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AN APPLICATION OF THE MARKOV CHAINS AND SIMULATION TO THE STUDY OF SOME PETROCHEMICAL PROCESSES*

by

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The possibility of applying the Markov chains theory and simulation to the study of petrochemical processes is presented. By associating a finite Markov chain to a petrochemical process, the quantities of raw materials and semi-products required for the fulfilment of a given production plan are established. The problem described is also tackled through the numercial simulation method which proves advantageous as the number of outfits increases.

1. Introduction

A number of practical problems can be approached in their whole complexity only by probabilistic methods. This complexity is due to the big volume of data, to the random character of variables and to the time variation of the characteristic parameters of the respective processes.

In this paper, we shall refer to the application of the Markov chains to the analysis of the industrial production processes with a continuous character. The described application concerns the technological installations in the petrochemical industry.

In K. V. RICHARDSON's work [1], we find the first ideas about the establishment of resources in the industrial processes using the Markov chains.

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* A paper presented at the Colloquy on Constructive Theory of Functions, September 6–12, 1973, Cluj, Romania.

In the following, we shall present some notions of the finite Markov chains theory, preserving the adnotations in the study [2].

Consider a Markov chain with r states: $S_1, S_2, \ldots, S_r$. Its states can be divided into equivalence classes by introducing the partial order relation: $S_i \nearrow S_j$ (state S_i communicates with state S_j) if the process can pass from state S_i to state S_j. The maximal elements corresponding to the introduced partial order relation are called ergodic classes. The remaining elements form the transition classes.

The transition classes have the property that once abandoned, the process no longer returns to its states, while the ergodic classes have the property that its states once attained, the process no longer abandons the respective class.

The Markov chains may be classified as follows:

1) Markov chains without transition classes;

2) Markov chains with transition classes.

If all ergodic classes are made up of one element each, the chain is called an absorbing chain.

The transition matrix for an absorbing Markov chain with s transition states and $r - s$ absorbing states which are denoted first, is written in the form

$$P = \begin{Vmatrix} I & O \\ R & Q \end{Vmatrix}, \tag{1}$$

where I is a unit matrix of size $(r - s) \times (r - s)$, R is a matrix $s \times (r - s)$ whose elements are the probabilities of passing from a transition state to an absorbing one, Q is a matrix $s \times s$ whose elements are the transition probabilities between the transition states, O is a matrix $(r - s) \times s$ with all elements equal to zero. The fundamental matrix for an absorbing Markov chain is defined as follows: $N = (1 - Q)^{-1}$.

If we define n_j to be the total number of tims that the process is in a transition state, the elements of N have the following interpretation:

THEOREM [2]. *$N = \{M_i[n_j]\}$ for any transition states S_i and S_j.*

In other words, the element n_{ij} of matrix N gives the mean of the total number of tims that the process is in a transition state S_j in the hypothesis that the process started from state S_i.

For an initial probability vector π we have $\pi' N = \{M_\pi[n_j]\}$ where π' is made up of the first s components of π, corresponding to the transition states.

THEOREM [2]. *If b_{ij} is the probability that the process starting from a transition state S_i should reach an absorbing state S_j, then $\{b_{ij}\} = B = NR$.*

2. The Markov chain associated with production in a petrochemical process

In the industrial processes and especially in the petrochemical processes, the raw materials and semi-products undergo a number of processings on various installations. The materials processed on an installation I_i are directed towards the next installations I_j according to an established technological succession or, they can be directed towards the delivery points if the processings are completed.

Consider an installation I_i (fig. 1) receiving semi-products from installations $I_{\alpha_1}, I_{\alpha_2}, \ldots, I_{\alpha_p}$.

