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# NEW SUFFICIENT CONDITIONS FOR THE SOLVABILITY OF A NEW CLASS OF SYLVESTER-LIKE ABSOLUTE VALUE MATRIX EQUATIONS

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**Abstract.** In this article, some new sufficient conditions for the unique solvability of a new class of Sylvester-like absolute value matrix equations $AXB - |CXD| = F$ are obtained. This work is distinct from the published work by Li [Journal of Optimization Theory and Applications, 195(2), 2022]. Some new conditions were also obtained, which were not covered by Li. We also give an example in support of our result.

**MSC.** 15A06, 90C05, 90C30.

**Keywords.** New Sylvester-like absolute value matrix equation, sufficient condition, unique solution.

## 1. INTRODUCTION

Recently, Li [8] introduced the following new class of Sylvester-like absolute value matrix equations (AVME)

$$AXB - |CXD| = F, \tag{1}$$

where $A, B, C, D, F \in \mathbb{R}^{n\times n}$ are given and $X \in \mathbb{R}^{n\times n}$ to be determined. Eq. (1) is a special case of the following new generalized absolute value equations (NGAVE)

$$Ax - |Cx| = f, \tag{2}$$

with $A, C \in \mathbb{R}^{n\times n}$, $f \in \mathbb{R}^n$ are known and $x \in \mathbb{R}^n$ is unknown.

The generalized absolute value equations (GAVE) is defined as

$$Ax - B|x| = f. \tag{3}$$

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The generalized absolute value matrix equations (GAVME) is a generalization version of the GAVE (3) and is defined as

$$AX + B|X| = F. \tag{4}$$

The Sylvester-like AVME is defined as

$$AXB + C|X|D = F. \tag{5}$$

The new class of Sylvester-like AVME (1) is quite different from the Sylvester-like AVME (5).

The absolute value equations is powerful tools in the field of optimization, complementarity problems, convex quadratic programming and linear programming. For more about the absolute value equations, one may refer to ([1, 9, 10, 11, 19] and references therein).

In 2021, the NGAVE (2) was first considered by Wu [19] and discussed its different conditions for a unique solution. In 2020, Dehghan *et al.* [2] first considered the generalized absolute value matrix equations (4) and provided a matrix multi-splitting Picard-iterative method for solving the GAVME. In 2022, Kumar *et al.* [5, 6] provided two new conditions to ensure the unique solvability of the GAVME, the condition of Kumar *et al.* [6] is superior to the conditions of Xie [20] and Dehghan *et al.* [2]. In 2022, Tang *et al.* [17] further discussed the unique solvability of the GAVME and provided a Picard-type method for the solution of the GAVME. In 2021, Hashemi [3] first considered the Sylvester-like absolute value matrix equations (5) and discussed its unique solvability conditions. Wang *et al.* [18] provided new unique solvability conditions for the Sylvester-like AVME (5), which are different work from the Hashemi [3]. Inspired by the above works on different types of matrix equations, Li [8] first considered the new class of Sylvester-like AVME (1) and provided unique solvability conditions for (1).

In this article, we further discussed the unique solvability of the new class of Sylvester-like AVME (1). As it has non-differentiable and non-linear terms, studying the new class of Sylvester-like AVME is exciting and challenging. The Sylvester-like absolute value matrix equations have many uses in the field of interval matrix equations [13, 14] and robust control [15] and so on.

Notation. We will denote $\hat{D} = \mathrm{diag}(\hat{d}_i)$ with $0 \leq \hat{d}_i \leq 1$ is a diagonal matrix. $\sigma(.)$, $\sigma_{\max}(.)$ and $\sigma_{\min}(.)$ denote singular value, maximum singular value and minimum singular value, respectively. For the determinant of a matrix, we will use $\det(.)$, and $\rho(.)$ is used for the spectral radius of a matrix.

The remainder of this paper is structured in the following manner: Section 2 contains some useful results. In Section 3, we obtain the unique solution condition for the new class of Sylvester-like AVME (1). A numerical example in support of our results is provided in Section 4, and we conclude our discussion in Section 5.

## 2. PRELIMINARIES

In this section, we recall some definitions, lemmas and theorems for further use.

