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ACADÉMIE DE LA RÉPUBLIQUE SOCIALISTE DE ROUMANIE FILIALE DE CLUJ-NAPOCA

> MATHEMATICA – REVUE D'ANALYSE NUMÉRIQUE ET DE THÉORIE DE L'APPROXIMATION

L'ANALYSE NUMÉRIQUE LA THÉORIE DE L'APPROXIMATION

TOME 9

Nº 2

1980

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MATHEMATICA - REVUE D'ANALYSE NUMÉRIQUE ET DE THÉORIE DE L'APPROXIMATION

L'ANALYSE NUMÉRIQUE ET LA THÉORIE DE L'APPROXIMATION Tome 9, Nº 2, 1980, pp. 157—162

CONES IN A CONVEXITY SPACE;
ORDERED CONVEXITY SPACES

A convexity space (1;c) is an independent place of the intervence of the convexity space.

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GABRIILLA CRISTESCU

(Cluj-Napoca)

1. In this paper we define the notion of cone in a convexity space (in the sense of v. w. BRYANT and R. J. WEBSTER [2], [3], [4], [5]) and we study the properties of some types of cones. Also, we define the concept of ordered convexity space and, using the cones theory in a convexity space, we give an example of ordered convexity space in the sense of the definition we'll give. This order relation is similar to the order relation induced by a convex cone in a Banach space.

2. Preliminaries

We denote, as in [2], by A, B, ... sets, by a, b, ... both the elements of a set and the singleton sets. Thus we use, as in [2], the sign \subset instead of \in except when a set is a member of a family of sets. The notation $A \approx B$ means $A \cap B \neq \emptyset$ and (a, b, c, \ldots) stands for the set formed by the elements a, b, c, ...

Let X be a nonempty set. We endow X with an operation $\cdot: X \times X \to 2^X$, called the product or join of a and b when a, $b \subset X$, and the inverse operation $f: X \times X \to 2^X$, defined by $a/b = (x \subset X: a \subset bx)$, for all a, $b \subset X$. The couple (X, .) is called a convexity space if it satisfies the following axioms:

- i) $ab \neq \emptyset$, $a|b \neq \emptyset$; we will discuss a set A to A to A to A
- (ii) a(bc) = (ab)c;
 - (iii) $a/b \approx c/d \Rightarrow ad \approx bc$;
- (iv) aa = a = a/a;
- (v) $ab \approx ac \Rightarrow b = c \text{ or } b \approx ac \text{ or } c \approx ab$, for any $a, b, c, d \subset X$. Here ab denotes $a \cdot b$.

For a, $b \subset X$, ab means the open line segment having its ends in a and in b, and a/b the half-line having the origin in a and not containing b. The product and its inverse are extended to subsets of X by defining

$$AB = \bigcup_{\substack{a \in A \\ b \in B}} ab, \qquad A/B = \bigcup_{\substack{a \in A \\ b \in B}} a/b$$

A set $A \subset X$ is said to be convex if $AA \subset A$ and linear if $A \mid A \subset A$. If $A \subset X$ we denote by [A] the convex hull of A, i.e. the intersection of all convex sets which contain A and by $\{A\}$ the linear hull of A, i.e. the intersection of all linear sets which contain A. For example, [a, b] = $= ab \cup a \cup b$, $\{a, b\} = a \mid b \cup a \cup ab \cup b \cup b \mid a$. [a, b] is the closed line segment joining a and b and $\{a, b\}$ is the line which contains a and b.

The concepts of independent set, basis, dimension for a convexity space are defined in the natural way, as in liniar spaces (see [4]).

A convexity space (X, .) is said to be complete if whenever $A \subset X$ is convex, $a \subset A$ and $b \not\subset A$, there exists $c \subset [a, b]$ such that $ac \subset A$ and $bc \subset X \setminus A$. A set $H \subset X$, $H \neq \emptyset$, $H \neq X$, is a hyperplane in (X, ...)if H is linear and $\{H \cup x\} = X$ for any $x \subset X \setminus H$. The unordered pair (C, D), $C \subset X$, $D \subset X$, $C \neq \emptyset$, $D \neq \emptyset$ is said to be a convex pair in X if C, D are convex sets, $C \cap D = \emptyset$ and $C \cup D = X$. The reader may find in [3] many properties concerning the relation between hyperplanes and convex pairs.

A topological convexity space (X, τ) is a convexity space (X, τ) with a topology τ satisfying:

(it) $a \subset \overline{ab}$ for all $a, b \subset X$; (iit) if $ab \approx U \in \tau$ then there exist V, $W \in \tau$ with $a \subset V$, $b \subset W$ and such that $a'b' \approx U$ whenever $a' \subset V$, $b' \subset W$;

(iiit) if $a/b \approx U \in \tau$ then there exist V, $W \in \tau$ with $a \subset V$, $b \subset W$ and such that $a'/b' \approx U$ whenever $a' \subset V$, $b' \subset W$.