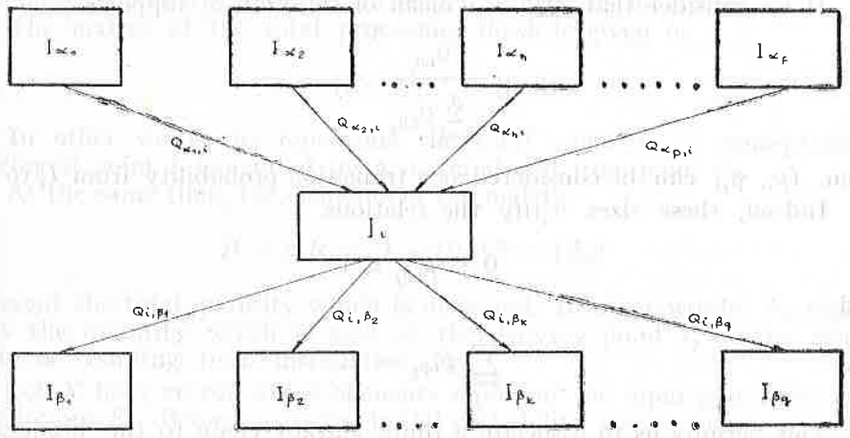

Fig. 1

Resulting from the processing are several semi-products which are directed towards the installations or delivery points $I_{\beta_1}, I_{\beta_2}, \ldots, I_{\beta_q}$.

We shall denote by Q_{ij} the quantity of the semi-product which is directed from installation I_i to installation I_j $(j \in \{\beta_1, \beta_2, \ldots, \beta_q\})$. Generally,

$$\sum_{h=1}^{p} Q_{\alpha_h,i} > \sum_{k=1}^{q} Q_{i,\beta_k} \tag{2}$$

due to the loss in outfits.

To secure, however the balance of inputs and outputs we introduce a fictitious installation $I_{\beta_{q+1}}$ or a fictitious delivery point which receives a quantity of semi-product equal to:

$$Q_{i,\beta_{q+1}} = \sum_{h=1}^{p} Q_{\alpha_h,i} - \sum_{k=1}^{q} Q_{i,\beta_k}. * \tag{3}$$

* For the sums in relations (2) and (3) to have sense, it is necessary that beforehand, all quantities Q_{ij} be expressed in the same measure units.

This, because some semi-products may be in a gasesous state (consequently expressed in volume units) or in a liquid state (consequently expressed, as usual, in weight units) or solid.

The distribution of the semi-products resulting from installation I_i can be characterized with the aid of rapports

$$p_{i,\beta_j} = \frac{Q_{i,\beta_j}}{\sum_{k=1}^{q} Q_{i,\beta_k}}. \tag{4}$$

In other words $100 \times p_{i,\beta_j}$ represents the percentage from a unit of semi-product processed on I_i and which is directed towards installation I_{β_j}. Generally, these rapports are time variables.

If we consider that p_{i,β_j} is a mean of selection of rapports

$$\frac{Q_{i,\beta_j}}{\sum_{k=1}^{q} Q_{i,\beta_k}},$$

then, (p_i, β_j) can be considered as a transition probability from I_i to I_{β_j}.

Indeed, these sizes verify the relations

$$0 \leqslant p_{i,\beta_j} \leqslant 1$$

and

$$\sum_{k=1}^{q} p_{i,\beta_k} = 1. \tag{5}$$

This permits us to associate a finite Markov chain to the production process.

It is considered that the installations are followed at various moments 1, 2, ... T. If T does not surpass a certain limit, for example 1 or 2 years, then the transition probabilities may be considered as constant (independent of time) because the specific consumptions have no significant variations in these time intervals. It results that the process is stationary. To show that the process is a Markov chain it should be pointed out that the probability for the process to be in a certain state depends only on the previous state. If we consider a particle successively covering various installations, the probability for it to pass from I_i to I_j depends on whether or not it was in installation I_i.

The states of the chain are the processing installations (real or fictitious) and the delivery points.

If I_i is a delivery point, it means that the products abandon the production process, thus I_i will represent in this case an absorbing state of the Markov chain.

If I_j represents a fictitious delivery point (irretrievable or retrievable losses for another installation outside the system) those stated above remain valid.

The fictitious installations $I_{\beta_{q+1}}$ can be considered as absorbing states if the losses are irretrievable or when the retrieval is made for being dispatched to an installation which is not part of the system.

The processing installations will represent the transition states for the Markov chain.

The petrochemical process can be characterized with the aid of a transition matrix from one installation to another, of the form (1), where Q will be a matrix of the transition probabilities between the processing installations; R will be the matrix of the transition probabilities from an installation to the delivery points, I the unit matrix expressing the fact that once delivered a product no longer returns to the system.

The matrix of the total processing flows is given by

$$N = (I - Q)^{-1} = ((n_{ij})).$$

In other words, n_{ij} represents the total quantity of semi-product arriving at point I_j generated by a material unit from point I_i.