Definition 1 ([16]). *Let $\mathcal{M} = \{M_1, M_2\}$ denote the set of matrices with $M_1, M_2 \in \mathbb{R}^{n\times n}$. A matrix $R \in \mathbb{R}^{n\times n}$ is called a row (or column) representative of $\mathcal{M}$, if $R_{j.} \in \{(M_1)_{j.}, (M_2)_{j.}\}$ (or $R_{.j} \in \{(M_1)_{.j}, (M_2)_{.j}\}$) j=1,2,...,n, where $R_{j.}$, $(M_1)_{j.}$, and $(M_2)_{j.}$ (or $R_{.j}$, $(M_1)_{.j}$, and $(M_2)_{.j}$) denote the $j^{th}$ row (or column) of $R$, $M_1$ and $M_2$, respectively.*

Definition 2 ([16]). *The set $\mathcal{M}$ holds the row (or column) $\mathcal{W}$-property if the determinants of all row (or column) representative matrices of $\mathcal{M}$ are positive.*

Definition 3 ([12]). *For given matrices $A_C, V \in \mathbb{R}^{n\times n}$, $V \geq 0$, the set of matrices $\mathbb{A} = \{A : |A - A_C| \leq V\}$, is known as interval matrix. An interval matrix $\mathbb{A}$ is regular if each $A \in \mathbb{A}$ is invertible.*

Lemma 4 ([4]). *The following results are hold for the square matrices $A, B, C, D \in \mathbb{R}^{n\times n}$ :*
(i) $\sigma(A \otimes B) = \sigma(A)\sigma(B)$.
(ii) $\rho(A \otimes B) = \rho(A)\rho(B)$.
(iii) $(|A \otimes B|) = |A| \otimes |B|$.
(iv) $(A \otimes B)(C \otimes D) = (AC \otimes BD)$.
(v) $vec(ABC) = (C^T \otimes A)vec(B)$.
(vi) *For non-singular matrices $A$ and $B$, $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$, where $\otimes$ is denote the Kronecker product and vec denote the vec operator.*

Theorem 5 ([7]). *If matrix $C$ is invertible, then the following assertions are equivalent:*
(i) *the NGAVE (2) has exactly one solution for any f;*
(ii) *$\{-I + AC^{-1}, I + AC^{-1}\}$ holds the column $\mathcal{W}$-property;*
(iii) *$(-I + AC^{-1})$ is invertible and $\{I, (-I + AC^{-1})^{-1}(I + AC^{-1})\}$ holds the column $\mathcal{W}$-property;*
(iv) *$(-I+AC^{-1})$ is invertible and $(-I+AC^{-1})^{-1}(I+AC^{-1})$ is a P-matrix;*
(v) *$(AC^{-1} + (I - 2\hat{D}))$ is invertible for any $\hat{D}$ ;*
(vi) *$\{(-I + AC^{-1})F_1 + (I + AC^{-1})F_2\}$ is invertible, where $F_1, F_2 \in \mathbb{R}^{n\times n}$ are two arbitrary non-negative diagonal matrices with $\mathrm{diag}(F_1 + F_2) > 0$.*

Theorem 6 ([7]). *If matrix $C$ is invertible, then the following assertions are equivalent:*
(i) *the NGAVE (2) has a unique solution;*
(ii) *$\{I + AC^{-1}, -I + AC^{-1}\}$ has the row $\mathcal{W}$-property;*
(iii) *$(I + AC^{-1})$ is invertible and $\{I, (-I + AC^{-1})(I + AC^{-1})^{-1}\}$ has the row $\mathcal{W}$-property;*
(iv) *$(I + AC^{-1})$ is invertible and $(-I + AC^{-1})(I + AC^{-1})^{-1}$ is a P-matrix;*

(v) $\{F_1(I + AC^{-1}) + F_2(-I + AC^{-1})\}$ *is invertible, where* $F_1, F_2 \in \mathbb{R}^{n\times n}$ *are two arbitrary non-negative diagonal matrices with* $\mathrm{diag}(F_1 + F_2) > 0$.

