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BOOK REVIEWS

Philippe G. Ciarlet, *Linear and Nonlinear Functional Analysis with Applications*, SIAM, Philadelphia, 2013, xiv + 832 pages, ISBN 978-1-611972-58-0 (alk. paper).

Here is a monumental book of functional analysis, linear and nonlinear, with a clear orientation towards its numerous applications. Written by a top researcher in numerical analysis, finite element methods, partial differential equations, computational mechanics, applied differential geometry, mathematical elasticity, plate and shell theories, the book offers the reader in a systematized framework and under a pedagogical presentation, almost everything he or she needs for research, both in the theoretical field of nonlinear analysis and in applied mathematics in any direction, in physics, engineering, biology, etc.

As the title mentions, the content of the book can be divided into two parts: linear functional analysis and its applications (Chapters 2–6) and nonlinear functional analysis and its applications (Chapters 7–9). Chapter 1 is a quick review of results from real analysis and the theory of functions. All the material that follows is self-contained, with complete proofs, some of which less accessible in the literature or more difficult to follow without a collateral knowledge.

Chapter 2: Normed Vector Spaces includes as examples the basic functional spaces $C(K;Y)$, $l^p, L^p(\Omega)$ and spaces of continuous linear operators, compact linear operators and continuous multilinear mappings. Topics presented here: compactness in finite-dimestional normed vector spaces, an application to the fundamental theorem of algebra and some basic results from the approximation theory, theorems of Korovkin, Bohman, Bernstein, Weierstrass, Fejér and Stone-Weierstrass. Convex sets and convex functions are also discussed.

In Chapter 3: Banach Spaces the reader can find Riesz's representation theorem in $L^p(\Omega)$, Banach's fixed point theorem and its applications to the existence of solutions of boundary value problems, the Ascoli-Arzelà theorem and its applications, the Cauchy-Peano theorem and Euler's method.

Chapter 4: Inner-Product Spaces and Hilbert Spaces contains the Cauchy-Schwarz-Bunyakovskii inequality, the projection theorem, direct sum theorem, Riesz representation theorem in a Hilbert space, Hilbert bases and Fourier series, eigenvalues and eigenvectors of self-adjoint operators and the spectral theorem for compact self-adjoint operators.

Chapter 6 reflects the great expertise of the author in the field of partial differential equations and the deep understanding of how functional analysis intervenes decisively in their study. The reader is guided with great pedagogical tact in the modern theory of PDEs and finds here everything he needs to know in the field: variational equations and inequalities, the Lax-Milgram lemma, Sobolev spaces, imbedding theorems, Babuška-Brezzi inf-sup theorem, Korn's inequality, Poincaré's lemma and several applications to elliptic equations, Stokes equations and models from elasticity.

Chapter 7: Differential Calculus in Normed Vector Spaces starts by introducing the Fréchet derivative and continues with a number of consequences that generalize to maps between normed vector spaces, well-known properties of real-valued functions of a real variable. The applications include extrema of real functionals in relation to differentiability and convexity, Newton-Kantorovich theorem, maximum principle for second-order linear elliptic operators, Lagrange interpolation and multipoint Taylor formulas. There are also given the implicit function theorem and some of its applications, Lagrange multipliers for general constrained optimization problems, and a brief introduction to saddle-points and Lagrangians.

Chapter 8: Differential Geometry in $\mathbb{R}^n$, although it appears as foreign to the general theme of the work, it brings to attention a series of important notions and results in applied mathematics, especially in mechanics and the theory of elasticity. It can be read independently by those interested in this field.

I deliberately left for the end Chapter 5: The "Great Theorems" of Linear Functional Analysis, and Chapter 9: The "Great Theorems" of Nonlinear Functional Analysis, because we can consider them as true syntheses of the most important results from the two parts, linear and nonlinear, of functional analysis, which any researcher in these fields should know. Thus, in Chapter 5, one may find Baire's theorem, Banach-Steinhaus theorem, Banach's open mapping theorem and closed graph theorem, Hahn-Banach theorem and its geometric forms, Banach-Saks-Mazur theorem and Banach-Eberlein-Šmulian theorem and some of their applications. In Chapter 9, the reader will discover results on minimizers of functionals, Ekeland's variational principle, Brouwer's, Schauder's, Schaefer's and Leray-Schauder's fixed point theorems, the Minty-Browder theorem on monotone operators, topological degree, Borsuk's and Borsuk-Ulam theorems and other theoretical results and some applications to p-Laplace operator, Navier-Stokes equations and in nonlinear elasticity.