3. Cones in a convexity space

Let (X,.) be a convexity space.

Definition 1. Let θ be a point of X and $S \subset X$. We call cone generated by S and having the vertex in θ the set

$$\mathcal{C}_0 S = (x \subset X : \exists y \subset S, \ x \subset y \cup \theta y \cup y / \theta \cup \theta)$$

In some places it is not necessary to put in evidence the set S which generates the cone. Thus we'll call cone a set K which was obtained as in definition 1, i.e. K is a cone if there exist a point $\theta \subset X$ and a set $S \subset X$ such that $K = \mathcal{Q}_{\theta}S$. It is obvious that a cone K can be obtained using an infinity of sets S.

If $S \subset \mathbb{R}^n$, $\theta = 0$, $\mathcal{Q}_{\theta}S$ is convex, closed and doesn't contain linear sets except the singleton sets then $\mathcal{Q}_0 S$ is a cone in sense of [9]. If $S \subset \mathbb{R}^n$, $\dot{\theta} = 0$, and $\mathcal{Q}_0 S$ is convex then $\mathcal{Q}_0 S$ is a wedge in sense of [7].

Definition 2. A cone $K \subset X$ is said to be a generating cone if for every $x \subset X$ there exist $u, v \subset K$ such that $\{0, x\} \approx \{u, v\}$, where θ is the vertex of the cone K.

:3

In linear spaces this definition is equivalent to the definition of a generating cone given using the group structure of the space: if Xis a Banach space then the convex cone $K \subset X$ (in sense of [9]) is a generating cone if for every $x \subset X$ there exist $u, v \subset K$ such that x == u - v. In order to proove this equivalence we suppose first that for every $x \subset X$ there exist $u, v \subset K$ such that x = u - v. Then, using the properties of vector operations in X we get that the line which contains the points u and v is parallel to the line which contains the origin of the space and the point x. Conversly, if $x \subset X$ and there exist u, $v \subset K$ such that $\{0, x\} \approx \{u, v\}$ than let $z \subset K$ such that $z/x \subset K$. Let $t \subset z/x$ and $v \subset K$ such that $\{t, y\} \approx \{0, x\}$ (this y exists because of the convexity of K). But on the line $\{t, y\}$ there exist a point s such that $\{0, s\} \approx \{x, z\}$. The points s and t satisfy the property that x = t - s.

Definition 3. The cone $K \subset X$ is a convex cone if KK = K. The theorems which follow are transpositions fo some classical results from the vectorial space case. Here is now a theorem of the Carathéodory type:

THEOREM 1. If (X,.) is a convexity space, $\dim X = n$, $\theta \subset X$ and $S \subset X$ such that $\mathcal{Q}_0 S$ is a convex cone and $\dim \mathcal{Q}_0 S = d$, then for every $x \subset \mathcal{C}_0 S$ there exist independent points $y_i \subset S$, $i = 1, 2, \ldots, p, p \leq d$, such that $x \subset K$, where K is the intersection of all convex cones having the vertex in θ and including the points y_i , $i = 1, 2, \ldots, p$.

Proof. $x \subset \mathcal{Q}_{\theta}S$ means that there exist $y \subset S$ such that $x \subset \theta y \cup S$ $(Jy)/(Jy)/\theta$. Applying for y the theorem 20 from [4] we get that there exist independent points, $y_1, y_2, \ldots, y_p \subset S$, $p \leq d$, such that $y \subset [0, y_1, \ldots, y_p]$. If we put $K = \mathcal{Q}_0[y_1, \ldots, y_p]$ then we get the cone required by theorem 1 required by theorem 1.

THEOREM 2. If (X, .) is a convexity space, τ a Hausdorff topology on X and K C X a cone containing interior points (a full cone) then K is a generating cone.

Proof. Let $x \subset X$ and $u \subset K$. This means that there exist a neighborhood V of the point u, $V \subset K$, $V \neq K$. Let $z \subset X$ such that $\{z, u\} \approx$ $\approx \{0, x\}$. Then there exist $v \subset \{z, u\} \cap V$. u and v are the points we looked for. ooked for.

4. Ordered convexity spaces

Definition 4. An ordered convexity space $(X, ., \leq)$ is a convexity space (X, .) with an order relation, \leq , satisfying:

(io) if $a \le b$ then for every $x \subset ab$, $a \le x \le b$;

(iio) if $a \le b$ then for every $x \subset a|b$ and $y \subset b|a$, $x \le a \le b \le y$ (iiio) if a and b are not comparable then any $x, y \subset \{a, b\}$ are also not comparable.

