 \phi_1)\|_2 \|(\xi^{(k)} - \xi^*)\|_2 \\ &\leq \|(I_1 - \phi_2^{-1} A_1 \phi_1)\|_2 \|(\xi^{(k)} - \xi^*)\|_2. \end{aligned}$$

If $\|I_1 - \phi_2^{-1} A_1 \phi_1\|_2 < 1$, then *Method* 1 is convergent. Therefore

$$\begin{aligned} \|I_1 - \phi_2^{-1} A_1 \phi_1\|_2 &= \|I_1 - \omega I_1 A_1 \alpha_1 D_1\|_2 \\ &= \|I_1 - \beta A_1 D_1\|_2. \end{aligned}$$

We have

$$\|I - \beta A_1 D_1\|_2 = \max\{|1 - \beta \nu_{\min}|, |1 - \beta \nu_{\max}|\}.$$

It follows that

$$\|I_1 - \beta A_1 D_1\|_2 = \begin{cases} |1 - \beta \nu_{\min}|, & \text{if } |1 - \beta \nu_{\min}| \geq |1 - \beta \nu_{\max}|, \\ |1 - \beta \nu_{\max}|, & \text{if } |1 - \beta \nu_{\max}| \geq |1 - \beta \nu_{\min}|. \end{cases}$$

Thus $\|I_1 - \beta A_1 D_1\|_2 < 1$ if and only if

$$(a) \begin{cases} |1 - \beta \nu_{\min}| < 1, \\ |1 - \beta \nu_{\min}| \geq |1 - \beta \nu_{\max}|, \end{cases}$$

and

$$(b) \begin{cases} |1 - \beta \nu_{\max}| < 1, \\ |1 - \beta \nu_{\max}| \geq |1 - \beta \nu_{\min}|. \end{cases}$$

From (a) and (b) we obtain the convergence condition of *Method* 1 that is $0 < \beta \leq \frac{2}{\nu_{min} + \nu_{max}}$ and $\frac{2}{\nu_{min} + \nu_{max}} \leq \beta < \frac{2}{\nu_{max}}$, from these two inequalities we obtain

$$0 < \beta < \frac{2}{\nu_{max}}.$$

□

4. CONCLUSION

We introduced an extended form of a fixed point method for solving the linear complementarity problem LCP(q, A_1) with parameter matrices ϕ_1 and ϕ_2. Also, we have shown how the iterative form relates to the parameter matrices ϕ_1 and ϕ_2. We have presented some convergence conditions and some sufficient convergence domains for the proposed method.

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REFERENCES

[1] Z.Z. Bai, *On the convergence of the multisplitting methods for the linear complementarity problem*, SIAM J. Matrix Anal. Appl., **21** (1999), pp. 67–78. https://doi.org/10.1137/S0895479897324032

[2] Z.Z. Bai, *Modulus-based matrix splitting iteration methods for linear complementarity problems*, Numer. Linear Algebra Appl., **17** (2010) no. 6, pp. 917–933. https://doi.org/10.1002/nla.680

[3] Z.Z. Bai, D. Evans, *Matrix multisplitting methods with applications to linear complementarity problems: parallel asynchronous methods*, Int. J. Comput. Math., **79** (2002), pp. 205–232. https://doi.org/10.1080/00207160211927

[4] M. Dehghan, M. Hajarian, *Convergence of SSOR methods for linear complementarity problems*, Oper. Res. Lett., **37** (2009) no. 3, pp. 219–223. https://doi.org/10.1016/j.orl.2009.01.013

[5] J.L. Dong, M.Q. Jiang, *A modified modulus method for symmetric positive-definite linear complementarity problems*, Numer. Linear Algebra Appl., **16** (2009) no. 2, pp. 129–143. https://doi.org/10.1002/nla.609

[6] X.M. Fang, *General fixed point method for solving the linear complementarity problem*, AIMS Math., **6** (2021) no. 11, pp. 11904–11920. https://doi:10.3934/math.2021691