In the end, the author provides Bibliographical Notes for each chapter sending to titles that may complement the text, while references to original papers are given in the footnotes spread throughout the chapters. The extensive Bibliography of 26 pages contains both new and old titles. A list of the Main Notations grouped thematically and a rich Index allow readers to orient themselves easily.

Like all other works of the author, this book is also very carefully written, with complete proofs and numerous remarks and comments. Many problems that come to complete the theory, some accompanied by hints, make this book a useful source for several courses at the last-year undergraduate or graduate levels. The author himself offers such suggestions based on his own teaching practice over years at several universities: Linear Functional Analysis, Linear and Nonlinear Boundary Value Problems, Differential Calculus and Applications, Introduction to Differential Geometry, Nonlinear Functional Analysis, and Mathematical Elasticity and Fluid Mechanics.

The book is also extremely useful to researchers in the field of functional analysis, partial differential equations and applied mathematics, offering an exact mathematical framework for the analysis of complex models.

Note that this monumental volume was recently complemented by a book on locally convex spaces and harmonic analysis: Philippe G. Ciarlet, Locally Convex Spaces and Harmonic Analysis. An Introduction, SIAM, Philadelphia, 2021, viii + 195 pages, ISBN 9781611976649 (paperback), ISBN 9781611976656 (ebook).

Radu Precup

Guojun Can, Chaoqun Ma, Jianhong Wu, *Data Clustering: Theory, Algorithms, and Applications*, 2nd ed., SIAM, Philadelphia, 2021, xxiii + 406 pp., ISBN 978-1-61197-632-8 (paperback), ISBN 978-1-61197-633-5 (ebook). Part of the Mathematics in Industry series.

The first edition of this monograph has grown and evolved from some collaborative projects for industrial applications undertaken by the Laboratory for Industrial and Applied Mathematics at York University, some of which were in collaboration with Generation 5 Mathematical Technologies, Inc. The motivation behind the creation of the first edition (2007) of the book was that there have been many clustering algorithms scattered in publications in very diversified areas such as pattern recognition, artificial intelligence, information technology, image processing, biology, psychology, and marketing. So the authors gathered and categorized the most important clustering algorithms from their point of view, stating that they made no attempt to provide a comprehensive coverage of the subject area.

Since the first edition, the advancement of technology in all the data related subject areas exploded. The second edition was published in 2020 to update the first edition with the latest developments. Chapter 19 was replaced because MATLAB lost the race compared to other programming languages as is not efficient for developing clustering algorithms; the new chapter now presents some Open Source Clustering Software from Python, R, Java, C++. Since the topic of Chapter 20 in the first edition is discussed now in a whole book, Chapter 20 was replaced by a new chapter, about Lightweight Java Clustering Framework. Chapters 21 and 22 were added presenting two applications of clustering algorithms.

The book is divided into four parts: basic concepts (clustering, data, and similarity measures), algorithms, programming languages and applications. We now briefly describe the content of each part of the book.

The first part of the book introduces all the concepts related to data clustering. The first chapter defines what data clustering is, describes the clustering process and list some resources related to clustering. In the second chapter the most important data types are discussed which are a major factor in choosing the appropriate clustering algorithm. The following two chapters deal with data such as the scale conversion and the standardization and transformation techniques. In Chapter 5 different data visualization techniques are presented like Sammon's Mapping, Tree Maps, t-SNE, etc. In the ending chapter of this part similarity and dissimilarity measures are introduced which are very important in data clustering because they are used to quantitatively describe the similarity/dissimilarity between two data points or two clusters.

The second part of the book presents some popular clustering algorithms based on some specific methodologies, such as hierarchical, center-based, search-based methods being the largest number of algorithms. It also includes chapters for fuzzy, graph-based, grid-based, density-based, model-based, subspace-based and scalable algorithms. There is also a chapter for some miscellaneous algorithms which did not fit into any other category and the last chapter of this part is discussing evaluation methods for clustering algorithms.

