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BOOK REVIEWS

Brad G. Osgood, *Lectures on the Fourier Transform and Its Applications*, AMS, 2019, 693 pp., ISBN 978-1-4704-4191-3 (paperback), ISBN 978-1-4704-4976-6 (ebook).

This is an excellently written and instrumental book for anyone who wants to deeply understand much of mathematics and use it successfully. In fact, at the end of the Preface, the author rightly writes: "I hope that this book offers a dose of mathematical know-how, honest and helpful, and with a light touch".

The book is impressive, with nine chapters, four appendices, a Preface along with Thanks and a Subject Index. It covers almost 700 pages. Each chapter ends with a Problems and Further Results Section. To solve problems, MATLAB codes are frequently invoked or, more rarely, Julia codes. For a subtle approach to difficult issues, the author points to sources in the literature from which one can start for a deeper understanding of the issue in question.

The writing style is very rigorous, but it is impregnated with various colloquial explanations. These make the work quite affordable.

Thus, in the first chapter, the author invites the reader to a brief review of some essential aspects related to the Fourier series. In the 2nd chapter he passes from the Fourier series to the Fourier transform and considers nonperiodic phenomena as a limit of the periodic ones with a period that tends to infinity. The chapter abounds with functions (some with more exotic names!) whose Fourier transforms are computed. The basic theorems that define the Fourier transform are extensively commented on. In chapter 3 we are introduced to what the product of convolution is. The fourth chapter deals with a special class of applications, namely distributions. Its title is Distributions and Their Fourier Transforms. Here the author deals with defining various distributions and then with the properties of their Fourier transforms. The fifth chapter is devoted to a special distribution, i.e., the Dirac one. The title of the sixth chapter is Sampling and Interpolation. The idea of the bandlimited signal is introduced along with sampling (under and oversampling) and aliasing (natural and nonuniform). It is often believed that the modern world began with DFT, i.e., Discrete Fourier Transform. This is the title of the seventh chapter which makes the transition from continuous to discrete. The title of the eighth chapter is Linear Time-Invariant Systems. It provides a brief analysis of the common points of FT and linear systems. Finally, the last chapter has the title: n-Dimensional Fourier Transform. It represents a direct extension of the fundamental ideas and the mathematical definitions tackled in the one-dimensional case.

We finally observe that the work is sprinkled with a series of historical notes (some funny!) about the evolution of the basic concepts in Fourier analysis and the illustrious mathematicians who produced them.

Călin Gheorghiu

Jeffrey Humpherys, Tyler J. Jarvis, Emily J. Evans, *Foundations of Applied Mathematics. Volume 1: Mathematical Analysis*, SIAM, Philadelphia, 2017, XX + 689 pp., ISBN 978-1-61197-489-8 (paperback), ISBN 978-1-61197-490-4 (ebook). Part of the Other Titles in Applied Mathematics series.

Published in the SIAM Series on Other Titles in Applied Mathematics, this book brings together important topics related to linear and nonlinear analysis. The overall picture of the book includes the rich experience of the three authors in different fields. As a former

Vice Chair of the SIAM Activity Group on Applied Mathematics Education and a two-term member of the SIAM Education Committee, Jeffrey Humpherys brings together his pedagogical background with Tyler Jarvis's experience in geometric problems arising from physics and Emily Evans' thinking as a software engineer. Thus, this book treats the concepts of linear and nonlinear analysis from a complex, even interdisciplinary perspective.

The book contains 15 chapters organized in 4 parts, an Appendix of 4 topics, a List of Notations, Bibliography and Index. The entire content is organized in an accessible manner and the introductory part also contains a diagram of the dependence among the chapters. Throughout the book, a great number of exercises and problems are proposed. These exercises and problems have different degrees of difficulty, being marked accordingly at the end of each chapter.

In Chapter 1 the authors start a rigorous study of the basics of linear algebra over both $\mathbb{R}$ and $\mathbb{C}$. This chapter contains a description of vector spaces, subspaces and the basic rules of vector algebra.

Chapters 2 and 3 continue the study of linear algebra presenting topics related to linear transformations, matrices, determinants, the LU decomposition, the QR decomposition, inner product spaces and least squares. The first three chapters are enriched with many examples of infinite-dimensional vector spaces.

Chapter 4 contains a short presentation of the spectral theory of linear operators on finite-dimensional vector spaces. A notable subject presented in this chapter is the singular value decomposition (SVD) theory.

The second part of this book starts with Chapter 5 that contains basic results of metric topology. The authors treat here the notions of completeness and compactness together with many examples of Banach spaces. An important result presented in this chapter is the continuous linear extension theorem.

Chapter 6 is devoted to the concept of differentiation. Here we find general results related to the Fréchet derivative in $\mathbb{R}^n$, the general Fréchet derivative and its properties, the Mean Value Theorem and Taylor's Theorem. All these notions are presented in the context of Banach spaces.

The last chapter of this second part (Chapter 7) presents the uniform contraction mappings theorem that is used to prove the convergence results for Newton's method. This chapter ends with the inverse and implicit function theorems.

