JOURNAL OF NUMERICAL ANALYSIS AND APPROXIMATION THEORY
J. Numer. Anal. Approx. Theory, vol. 52 (2023) no. 2, pp. 295–297,
ictp.acad.ro/jnaat

BOOK REVIEWS

Gabriele Ciaramella, Martin J. Gander, *Iterative Methods and Preconditioners for Systems of Linear Equations*, SIAM, Philadelphia, 2022, X + 275 pp., ISBN 978-1-611976-89-2 (paperback), ISBN 978-1-61197-690-8 (ebook). Part of the Fundamentals of Algorithms series.

Published in the SIAM Series on Fundamentals of Algorithms, this book presents iterative methods and preconditioning for solving the Laplace equation as a model problem for elliptic partial differential equations (PDEs). The second author is a renowned expert in numerical analysis of PDEs and has been teaching many courses throughout the years covering the material of this book at the University of Geneva and at McGill University. The lecture notes have been developed and refined together with the first author to make a very clear textbook which can be used as teaching material for graduate or advanced undergraduate courses. All the methods are presented in a self-contained manner with a complete analysis and are accompanied by many numerical examples in Matlab (the code is freely available online). Historical contributions are presented in an erudite way and relevant quotes from the seminal works are used as a starting point in each section. Each chapter ends with a set of theoretical and computational problems.

The book starts in *Chapter 1* with a historical introduction to iterative methods and then presents the Laplace equation and its discretization with finite differences as the model problem to be solved with each method that will be discussed. Stationary iterative methods are introduced in *Chapter 2* by discussing the fundamental ideas of error, residual, convergence, convergence factor, convergence rate, splitting. Classical methods such as Jacobi, Gauss-Seidel, successive over-relaxation (SOR), Richardson are analyzed in detail. Numerical experiments are given for every method to illustrate the approximation of the model problem (Laplace equation) together with convergence. An emphasis is put on showing how that the convergence depens on the mesh size. For these methods it is shown theoretically and numerically that the convergence deteriorates as the mesh is refined.

Chapter 3 introduces Krylov methods by discussing first the steepest descent (gradient descent) method for quadratic functions with a symmetric positive definite matrix. The idea of orthogonal search directions is then developed for the Conjugate Gradient method, whose discussion includes convergence in a finite number of steps, best approximation property, convergence estimate, efficient implementation using only one matrix-vector product per iteration step. The chapter continues with the Arnoldi iteration, the Lanczos algorithm and generalized minimal residual (GMRES), for which three different methods to obtain error bounds are presented (using the spectrum, using the numerical range and using the pseudospectrum). Numerical experiments show that when decreasing the mesh size, the convergence deteriorates for these methods.

Preconditioning is the topic of *Chapter 4*, which starts with introduction on left and right preconditioning, implementing preconditioning, flexible GMRES and algebraic preconditioning methods. The main focus of this chapter is on domain decomposition methods (Schwarz, Dirichlet-Neumann, Neumann-Neumann) followed by the multigrid method. Convergence results using Fourier analysis are presented together with numerical experiments on the model Laplace problem showing that the convergence of these methods is *independent* on the mesh size. *Chapter 5* finally discusses the optimal control problem for the Laplace equation where saddle-point problems arising from constrained optimization are solved with iterative methods. The book ends with an *Appendix*, containing some technical results and results used to prove existence and uniqueness on the continuous level, and a useful *Index*.

Mihai Nechita

YUTAKA YAMAMOTO, *From Vector Spaces to Function Spaces. Introduction to Functional Analysis with Applications*, SIAM, Philadelphia, 2012, XIV + 268 pp., ISBN 978-1-61197-230-6 (paperback), ISBN 978-1-61197-231-3 (ebook). Part of the Other Titles in Applied Mathematics series.