[7] A. Frommer, G. Mayer, *Convergence of relaxed parallel multisplitting methods*, Linear Algebra Appl., **119** (1989), pp. 141–152. https://doi.org/10.1016/0024-3795(89)90074-8

[8] A. Hadjidimos, M. Lapidakis, M. Tzoumas, *On iterative solution for linear complementarity problem with an H_+-matrix*, SIAM J. Matrix Anal. Appl., **33** (2012) no. 1, pp. 97–110. https://doi.org/10.1137/100811222

[9] A. Hadjidimos, L.L. Zhang, *Comparison of three classes of algorithms for the solution of the linear complementarity problem with an H_+-matrix*, J. Comput. Appl. Math., **336** (2018), pp. 175–191. https://doi.org/10.1016/j.cam.2017.12.028

[10] N.W. Kappel, L.T. Watson, *Iterative algorithms for the linear complementarity problem*, Int. J. Comput. Math., **19** (1986) nos. 3-4, pp. 273–297. https://doi.org/10.1080/00207168608803522

[11] S. Liu, H. Zheng, W. Li, *A general accelerated modulus-based matrix splitting iteration method for solving linear complementarity problems*, Calcolo, **53** (2016) no. 2, pp. 189–199. https://doi.org/10.1007/s10092-015-0143-2

[12] O. Mangasarian, *Solution of symmetric linear complementarity problems by iterative methods*, J. Optim. Theory Appl., **22** (1977), pp. 465–485. https://doi.org/10.1007/BF01268170.

[13] K.G. Murthy, *On the number of solutions to the complementarity problem and spanning properties of complementarity cones*, Linear Algebra Appl., **5** (1972), pp. 65–108. https://doi.org/10.1016/0024-3795(72)90019-5
[14] S.K. Neogy, R. Bapat, A.K. Das, T. Parthasarathy, *Mathematical Programming and Game Theory for Decision Making*, World Scientific, 2008. https://doi.org/10.1142/6819
[15] S.K. Neogy, A.K. Das, R. Bapat, *Modeling, Computation and Optimization*, World Scientific, 2021. https://doi.org/10.1142/7303
[16] H.S. Najafi, S.A. Edalatpanah, *Modification of iterative methods for solving linear complementarity problems*, Eng. Comput., **30** (2013) no. 7, pp. 910–923. https://doi.org/10.1108/EC-10-2011-0131
[17] S.K. Neogy, A.K. Das, A. Gupta, *Generalized principal pivot transforms, complementarity theory and their applications in stochastic games*, Optim. Lett., **6** (2012) no. 2, pp. 339–356. https://doi.org/10.1007/s11590-010-0261-3
[18] S.K. Neogy, S. Sinha, A.K. Das, A. Gupta, *Optimization models for a class of structured stochastic games*, in Mathematics in Science and Technology: Mathematical Methods, Models and Algorithms in Science and Technology (2011), pp. 448–470. https://doi.org/10.1142/9789814338820_0016
[19] S.K. Neogy, A.K. Das, S. Sinha, A. Gupta, *On a mixture class of stochastic game with ordered field property*, in Mathematical Programming and Game Theory for Decision Making, World Scientific (2008), pp. 451–477. https://doi.org/10.1142/9789812813220_0025
[20] X.J. Shi, L. Yang, Z.H. Huang, *A fixed-point method for linear complementarity problem arising from American option pricing*, Acta Math. Appl. Sin. Engl. Ser., **32** (2016), pp. 921–932. https://doi.org/10.1007/s10255-016-0613-6
[21] M.H. Xu, G.F. Luan, *A rapid algorithm for a class of linear complementarity problems*, Appl. Math. Comput., **188** (2007) no. 2, pp. 1647–1655. https://doi.org/10.1016/j.amc.2006.11.184
[22] H. Zheng, W. Li, S. Vong, *A relaxation modulus-based matrix splitting iteration method for solving linear complementarity problems*, Numer. Algor., **74** (2017) no. 1, pp. 137–152. https://doi.org/10.1007/s11075-016-0142-7

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