At the same time, the elements of the matrix

$$B = NR = (I - Q)^{-1}R = ((b_{ij})$$

represent the total quantity which is delivered. In other words, b_{ij} represents the quantity which is sold at the delivery point I_j of the semi-products resulting from installation I_i.

Let X be a vector whose elements represent the input quantities into installations E_i. It represents the initial probability vector for the associated Markov chain.

Denote by Y the sales (the quantities arriving at the sale points). The following relation takes place

$$Y = X(I - Q)^{-1}R. \tag{6}$$

Resulting thereof is that the input quantities being given one can infer the quantities which will arrive at the sale points.

Reciprocally, resulting from relation (6) is that

$$X = YR^{-1}(I - Q), \tag{7}$$

which means to establish the corresponding inputs of a given delivery point.

In relation (7) matrix R, in most frequent cases, is not quadratic, thus it has no inverse R^{-1}. For this reason, in practical applications it is considered that from each installation which does not precede directly a delivery point, an infinitesimal part ε of the semi-product arrives at a fictitious delivery point.

In this way, matrix R will become quadratic with elements different from zero, only on the diagonal and whose inverse is trivial.

3. An example of petrochemical process modelled with the aid of the Markov chains

Those presented above have been applied at the Borzești Petrochemical Plant*, namely at the installations producing polyvinylchloride. 34 raw materials (methyl alcohol, mercuric chloride, sodium sulphite, sulphuric acid, etc.) and 11 semi-products (acetylene, vinyl chloride, trichloretylene, hydrochloric acid, etc.) which are obtained on 11 installations are necessary for the manufacturing of this product.

By using the Markov chains theory, the problem of the requirement of materials for a given plan has been solved, in the hypothesis of some perfect correlations between the plan and the proportions in which the products are resulting due to chemical reactions. For example, if the proportions in which the products are resulting are $r_1, r_2, \ldots, r_p$, the components of the plan vector $y = (y_1, y_2, \ldots, y_p)$ must satisfy relation:

$$\frac{y_1}{r_1} = \frac{y_2}{r_2} = \ldots = \frac{y_p}{r_p} = \lambda. \tag{8}$$

Generally, this perfect correlation is impossible. For this reason, some products are resulting in undesired quantities, a fact which may lead to exaggerated stocking expenses. To avoid it, some initial orders are diminished, hence a number of penalties may result due to the not-delivery of the respective product. The problem is to establish the production plan rendering minimum the sum of these expenses (stocking and penalty).

Let y_h^* be the quantities demanded by the beneficiaries and which does not observe relation (8) and y_h the quantities to be effectively produced by the enterprise and which observes relation (8). The difference is considered

$$\delta_h = y_h^* - y_h = y_h^* - \lambda r_h.$$

If $\delta_h > 0$ a stock of finite product results and, if $\delta_h < 0$ a shortage of finite product.

Cost c_h is attached, having the following expression:

$$c_h = D_h \quad \text{if} \quad \delta_h > 0,$$

$$c_h = P_h \quad \text{if} \quad \delta_h < 0,$$

where D_h represents the stocking cost for a unit of product h and, P_h is the penalty cost for the shortage of a unit of product h.

With these adnotations, the optimum function becomes:

$$F = \sum_h \delta_h c_h$$

which should be minimized.

* The numerical example cannot be reproduced in this paper because of the ig number of calculi.

4. Simulating by the Monte-Carlo method the technological flow in a petrochemical installation

In applying the Markov chains, the volume of calculations is proportional to the square of the number of installations. To reduce the volume of calculations in case of a big number of installations (the order of hundreds) the numerical simulation method can be used in which the volume of calculations generally has a growing linear tendency in relation to the number of installations.

We shall illustrate the numerical simulation method by considering the electrolysis process of solution NaCl from which various semi-products are resulting (hydrochloric acid, hydrogen, etc.) which are among the 11 semiproducts necessary for obtaining the vinyl polychloride.

The elementary scheme of the process of obtaining the hydrochloric acid is given in fig. 2.

The transition probabilities p_{ij} deduced according to relation (4) were written on the arces of the graph.

A particle is considered which enters „the brine purifying" installation. This particle may further pass either to „electrolysis" with a 0.95 probability or to „residue" with a 0.05 probability. We generate a random number $N_1 \in [0; 1]$ with RANDU subroutine of SSP*. If N_1 is comprised between 0 and 0.95, the particle is directed towards „electrolysis". Otherwise, the particle is directed towards „residue".