THEOREM 7 ([7]). *Let all diagonal entry of* $AC^{-1} + I$ *have the same sign as the corresponding entries of* $AC^{-1} - I$. *Then the NGAVE* (2) *has exactly one solution for any* $f$ *if any one of the following conditions is true:*
(i) $AC^{-1} - I$ *and* $AC^{-1} + I$ *are strictly diagonally dominant by columns;*
(ii) $AC^{-1} - I$, $AC^{-1} + I$ *and all their column representative matrices are irreducibly diagonally dominant by columns.*

THEOREM 8 ([7]). *If matrix* $C$ *is non-singular, then the NGAVE* (2) *has exactly one solution for any* $f$, *if the interval matrix* $[AC^{-1} - I, AC^{-1} + I]$ *is regular.*

THEOREM 9 ([7]). *Let matrix* $C$ *is non-singular, then the NGAVE* (2) *has unique solution for any* $f$ *if* $\sigma_{\min}(AC^{-1}) > 1$.

THEOREM 10 ([7]). *The NGAVE* (2) *has exactly one solution if and only if* $det(A + C) \neq 0$ *and for any* $\hat{D}$, *matrix* $A - C + 2\hat{D}C$ *is non-singular.*

THEOREM 11 ([19]). *Let matrix* $A$ *is invertible. The NGAVE* (2) *has exactly one solution if* $\rho((I - 2\hat{D})CA^{-1}) < 1$, *for any diagonal matrix* $\hat{D}$.

THEOREM 12 ([8]). *Let* $\sigma_{\max}(C)\sigma_{\max}(D) < \sigma_{\min}(A)\sigma_{\min}(B)$, *then the new Sylvester-like AVME* (1) *has exactly one solution.*

## 3. MAIN RESULTS

This section provides some unique solvability conditions for the new Sylvester-like AVME. First, we see the following result for the NGAVE (2).

THEOREM 13. *Let matrix* $A$ *is non-singular. The NGAVE* (2) *has exactly one solution if* $\rho(|C| \cdot |A^{-1}|) < 1$.

*Proof.* For any diagonal matrix $\hat{D}$, we have $(I-2\hat{D})CA^{-1} \leq |(I-2\hat{D})CA^{-1}| \leq |I - 2\hat{D}| \cdot |CA^{-1}| \leq |CA^{-1}| \leq |C| \cdot |A^{-1}|$. Since $(I - 2\hat{D})CA^{-1} \leq |C| \cdot |A^{-1}|$, this implies that $\rho((I - 2\hat{D})CA^{-1}) \leq \rho(|C| \cdot |A^{-1}|) < 1$.
Hence, based on Theorem 11, $\rho(|C|\cdot|A^{-1}|) < 1$ implies the unique solvabilty of the NGAVE (2). □

For presenting some unique solvability conditions for the new Sylvester-like AVME (1), we first write Eq. (1) into the equivalent NGAVE form (2), and use the results of the NGAVE.
So by taking $S = B^T \otimes A$, $T = D^T \otimes C$, $f = vec(F)$ and $x = vec(X)$, where 'vec' is vec operator and '$\otimes$' is the Kronecker product. Then, the new Sylvester-like AVME (1) can be written as the following NGAVE form

$$Sx - |Tx| = f. \tag{6}$$

Now see the following results for the new Sylvester-like AVME.

Theorem 14. *Let $C$, $D$ be non-singular matrices. The new Sylvester-like AVME* (1) *has exactly one solution if* $\sigma_{\min}(D^{-1}B)\sigma_{\min}(AC^{-1}) > 1$.

*Proof.* To prove the above Theorem, we use Eq. (6) and Theorem 9. If $\sigma_{\min}(ST^{-1}) > 1$, then the Sylvester-like AVME (1) has unique solution.

Now, $\sigma_{\min}(ST^{-1}) = \sigma_{\min}((B^T \otimes A)(D^T \otimes C)^{-1}) = \sigma_{\min}((B^T \otimes A)(D^{-T} \otimes C^{-1})) = \sigma_{\min}(B^T D^{-T} \otimes AC^{-1}) = \sigma_{\min}((D^{-1}B)^T \otimes AC^{-1}) = \sigma_{\min}((D^{-1}B) \otimes AC^{-1}) = \sigma_{\min}(D^{-1}B)\sigma_{\min}(AC^{-1}) > 1$.