The third part consists of two chapters. The first one describes the most popular software related to clustering in the most widely used programming languages, such as R, Python, Java and C++. The second chapter of the third part presents a lightweight Java clustering framework which was written by the authors of this book.

The final, fourth part of the book presents two applications for clustering: one about gene expression and the other about variable annuity policies.

Anyone familiar with elementary linear algebra, calculus, and basic statistical concepts can easily read and understand the presented concepts. The monograph is intended not only for statistics, applied mathematics, and computer science senior undergraduates and graduates, but also for research scientists who need cluster analysis to deal with data. It can be a great resource for any data scientist or machine learning engineer who is dealing with unsupervised learning problems.

Imre Boros

Michael W. Mahoney, John C. Duchi, Anna C. Gilbert, *The Mathematics of Data*, AMS/IAS/SIAM, 2018, xii + 325 pp., ISBN 978-1-4704-357-52 (alk. paper). Part of the IAS/Park City Mathematics Series Volume 25.

"The Mathematics of Data" was the topic for the 26th annual Park City Mathematics Institute (PCMI) summer session, held in July 2016. Lecture notes from the Graduate Summer School are published each year in the IAS/Park City Mathematics Series. Each chapter was written by a different author, so it has its own unique style, including notational differences.

The first lecture, “Lectures on Randomized Numerical Linear Algebra” by Petros Drineas and Michael W. Mahoney, goes through the basics of linear algebra and discrete probability, and in the ways in which they interact in many large-scale data applications. Several randomized algorithms are introduced: for matrix multiplication, for least-squares regression problems, and for low-rank approximation.

The second lecture, “Optimization Algorithms for Data Analysis” by Stephen J. Wright, starts with the description of some canonical problems in data analysis and their formulation as optimization problems, and continues with some preliminaries for optimization. Then Gradient based methods are discussed, including the case of regularized objectives, the Prox-Gradient Methods and some accelerated methods like Nesterov, Heavy-Ball, Conjugate Gradient. The last section is about Newton methods which are analyzed on convex and nonconvex functions.

The third lecture, “Introductory Lectures on Stochastic Optimization” by John C. Dutchi, covers the basic analytical tools and algorithms necessary for stochastic optimization. In the first section are introduced the convex analytic tools necessary for the development of the optimization algorithms. Then subgradients are introduced and their application to certain stochastic optimization problems. In the closing section, the optimality guarantee is studied for several methods, two standard techniques are used to provide lower bounds on the ability of any algorithm to solve stochastic optimization problems.

The fourth lecture, “Randomized Methods for Matrix Computation” by Per-Gunner Martinsson, describes a set of randomized methods for efficiently computing low rank approximation to a given matrix. Such an approximation is useful for: storing the approximated matrix more frugally, efficient computation of matrix vector products, data interpretation, and much more. This lecture describes randomized algorithms that obtain better worst-case running time, both in the randomized low-rank approximation methods (RAM) and a streaming model. It demonstrates how randomness can be used to obtain improved communication properties for algorithms, and also several data-driven decomposition such as Nyström method, the Interpolative Decomposition, and the CUR decomposition.

The fifth lecture, “Four Lectures on Probabilistic Methods for Data Science” by Roman Vershynin, describes a sample of useful tools of high-dimensional probability, focusing on the classical and matrix Bernstein’s inequality and the uniform matrix deviation inequality, presenting applications of these tools for dimension reduction, network analysis, covariance estimation, matrix completion, and space signal recovery.

The sixth lecture is entitled “Homological Algebra and Data” by Robert Ghrist. It approaches topological data analysis from the perspective of homological algebra, where homology is an algebraic compression scheme that excises all but the essential topological features from a class of data structures. The lecture provides an example of how methods from pure mathematics, in this case topology, might be fruitfully used in data science and the mathematics of data. The lecture also introduces methods and perspectives of Applied Topology for students and researchers in areas including data science, neuroscience, complex systems, and statistics.

The area of the mathematics of data is not sufficiently mature to say the final word, the lectures tried to capture the major trends sufficiently broadly and at a sufficiently introductory level that this volume could be used as teaching resource for students with backgrounds in any of the wide range of areas related to the mathematics of data.

Imre Boros