Let us admit that $N_1 \in [0; 0.95]$. We generate again a random number $N_2 \in [0; 1]$. If $N_2 \in [0; 0.20]$ from the „electrolysis" the particle is directed towards the „decomposition 1" installation. If $N_2 \in (0.20; 0.57]$ the particle will go to the „decomposition 2" installation, etc.

The described method is repeated until we come to an absorption point after which the cycle is resumed.

Due to the recirculations it is possible that the same particle should pass several times through an installation.

The number of outputs N^j is registered at each delivery point I_j. This permits for each simulation cycle of rank N to estimate the delivery probability:

$$\tilde{p}_j^N = \frac{N^j}{N}.$$

For example, in a concrete case analysed after 10 simulation cycles, the particle arrived ten times at the delivery point „residue", seven times at the delivery point „lye" and only once at the delivery point „chlorine",

* Scientific Subroutine Package (program deck for the IBM 360 computer).

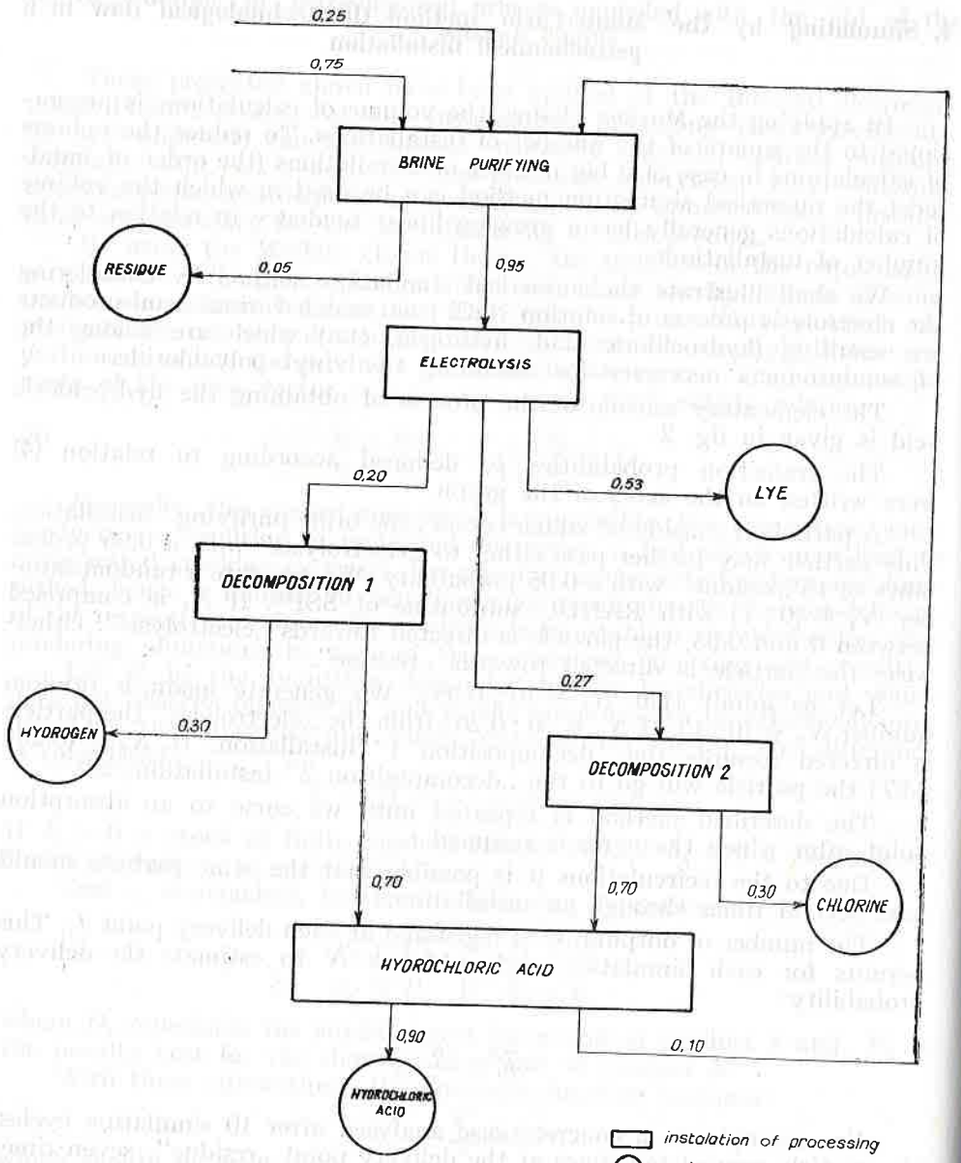

Fig. 2

respectively ,,hydrogen'' and ,,final hydrochloric acid''. In means that the probabilities estimated after 10 cycles will be:

$$\tilde{p}^{10}\ (\text{residue}) = 0,$$

$$\tilde{p}^{10}\ (\text{lye}) = \frac{7}{10} = 0{,}7,$$

$$\tilde{p}^{10}\ (\text{chlorine}) = \tilde{p}^{10}\ (\text{hydrogen}) = \tilde{p}^{10}\ (\text{hydrochloric acid}) = \frac{1}{10} = 0{,}1.$$

According to the weak law of big numbers, for $N \to \infty$ we have:

$$\lim_{N\to\infty} \tilde{p}_j^N = p_j,$$

where p_j will represent the absorption probability at the delivery point I_j. This probability shows a relation between inputs and outputs, of course by a certain degree of precision which is established beforehand.

Practically, a number N_0 of simulation cycles is established. The absorption probabilities are estimated for each cycle. The probability $\tilde{p}_j^{N_0}$ of the last cycle is considered. A relatively feasible error u_j being established from the beginning, the absolutely feasible error $\varepsilon_{j,a}^{N_0}$ is calculated with the aid of relation:

$$\varepsilon_{j,a}^{N_0} = u_j \cdot \tilde{p}_j^{N_0}.$$

We also calculate the effective error

$$\varepsilon_{j,e}^{N_0} = \frac{k \cdot \sigma_j}{\sqrt{N_0}}$$

where k is a verisimilitude coefficient (deduced with the aid of Laplace's function in keeping with the probability that the feasible error is not surpassed) and σ_j is the quadratic mean deviation of the absorption probability for the delivery point I_j.
If

$$\varepsilon_{j,a}^{N_0} \geqslant \varepsilon_{j,e}^{N_0}$$

the effectuation of the simulation cycles stops and the probabilities of the last cycle are considered absorption probabilities. Otherwise, the number of cycles is supplemented by a value δ and the calculations are repeated for $N_1 = N_0 + \delta$.

In the examined concrete case we have chosen $N_0 = 30$ cycles; $\delta = 10$ cycles; $u = 0.05$; $k = 2$ (which ensures a probability of 97% that the feasible error is not surpassed). We have considered as the main final product the hydrochloric acid for which we obtained the absorption probability (annex 1)

$$p(\text{HCl}) \cong 0{,}255$$

corresponding to which is a feasible error

$$\varepsilon_a(\mathrm{HCl}) = 0{,}05 \times 0{,}255 = 0{,}01275.$$

After 110 simulation cycles we obtained an effective error

$$\varepsilon_e(\mathrm{HCl}) = 0{,}012440 < \varepsilon_a(\mathrm{HCl}).$$

In case that several important products were considered, the verifying tests should be made for each product.

On the basis of the relations obtained between inputs and outputs, the quantities of raw material corresponding to some given inputs (and inversely) can be calculated. At the same time, on the basis of the transition probabilities from one installations to another, the extent to which the installations are being loaded can be established (annex 1).

REFERENCES

[1] Richardson, K. V., *A Markov Chain Model for Detemining Resources for Industrial Processing*. Operational Research Quarterly, **21**, *1*, 119–125 (1970).

[2] Kemeny, J. G., Snell, J. L., *Finite Markov Chains*. Van Nostrand, New York, 1959.