The book is a panoramic introduction to functional analysis with emphasis on those aspects of interest in applied mathematics, in science and engineering. In an accessible manner that does not ignore the specific rigor of mathematics, notions and results such as Banach and Hilbert spaces, distribution theory, Fourier and Laplace analysis and Hardy spaces are presented. Applications to linear systems and control theory are also presented. The book reflects the author's conception that a conceptual understanding is not only indispensable but also of great importance in the manipulation of computational methods. A special attention is paid to the background motivations of analytical concepts and methods, extremely important for those who are oriented towards applied mathematics but want a rigorous foundation of their work techniques.

The book is structured in 10 chapters, an Appendix of 8 topics, a Table of Laplace Transforms, a list of Solutions to Exercises and Problems, Bibliographic Notes, Bibliography and Index.

Chapter 1 introduces the notions of finite-dimensional vector space, linear mapping, matrix, subspace, quotient space, duality and dual space. All these concepts are well motivated and illustrated through examples of linear algebra, polynomials and differential equations.

Chapter 2 is dedicated to linear spaces and Banach spaces. Here are stated and proved Banach's Open Mapping and Closed Graph Theorems, Baire's Category Theorem and the Uniform Boundedness Principle.

Chapter 3 is on inner product and Hilbert spaces, with some fundamental examples and properties such as the theorem about the minimum-norm approximation, orthogonal complements and the projection theorem, orthogonal expansion and abstract Fourier series, the problem of best approximation.

Chapter 4, Dual Spaces, presents dual spaces and their norms, The Riesz-Fréchet Theorem, weak and weak* topologies, duality between subspaces and quotient spaces.

The contents of Chapter 5 are as follows: the space of linear operators between two Banach spaces, dual mappings, inverse operators, spectra and resolvents, adjoint operators in Hilbert spaces, examples of adjoint operators, Hermitian operators, compact operators and spectral resolution.

Chapter 6 deals with the Schwartz distributions. After giving the motivation, the author defines the notion of a distribution and the space of distributions, differentiation of distributions, support of distributions, convergence of distributions, convolution and its system theoretic interpretation, and an application.

Chapter 7, Fourier Series and Fourier Transform, starts with the classical Fourier series of periodic functions and continues with the less discussed in the literature theory of Fourier series expansion of distributions. The next topics are the space of rapidly decreasing functions, the space of tempered distributions, the Fourier transform and convolution. An application to the sampling theorem in signal processing is included.

Chapter 8, Laplace Transform, deals with the Laplace transform for distributions together with some examples, its connection with convolution and distributional derivative, the inverse Laplace transform and final-value theorem.

Through Chapters 9 and 10, the author guides the reader towards his own field of research, namely control and systems theory. At this point the book distances itself from all introductory books to functional analysis and reveals the target for which this work was designed. Thus, Chapter 9 is an easy introduction to the theory of Hardy spaces which plays a fundamental role in modern control theory. The contents are as follows: Hardy spaces; Poisson kernel and boundary values; canonical factorization; shift operators; Nehari approximation and generalized interpolation; application of Sarason's Theorem; Nehari's Theorem-Supplements. Chapter 10, Applications to Systems and Control, has the following sections: linear systems and control; control and feedback; controllability and observability; input/output correspondence; realization; H^∞ control; solution to the sensitivity minimization problem; general solution for distributed parameter systems; supplementary remarks.

Throughout the book, a reasonable number of exercises and problems are proposed whose solutions are given in an Appendix at the end of the book.

Written in an airy manner, with care for a good understanding of the notions, results and their applicability, this book achieves the double goal to "provide young students with an accessible account of a conceptual understanding of fundamental tools in applied mathematics" and to "give those who already have some exposure in applied mathematics, but wish to acquire a more unified and streamlined comprehension of this subject, a deeper understanding through background motivations."

Certainly, the book is extremely useful for students and young researchers in applied mathematics, in mathematical sciences and engineering. It is an indispensable material especially for those interested in the modern theory of system control.

Radu Precup