This completes the proof. □

Remark 15. *In some instances, our result performs better compared to the condition of Theorem 12; see example in Section 4.*

Theorem 16. *Let $S = B^T \otimes A$, $T = D^T \otimes C$. The new Sylvester-like AVME* (1) *has exactly one solution if and only if* $det(S+T) \neq 0$ *and for any $\hat{D}$, matrix $S - T + 2\hat{D}T$ is non-singular.*

*Proof.* The proof of the above Theorem directly holds by Theorem 10 and Eq. (6). □

Theorem 17. *Let 0 is not an eigenvalue of the matrices $A$ and $B$. The new Sylvester-like AVME* (1) *has exactly one solution if* $\rho(|D^T| \cdot |B^{-T}|) \cdot \rho(|C| \cdot |A^{-1}|) < 1$.

*Proof.* Since $S^{-1} = B^{-T} \otimes A^{-1}$, then

$|T| \cdot |S^{-1}| = |D^T \otimes C| \cdot |B^{-T} \otimes A^{-1}| = |D^T| \otimes |C| \cdot |B^{-T}| \otimes |A^{-1}| = |D^T| \cdot |B^{-T}| \otimes |C| \cdot |A^{-1}|$.

Based on spectral radius property of the matrix, we have $\rho(|D^T| \cdot |B^{-T}| \otimes |C| \cdot |A^{-1}|) = \rho(|D^T| \cdot |B^{-T}|) \cdot \rho(|C| \cdot |A^{-1}|)$.

Based on Eq. (6) and Theorem 13, if $\rho(|D^T| \cdot |B^{-T}|) \cdot \rho(|C| \cdot |A^{-1}|) < 1$, then the new Sylvester-like AVME (1) has unique solution for any F. □

The new Sylvester-like AVME (1) is equivalently written as NGAVE form (6). So now we use the results of the NGAVE for the new Sylvester-like AVME.

Now, $ST^{-1} = (B^T \otimes A)(D^T \otimes C)^{-1} = (B^T \otimes A)(D^{-T} \otimes C^{-1}) = (B^T D^{-T} \otimes AC^{-1}) = (D^{-1}B)^T \otimes (AC^{-1})$.

So, based on Theorems 5 to 8 we have the following results, see Theorems 18 to 21 respectively.

Theorem 18. *Let $C$, $D$ be non-singular matrices, then the following statements are equivalent:*

(i) *the new Sylvester-like AVME* (1) *has exactly one solution;*

(ii) $\{(D^{-1}B)^T \otimes (AC^{-1}) - I, (D^{-1}B)^T \otimes (AC^{-1}) + I\}$ *holds the column $\mathcal{W}$-property;*

(iii) $((D^{-1}B)^T \otimes (AC^{-1}) - I)$ *is invertible and* $\{I, ((D^{-1}B)^T \otimes (AC^{-1}) - I)^{-1}((D^{-1}B)^T \otimes (AC^{-1}) + I)\}$ *holds the column $\mathcal{W}$-property;*

(iv) $((D^{-1}B)^T \otimes (AC^{-1}) - I)$ *is invertible and* $((D^{-1}B)^T \otimes (AC^{-1}) - I)^{-1}((D^{-1}B)^T \otimes (AC^{-1}) + I)$ *is a P-matrix;*

(v) $((D^{-1}B)^T \otimes (AC^{-1}) + (I - 2\hat{D}))$ *is invertible for any* $\hat{D}$*;*

(vi) $\{((D^{-1}B)^T \otimes (AC^{-1}) - I)F_1 + ((D^{-1}B)^T \otimes (AC^{-1}) + I)F_2\}$ *is invertible, where* $F_1, F_2 \in \mathbb{R}^{n\times n}$ *are two arbitrary non-negative diagonal matrices with* $\mathrm{diag}(F_1 + F_2) > 0$.