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TABLE OF THE ELECTROLYSIS SIMULATION CYCLES

ANNEX 1

No	Brine	Electrolyss	Decomposition 1	Decomposition 2	Hydrochloric acid import	Residue	Lye	Hydrogen	Chlorine	Pure Hydrochloric acid	Probability of having a residue	Probability of obtaining lye	Probability of obtaining hydrogen	Probability of obtaining chlorine	Probability of obtaining hydrochloric acid	Probability of having recirculations
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	47.	79.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	1.000	0 0	0.0	0.0	0.0
2	56.	24.	0.	40.	24.	0.	0.	0.	0.	1.	0.0	0.500	0.0	0.0	0.500	0.0
3	83.	83.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.667	0.0	0.0	0.333	0.0
4	52.	69.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.750	0.0	0.0	0.250	0.0
5	45.	49.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.800	0.0	0.0	0.200	0.0
6	89.	95.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.833	0.0	0.0	0.167	0.0
7	70.	69.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.857	0.0	0.0	0.143	0.0
8	77.	45.	0.	78.	0.	0.	0.	0.	1.	0	0.0	0.750	0.0	0.125	0.125	0.0
9	60.	57.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.778	0 0	0.111	0.111	0.0
10	2.	1.	86.	0.	0.	0.	0.	1	0	0	0.0	0.700	0 100	0.100	0.100	0.0
11	5.	57.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.727	0.091	0.091	0.091	0.0
12	1.	93.	0.	0.	0.	0.	1.	0	0.	0	0.0	0.750	0.083	0.083	0.083	0.0
13	51.	65.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.769	0.077	0.077	0.077	0.0
14	32.	8.	57.	0.	72.	0.	0.	0.	0.	1	0.0	0.714	0.071	0.071	0.143	0.0
15	18.	62.	0.	0.	0.	0.	1.	0.	0.	0	0.0	0.733	0.067	0.067	0.133	0.0
16	8.	97.	0.	0.	0.	0.	1.	0.	0	0.	0.0	0.750	0.063	0.063	0.125	0.0
17	4.	54.	0.	0.	0.	0.	1	0	0	0	0.0	0.765	0.059	0.059	0.118	0.0
18	92.	63.	0.	0.	0.	0.	1	0	0	0	0.0	0.778	0.056	0.056	0.111	0.0
19	50.	27	0.	19.	65.	0.	0.	0.	0.	1	0.0	0.737	0.053	0.053	0.158	0.0
20	20	38.	0	49	47	0.	0	0	0	1	0.0	0 700	0 050	0 050	0 200	0.0
21	42.	28.	0.	93	0	0	0	0	1	0	0.0	0 667	0.048	0 095	0 190	0 0
22	4	83	0	0	0	0	1	0	0	0	0 0	0 682	0 045	0 091	0.182	0.0
23	65.	41	0	65	23	0	0	0	0	1	0.0	0 652	0.043	0 087	0 217	0.0
24	49	85.	0	0	0.	0	1	0	0	0	0 0	0 667	0 042	0 083	0.208	0 0
25	74	74	0	0	0	0	1	0	0	0	0.0	0 680	0 040	0 080	0 200	0.0
26	78	4	25.	0	11.	0	0	0	0.	1	0.0	0.654	0 038	0.077	0 231	0.0
27	44.	61	0.	0	0.	0.	1.	0	0.	0	0 0	0.667	0.037	0.074	0.222	0.0
28	73	87	0	0	0	0	1	0	0	0	0 0	0 679	0.036	0.071	0.214	0 0
29	62	90	0	0.	0	0.	1	0	0	0	0 0	0.690	0 034	0 069	0 207	0.0
30	81	73	0	0	0	0	1	0	0	0	0.0	0.700	0.033	0 067	0 200	0.0
															EPS = 0.033408	
31	14	20	0	99	0	0	0	0	1	0	0.0	0.677	0.032	0.097	0.194	0.0
32	13	89	0	0	0	0	1	0	0	0	0.0	0.688	0.031	0.094	0.188	0.0
33	12	74	0	0	0	0	1	0	0	0	0.0	0.697	0 030	0.091	0.182	0.0
34	32	25	0	64	57	0	0	0	0	1	0.0	0676	0 029	0.088	0 206	0 0
35	72	15	42	0	16	0	0	0	0	1	0.0	0 657	0 029	0 086	0 229	0.0
36	23	87	0	0	0	0	1	0	0	0	0 0	0 667	0 028	0 083	0.222	0 0
37	18	25	0	93.	0	0	0	0	1	0	0.0	0 649	0.027	0 108	0216	0 0
38	31.	51	0	0	0	0	1	0	0	0	0.0	0.658	0.026	0 105	0 211	0 0
39	28	3	72	0	0	0	0	1	0	0	0.0	0.641	0.051	0 103	0 205	0 0
40	1	60	0	0	0	0	1	0	0	0	0 0	0 650	0 050	0 100	0 200	0 0
															EPS = 0 025169	
41	51	61	0	0	0	0	1	0	0	0	0.0	0.659	0.049	0 098	0 195	0.0
42	12	20	11	0	92	0	0	0	0	1	0.0	0.643	0.048	0.095	0.214	0.0
43	48.	63	0.	0.	0.	0	1	0	0	0	0.0	0.651	0.047	0.093	0.209	0 0