The third part of this book starts with Chapters 8 and 9 that address the problem of integration. The authors discuss here in detail about Daniel-Lebesgue integral, Fubini's theorem and Leibniz's integral rule together with concepts related to measure zero and measurability.

Chapter 10 completes the previously presented ideas and extends some notions in the case of manifolds. It is important to mention that Chapters 10 and 11 contain a short introduction to the fundamental tools of complex analysis (related to parametrized curves, surfaces, line integrals, Green's theorem or Cauchy's theorem).

The last part of this book contains important results related to spectral theory and iterative methods. In Chapter 12 we find basic results related to projections, generalized eigenvectors, spectral resolution and decomposition and spectral mapping theorem.

Chapter 13 contains different methods for linear systems, important results related to minimal polynomial, Krylov subspaces, the Arnoldi iteration and GMRES methods. This chapter ends with the Schur's lemma and QR decomposition which helps in computing the eigenvalues of a matrix.

Chapter 14 deals with the notions of spectra and pseudospectra. This concepts are fundamental in the modern theory of linear algebra. Also, in this chapter the authors include a proof of the Kreiss Matrix Theorem.

This excellent book ends with Chapter 15 devoted to applied ring theory, focused on the algebraic structure of polynomials and matrices. An important result presented in this chapter shows the connection between Lagrange interpolation and the spectral decomposition of a matrix. The topics presented above open the way to the second volume of this book, *Foundations of Applied Mathematics, Volume 2: Algorithms, Approximation and Optimization.*

Certainly, the book is extremely useful for students and young researchers in theoretical and applied mathematics. It is an important material that includes - in addition to standard notions of linear and nonlinear analysis - several concepts related to modern applied mathematical analysis.

Eduard Grigoriciuc

Jeffrey Humpherys, Tyler J. Jarvis, *Foundations of Applied Mathematics. Volume 2: Algorithms, Approximation, Optimization*, SIAM, Philadelphia, 2020, XVIII + 788 pp., ISBN 978-1-61197-605-2 (paperback), ISBN 978-1-61197-606-9 (ebook).

The second volume of this book series aims to present, based on a modern approach, the fundamental ideas of applied and computational mathematics. The main topics discussed in this book cover algorithms, approximation, and optimization. This volume can be used as a two-semester course for advanced undergraduates or beginning graduate students in mathematics, data science, or machine learning, or it can be seen as part of a larger curriculum in computational mathematics. Each section contains five to seven exercises for students, marked according to their difficulty.

The book is divided into four parts: Algorithms, Approximation, Interlude, and Optimization. We briefly describe each part of this book.

The first part is dedicated to Algorithms. Chapter 1 presents introductory aspects related to Algorithms and Analysis, such as complexity, leading-order behavior, cost summation techniques and counting problems to analyze the spatial and temporal complexity of an algorithm, nested loops, or the divide and conquer algorithm. Chapter 2 examines in detail the asymptotic behavior of combinatorial functions and algorithms, based on the Gamma function, Stirling's approximation, Beta function, and Laplace's method. Chapter 3 deals with data structures: arrays, pointers, and graph-based data structures (undirected and directed rooted trees, linked lists, stacks, queues, search trees). Complexity bounds for several important algorithms are also discussed. Chapter 4 discusses combinatorial optimization problems and some techniques for attacking them, such as dynamic programming, greedy algorithms, graph search algorithms, and Huffman encoding. The concept of NP (nondeterministic polynomial time) problems is briefly presented at the end of this chapter, with examples including the knapsack and traveling salesman problems. Chapter 5 covers key aspects of probability theory, such as discrete probability, discrete random variables, discrete distributions and their generalization to the continuous case. Chapter 6 discusses probabilistic sampling and statistical estimation, based on two very important results: the law of large numbers and the central limit theorem. Chapter 7 covers random algorithms, including methods that use random sampling to estimate various quantities (Monte Carlo methods), methods that produce efficient data structures (hashing), and techniques that serve as important optimization tools (simulated annealing, genetic algorithms).

The second part of the book is devoted to the Approximation Theory. Chapter 8 is dedicated to harmonic analysis, focusing mainly on Fourier analysis and wavelet analysis. Chapter 9 is about polynomial approximation and interpolation, the main aspects presented being related to the Weierstrass approximation theorem, Lagrange interpolation, orthogonal polynomials, and numerical integration based on polynomial approximation.

The third part of the book contains two chapters. Chapter 10 revisits derivatives in higher dimensions, while Chapter 11 is devoted to the fundamentals of numerical computing such as floating-point arithmetic, condition numbers, and stability of numerical algorithms.

