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ON THE APPROXIMATIVE SOLVING OF THE INTEGRAL EQUATIONS

by

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A large number of technical and scientific problems requires the solving of the nonlinear integral equations, but the finding of the exact solutions is generally a hard problem. In the latest time several papers and lectures are devoted to the theory and application of the method of the approximative solving of these equations. In Balázs's paper [2], the method ([3], [4]) of chords is applied for the equation

$$x(s) = \int_0^1 K[s, t, x(t)]\,dt, \tag{1}$$

where $K \in C([0, 1] \times [0, 1] \times R)$. As divided difference of the operator $P : C[0, 1] \to C[0, 1]$

$$P(x) = x(s) - \int_0^1 K[s, t, x(t)]dt,$$

the mapping $[x_i, x_j ; P] \in \mathfrak{L}(C[0, 1], C[0, 1])$

$$[x_i, x_j ; P]h = h(s) - \int_0^1 [x_i, x_j, K]_{(t)} h(t)dt$$

is considered, where

$$[x_i, x_j ; K]_{(t)} = \frac{K(s, t, x_i(t)) - K(s, t, x_j(t))}{x_i(t) - x_j(t)} .$$

We can se obtain a divided difference of the second order

$$[x_i,\ x_j,\ x_k;\ P]hk = -\int_0^1 \frac{[x_i,\ x_j;\ K]_{(t)} - [x_j,\ x_k;\ K]_{(t)}}{x_i(t) - x_k(t)} h(t)k(t)dt.$$

Using the rezults of the paper [2] we give a theorem of existence of the solution for the equation.

(2) $$x(s) = \int_0^1 K_1[s,\ t,\ x(s),\ x(t)]dt,$$

where $K_1 \in C([0,\ 1] \times [0,\ 1] \times R^2)$. We have

$$P_1(x) = x(s) - \int_0^1 K_1[s,\ t,\ x(s),\ x(t)]dt,$$

$$[x_i,\ x_j;\ P_1]h = h(s) - \int_0^1 \{[x_i,\ x_j;\ K_1]_{(s)}h(s) + [x_i,\ x_j;\ K_1]_{(t)}h(t)\}dt,$$

where

$$[x_i,\ x_j,\ K_1]_{(s)} = \frac{K_1[s,\ t,\ x_i(s),\ x_i(t)] - K_1[s,\ t,\ x_j(s),\ x_i(t)]}{x_i(s) - x_j(s)},$$

$$[x_i,\ x_j;\ K_1]_{(t)} = \frac{K_1[s,\ t,\ x_j(s),\ x_i(t)] - K_1[s,\ t,\ x_j(s),\ x_j(t)]}{x_i(t) - x_j(t)},$$

We observe that the divided difference of the second order of the P_1, is formed with the divided differences of second order of the function K_1.

We consider the integral equation

$$h(s)f(s) - \int_0^1 [x_i,\ x_j;\ K_1]_{(t)}h(t)dt = y_1(s)$$

Now we can apply the theorem of [2]; and we can give the

THEOREM *If for the equation (2) there exist the functions* $x_{-1},\ x_0 \in C[0,\ 1]$ *so that the following conditions are satisfied:*

(i) *for the kernel* $[x_i,\ x_j;\ K_1]_{(t)}$ *there exists the rezolvent kernel* $R(s,\ t)$ *with*

$$\int_0^1 |R(s,\ t)|dt \leqslant A \qquad (0 \leqslant s \leqslant 1);$$

(ii) $\|x_0 - x_{-1}\| \leqslant \eta_{-1},\ \|[x_0,\ x_{-1};\ P_1]^{-1}P_1(x_0)\| \leqslant \eta_0;$

(iii) $|[u,\ v,\ w;\ K_1| \leqslant M,\ (s,\ t) \in [0,\ 1] \times [0,\ 1]$

for every $u,\ v,\ w$ *of the sphere* S *defined by the* $\|x - x_0\| \leqslant 2\eta_{-1}$;

(iv) $h_0 = (A + 1)(\eta_0 + \eta_{-1})M \leqslant \frac{1}{4}$,

Then the equation (2) has a solation x^* *in* S *being the limit of the sequence given by*

$$x_{n+1} = x_n - [x_n,\ x_{n-1};\ P_1]^{-1}P_1(x_n).$$

The rapidity of the convergence being given by the

$$\|x_n - x^*\| \leqslant \frac{1}{2^{s_{n-1}}} \cdot q^{s_{n-1}}(4h_0)^{s_n}\eta_0$$

where $0 < q < 1$, *and* s_n *is the partial sum of the Fibonacci's series with* $u_1 = u_2 = 1$.

This result can be applied to the Chandrasekar equation which arises in radiative transfer ([1] pp. 277).

REFERENCES

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