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
44	47.	10.	37.	0.	34.	0.	0.	0.	0.	1.	0.0	0.636	0.045	0.091	0.227	0.0
45	71.	19	76.	0.	0	0.	0.	1.	0.	0.	0.0	0.622	0.067	0.089	0.222	0.0
46	88.	43.	0.	61.	79.	0.	0.	0.	0.	1.	0.0	0.609	0.065	0.087	0.239	0.0
47	27.	51.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.617	0.064	0.085	0.234	0.0
48	61.	9.	3.	0.	43.	0.	0.	0.	0.	1.	0.0	0.604	0.063	0.083	0.250	0.0
49	26.	74.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.612	0.061	0.082	0.245	0.0
50	6.	73.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.620	0.060	0.080	0.240	0.0
														EPS = 0.024603		
51	84.	42.	0.	1.	26.	0.	0.	0.	0.	1.	0.0	0.608	0.059	0.078	0.255	0.0
52	44.	36	0.	16.	71.	0.	0.	0.	0.	1.	0.0	0.596	0.058	0.077	0.269	0.0
53	85.	71.	0	0.	0	0.	0.	1.	0.	0.	0.0	0.604	0.057	0.075	0.264	0.0
54	61.	29	0.	28.	4.	0.	0.	0.	0.	0.	0.0	0.593	0.056	0.074	0.259	0.019
55	72.	95.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.600	0.055	0.073	0.255	0.018
56	17	53.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.607	0.054	0.071	0.250	0.018
57	59.	83.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.614	0.053	0.070	0.246	0.018
58	64.	38.	0.	52.	72.	0.	0.	0.	0.	1.	0.0	0.603	0.052	0.069	0.259	0.017
59	63.	32.	0.	30.	88.	0.	0.	0.	0.	1.	0.0	0.593	0.051	0.068	0.271	0.017
60	59.	60.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.600	0.050	0.067	0.267	0.017
														EPS = 0.024602		
61	25.	13.	55.	0.	12.	0.	0.	0.	0.	1.	0.0	0.590	0.049	0.066	0.279	0.016
62	77.	55.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.597	0.048	0.065	0.274	0.016
63	42.	53.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.603	0.048	0.063	0.270	0.016
64	42.	72.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.609	0.047	0.063	0.266	0.016
65	53.	68.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.615	0.046	0.062	0.262	0.015
66	32.	83.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.621	0.045	0.061	0.258	0.015
67	9.	9.	72.	0.	0.	0.	0.	1.	0.	0.	0.0	0.612	0.060	0.060	0.254	0.015
68	51.	65.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.618	0.059	0.059	0.250	0.015
69	25.	70.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.623	0.058	0.058	0.246	0.014
70	91.	23.	0.	15.	83.	0.	0.	0.	0.	1.	0.0	0.614	0.057	0.057	0.057	0.014
														EPS = 0.019759		
71	61.	21.	0.	76.	0.	0.	0.	0.	1.	0.	0.0	0.606	0.056	0.070	0.254	0.014
72	65.	8.	63.	0.	9.	0.	0.	0.	0.	0.	0.0	0.597	0.056	0.069	0.250	0.028
73	85.	27.	0.	99.	0.	0.	0.	0.	1.	0.	0.0	0.589	0.055	0.082	0.247	0.027
74	49.	6.	96.	0.	0.	0.	0.	1.	0.	0.	0.0	0.581	0.068	0.081	0.243	0.027
75	22.	68.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.587	0.067	0.080	0.240	0.027
76	10.	47.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.592	0.066	0.079	0.237	0.026
77	93.	32.	0.	55.	41.	0.	0.	0.	0.	1.	0.0	0.584	0.065	0.078	0.247	0.026
78	54.	57.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.590	0.064	0.077	0.244	0.026