*Proof.* The proof of the above Theorem is directly held by Theorem 5 and Eq. (6). □

THEOREM 19. *Let* $C$*,* $D$ *be non-singular matrices, then the following statements are equivalent:*

(i) *the new Sylvester-like AVME* (1) *has exactly one solution;*

(ii) $\{(D^{-1}B)^T \otimes (AC^{-1}) + I, (D^{-1}B)^T \otimes (AC^{-1}) - I\}$ *has the row* $\mathcal{W}$*-property;*

(iii) $((D^{-1}B)^T \otimes (AC^{-1}) + I)$ *is invertible and* $\{I, ((D^{-1}B)^T \otimes (AC^{-1}) - I)((D^{-1}B)^T \otimes (AC^{-1}) + I)^{-1}\}$ *has the row* $\mathcal{W}$*-property;*

(iv) $((D^{-1}B)^T \otimes (AC^{-1}) + I)$ *is invertible and* $((D^{-1}B)^T \otimes (AC^{-1}) - I)((D^{-1}B)^T \otimes (AC^{-1}) + I)^{-1}$ *is a P-matrix;*

(v) $\{F_1((D^{-1}B)^T \otimes (AC^{-1}) + I) + F_2((D^{-1}B)^T \otimes (AC^{-1}) - I)\}$ *is invertible, where* $F_1, F_2 \in \mathbb{R}^{n\times n}$ *are two arbitrary non-negative diagonal matrices with* $\mathrm{diag}(F_1 + F_2) > 0$.

*Proof.* With the help of Theorem 6 and Eq. (6), our result will be true. □

THEOREM 20. *Let all diagonal entries of the matrix* $(D^{-1}B)^T \otimes (AC^{-1}) + I$ *have the same sign as the corresponding entries of the matrix* $(D^{-1}B)^T \otimes (AC^{-1}) - I$*. Then the new Sylvester-like AVME* (1) *has exactly one solution for any* $F$ *if any one of the following conditions is true:*

(i) $(D^{-1}B)^T \otimes (AC^{-1}) - I$ *and* $(D^{-1}B)^T \otimes (AC^{-1}) + I$ *are strictly diagonally dominant by columns;*

(ii) $(D^{-1}B)^T \otimes (AC^{-1}) - I$*,* $(D^{-1}B)^T \otimes (AC^{-1}) + I$ *and all their column representative matrices are irreducibly diagonally dominant by columns.*

*Proof.* Applying Eq. (6) directly into Theorem 7, we get our result. □

THEOREM 21. *If matrices* $C$ *and* $D$ *are non-singular, then the new Sylvester-like AVME* (1) *has a unique solution for any* $F$*, if the interval matrix* $[(D^{-1}B)^T \otimes (AC^{-1}) - I, (D^{-1}B)^T \otimes (AC^{-1}) + I]$ *is regular.*

*Proof.* We come to our result with the help of Eq. (6) and Theorem 7. □

## 4. A NUMERICAL EXAMPLE

In support of our result, we are considering a small example here. However, our result is also applicable to a larger problem. Let's consider the following matrices for the new class of Sylvester-like AVME (1)

$$A = \begin{bmatrix} 3 & -4 & 1 \\ 5 & 4 & 1 \\ -3 & 5 & 1 \end{bmatrix},\ B = \begin{bmatrix} -6 & 4 & 2 \\ 3 & 2 & 4 \\ -2 & -5 & 7 \end{bmatrix}\ C = \begin{bmatrix} 5 & -4 & 1 \\ 2 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & -4 & 0 \\ 2 & 3 & 2 \\ -1 & 1 & 1 \end{bmatrix} \quad F = \begin{bmatrix} -385 & -138 & -56 \\ -104 & -88 & 103 \\ 114 & -274 & 61 \end{bmatrix}$$

It is clear that the condition of Theorem (14) hold.

$\sigma_{\min}(D^{-1}B)\sigma_{\min}(AC^{-1}) = 1.1224 \times 0.9154 = 1.027445 > 1.$

But condition of the Theorem 12 of [8] is not satisfying here, since $\sigma_{\max}(C) \cdot \sigma_{\max}(D) = 7.6562 \times 6.5791 = 50.3709 \nless \sigma_{\min}(A)\sigma_{\min}(B) = 1.3270 \times 5.0844 = 6.74699$.

Moreover, the unique solution of (1) is:

$$X = \begin{bmatrix} 4 & -3 & 1 \\ -4 & 2 & 2 \\ 3 & -1 & 5 \end{bmatrix}.$$

## 5. CONCLUSIONS

In this article, we considered the new class of Sylvester-like AVME $AXB - |CXD| = F$ and obtained new sufficient results for ensuring the unique solvability of the new class of Sylvester-like AVME (1). We also provided an example in support of our result. Further, the numerical methods for solving the new class of Sylvester-like AVME are also an exciting topic in the future.

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