The last part of the book is dedicated to Optimization. Chapter 12 presents unconstrained optimization problems where the focus is on minimizing or maximizing a multivariable objective function. Algorithms for finding a zero of a function such as the bisection method, Newton's method, and the secant method are discussed throughout this chapter. This chapter also presents the first- and second-order necessary conditions and the second-order sufficient condition for finding an optimizer and it discusses a class of algorithms for solving unconstrained optimization problems in higher dimensions, called gradient descent methods. Chapter 13 is dedicated to linear optimization, where both the objective and the constraints are linear. The chapter begins with introductory notions of convex and affine sets, with its main focus on the simplex method and the weak and strong duality theorems. Chapter 14 is devoted to nonlinear constrained optimization, focusing on two types of nonlinear constraints: equality and inequality constraints. Similar results to those in Chapter 12 for the equality constraints are presented here, including Lagrange's first- and second-order necessary conditions, as well the second-order sufficient condition. Numerical methods for constrained optimization are also presented, such as conditional gradient, gradient projection, and Newton's method with constraints. Chapter 15 focuses on convex analysis and optimization. It begins by presenting important properties of convex functions and several key inequalities, including Jensen's inequality. The rest of the chapter deals with convex optimization problems, many of which satisfy the strong duality, a case in which the dual problem is easier to solve than the original one. Numerical methods for solving this kind of problem are also presented. Chapter 16 addresses dynamic optimization, where several decisions are made at various points in time, each influencing subsequent decisions, the goal being to optimize over the entire time horizon. Both finite- and infinite-horizon problems are discussed. A result that gives a necessary condition for the unique solution of a class of infinite-horizon problems, called Blackwell's theorem, is presented. The last chapter deals with the case of unknown and uncertain outcomes of the decisions made, called stochastic dynamic optimization. The topics discussed include Markov decision processes and the bandit problem along with a powerful heuristic method to approximate its solution, called Thompson sampling.

This book is a valuable resource for both students and teachers in the field of applied and computational mathematics. The wide variety of examples and exercises, along with the connections between different concepts, helps students retain what they have learned and gain a deeper understanding of the subjects presented.

Andra Malina

Ed Bueler, *PETSc for Partial Differential Equations: Numerical Solutions in C and Python*, SIAM, Philadelphia, 2020, XVI + 391 pp., ISBN 978-1-61197-630-4 (paperback), ISBN 978-1-61197-631-1 (ebook).

This book is a great resource for researchers, engineers, and computational scientists working on solving partial differential equations (PDEs) using the Portable, Extensible Toolkit for Scientific Computation (PETSc). Developed by Argonne National Laboratory, PETSc is the world's most widely used parallel numerical software library for PDEs and sparse matrix computations, containing grid/mesh distribution tools, time-stepping schemes for ordinary differential equations (ODEs), nonlinear iterations, iterative linear solvers and preconditioners.

The book's clear and informative preface sets the stage for understanding both the motivation and philosophy behind PETSc. A particularly striking quote by Nick Trefethen highlights a key principle: ”As machines become more powerful, the efficiency of algorithms grows more important, not less.” This emphasis on algorithmic efficiency underpins the book’s approach to computational science.

The book is structured into two parts: *Essentials* and *Constructions*, ensuring a logical progression from foundational concepts to more advanced applications.

Part I: Essentials introduces PETSc and its fundamental numerical techniques. Chapter 1 provides a high-level overview of PETSc. Chapter 2 covers linear solvers and preconditioning, key aspects of numerical linear algebra. Chapter 3 solves the Poisson equation on a square discretized with finite differences and highlights the importance of preconditioning. Chapter 4 discusses nonlinear solvers, particularly Newton’s method. Chapter 5 explores time-stepping methods for ODEs, emphasizing the importance of stability. Chapter 6 presents preconditioners for PDEs, including multigrid methods which are heavily used further in the book. Chapter 7 analyzes solver optimality in terms of computational complexity in the number of degrees of freedom. Chapter 8 concludes this part with a discussion on parallel scalability. These chapters collectively build a strong foundation for understanding and implementing PETSc-based solutions.

Part II: Constructions delves into more advanced topics and practical applications. Chapters 9 introduces finite element methods on a structured mesh for a nonlinear problem Helmholtz problem. Chapter 10 continues with a naive approach regarding unstructured meshes, assembling matrices and multigrid, and discusses a strategy for a more efficient approach. Chapter 11 discusses finite volume methods for advection-diffusion problems and how to deal with numerical instabilities in the case of high Péclet numbers. Chapter 12 explores variational inequalities and the obstacle problem. Chapters 13 and 14 showcase practical implementations using the Firedrake library for solving the Poisson and Stokes equations in Python. The book ends with an Appendix containing so-called *numerical facts of life* – distilled bits of knowledge that capture the essence of a practitioner’s experience in numerically solving PDEs.

Throughout the book, readers benefit from well-structured exercises at the end of each chapter, reinforcing the material and encouraging hands-on learning. Additionally, all the example codes are available on the author’s GitHub, organized by chapter. The author emphasizes several key principles for the implementations: every problem should be treated as nonlinear; numerical computations should be inherently parallel; solver choices should remain flexible and adaptable; performance hinges on a careful preconditioner selection. Another strength of the book is its in-depth use of preconditioning, a crucial factor in efficient numerical computations.

PETSc for Partial Differential Equations is a meticulously structured and detailed book that serves as both an introduction and a reference for PETSc users, including those of finite element libraries that integrate PETSc – such as Firedrake, FEniCS, libMesh, MOOSE, and deal.II. It is particularly valuable for those working on large-scale scientific computations requiring parallel efficiency and advanced numerical methods.

Mihai Nechita