79	52.	2.	43.	0.	40.	0.	0.	0.	0.	1.	0.0	0.582	0.063	0.076	0.253	0.025
80	57.	81.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.587	0.063	0.075	0.250	0.025
														EPS = 0.016512		
81	75.	21.	0.	50.	8.	0.	0.	0.	0.	0.	0.0	0.580	0.062	0.074	0.247	0.037
82	2.	37.	0.	4.	91.	0.	0.	0.	0.	1.	0.0	0.573	0.061	0.073	0.256	0.037
83	10.	35	0.	25.	32.	0.	0.	0.	0.	1.	0.0	0.566	0.060	0.072	0.265	0.036
84	66.	6.	45.	0.	16.	0.	0.	0.	0.	1.	0.0	0.560	0.060	0.071	0.274	0.036
85	90.	96.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.565	0.059	0.071	0.271	0.035
86	68.	42.	0.	41.	68.	0.	0.	0.	0.	1.	0.0	0.558	0.058	0.070	0.279	0.035
87	44.	51.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.563	0.057	0.069	0.276	0.034
88	4.	68.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.568	0.057	0.068	0.273	0.034
89	71.	19.	71.	0.	0.	0.	0.	1.	0.	0.	0.0	0.562	0.067	0.067	0.270	0.034
90	56.	96.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.567	0.067	0.067	0.067	0.033
														EPS = 0.016500		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
91	76.	94	0	0	0.	0	1	0	0	0	0.0	0.571	0.066	0.066	0.264	0.033
92	78	20	14	0.	10.	0.	0	0	0	0	0.0	0.565	0.065	0.065	0.261	0.043
93	28.	83	0	0.	0	0	1	0	0	0	0.0	0.570	0.065	0.065	0.258	0.043
94	42.	7.	64.	0.	22	0.	0.	0	0	1	0.0	0.564	0.064	0.064	0.266	0.043
95	58.	44	0	48	88	0	0	0	0	1	0.0	0.558	0.063	0.063	0.274	0.042
96	96	0	0	0	0	1.	0.	0	0	0	0.010	0.552	0.063	0.063	0.271	0.042
97	88	66.	0	0.	0	0	1	0	0	0	0.010	0.557	0.062	0.062	0.268	0.041
98	1	13	68	0	95.	0.	0	0	0	1	0.010	0.551	0.061	0.061	0.276	0.041
99	54	71	0	0	0	0.	1.	0	0	0	0.010	0.556	0.061	0.061	0.273	0.040
100	35.	73.	0	0	0	0.	1	0	0	0	0.010	0.560	0.060	0.060	0.070	0.040
														EPS = 0.015230		
101	25.	88.	0.	0.	0.	0.	1.	0	0	0	0.010	0.564	0.059	0.059	0.267	0.040
102	6.	42	0.	100.	0	0	0	0	1	0	0.010	0.559	0.059	0.069	0.265	0.039
103	15.	92.	0	0	0	0	1	0	0	0	0.010	0.563	0.058	0.068	0.262	0.039
104	21	97.	0	0	0	0.	1	0	0	0	0.010	0.567	0.058	0.067	0.260	0.038
105	93.	85.	0.	0	0	0.	1	0	0	0.	0.010	0.571	0.057	0.067	0.257	0.038
106	73.	72.	0.	0.	0.	0	1	0	0	0	0.009	0.575	0.057	0.066	0.255	0.038
107	75.	1.	33.	0.	90.	0.	0.	0	0	1.	0.009	0.570	0.056	0.065	0.262	0.037
108	45.	60.	0.	0.	0.	0	1	0	0.	0	0.009	0.574	0.056	0.065	0.259	0.037
109	51.	64.	0.	0.	0.	0.	1.	0.	0.	0.	0.009	0.578	0.055	0.064	0.257	0.037
110	30.	98.	0.	0.	0	0.	1	0.	0.	0.	0.009	0.582	0.055	0.064	0.255	0.036
														EPS = 0.012440		

The quantities of raw materials and finite products corresponding to 1253 tons of hydrochloric acid

Raw material

Salt = 1185.8750 tons
Water = 3557.6250 tons

Finite products

Residue = 44.7500 tons
Lye = 2864.0000 tons
Hydrogen = 268.5000 tons
Chlorine = 313.2500 tons

Loading on installations

Installation

Brine purifying = 5101.5000 tons
Electrolysis = 4877.7500 tons
Decomposition 1 = 895.0000 tons
Decomposition 2 = 1118.7500 tons
Hydrochloric acid = 1432.0000 tons