It's easy to verify that $(\mathbf{R}^n, \cdot, \leq)$, where \cdot is defined by $a \cdot b =$ $= (x \subset \mathbb{R}^n : \exists \lambda, \ 0 < \lambda < 1, \ x = \lambda a + (1 - \lambda)b)$ and \leq is the order relation induced by the cone of the nonnegative elements from R", is an ordered convexity space.

We remark that the axioms (io) and (iio) show that a linear set $\{a, b\}$, where $a \leq b$, in an ordered convexity space is isomorphic to a line containing two points which are comparable by the order relation induced by the cone of nonnegative elements from R" or to a part of such a line.

THEOREM 3. If (X, .) is a convexity space and $K \subset X$ is a convex cone which doesn't contain linear sets (except the singleton sets) then the binary relation \leq_{κ} defined on X by:

 $1^{\circ} x \leq_{\kappa} x \text{ for any } x \subset X;$

 2° x, $y \subset X$, $x \leq_K y$ if there exist $z \subset y/x$ such that $z/y \subset K$, is a partial order on X, and (X, \cdot, \leq_K) is an ordered convexity space.

Proof. 1° implies that \leq_K is reflexive.

Antisymmetry. Let $x, y \subset K$ with $x \leq_K y$, $y \leq_K x$ and $x \neq y$. Than $y \mid x \subset K$ and $x \mid y \subset K$. From the convexity of K it follows that $xy \subset K$, than we have $\{x, y\} = x \cup y \cup xy \cup x/y \cup y/x \subset K$ which is a contradiction with the hypotesis that K doesn't contain lines.

If x, $y \subset X \setminus K$ with $x \leq_K y$, $y \leq_K x$ and $x \neq y$ than there exist $z_1 \subset y/x$ and $z_2 \subset x/y$ such that $z_1/y \subset K$ and $z_2/x \subset K$. But

 $z_1/y = z_1/x \subset y/x \subset \{x, y\}, \qquad z_2/x = z_2/y \subset x/y \subset \{x, y\}.$

Let $u \subset z_1/y$ and $v \subset z_2/x$. Than $uv \subset K$ because of the convexity of K. Since $z_2 \subset vx$, $z_1 \subset uy$, $y \subset z_1x$ and $x \subset z_2y$ we have $y \subset uxy$ and $x \subset vxy$ then $xy \subset uvxy$. This can take place only if $xy \subset uv$. But $xy \subset uv$. $\subset X \setminus K$ and $uv \subset K$, which is a contradiction with the convexity of K. The case $x \subset X$, $y \subset K$ is similar.

Transitivity. Let x, y, $z \subset K$, $x \leq_K y$, $y \leq_K z$. Than we have $y/x \subset K$, $z/y \subset K$. Than $(y/x)(z/y) \subset K$. If $y \subset \{z, x\}$ than $x \leq_K z$. If $y \not\subset \{z, x\}$ let $u \subset z/y$ and $v \subset y/x$. Hence $z \subset uy$ and $y \subset vx$ and we have $zy \cup uvxy = uvx$. This means that $z \subset uvx$ and than there exist $w \subset z/x \cap uv$. But $uv \subset z/x \cap uv$. $\subset (y/x)(z/y) \subset K$, than $w \subset K$. Because u and v was arbitrarily choosed on z/y respectively on y/x, by a convenient choice of these points we find that every $w \subset z/x$, $w \subset K$, and than $z/x \subset K$, hence $x \leq_K z$. If $x, y, z \subset$ $\subset X \setminus K$, $x \leq_K y$, $y \leq_K z$ than there exist $a \subset y/x$ and $b \subset z/y$ such that $a/y \subset K$ and $b/z \subset K$. A similar argument for a and b proves that $z/x \subset K$ C(y/x)(z/y) and hence there exist $c \subset z/x$ such that $c/z \subset K$, than we have also $x \leq_{\mathbf{K}} z$.

We use analogous arguments for the cases x, $y \subset X \setminus K$, $z \subset K$ and $x \subset X \setminus K$, y, $z \subset K$. The axioms (io), (iio), (iiio) are now obvious.

Remark. If X is a Banach space and K a convex cone in X the order relation defined in theorem 3 is equivalent to: if $x, y \subset K$ or $x \subset X \setminus K$, $y \subset K$, $x \leq_K y$ if $y - x \subset K$, and if $x, y \subset X \setminus K$, $x \leq_K y$ if $y - x \subset K^\circ$.

LEMMA 1. If (X, \cdot, τ) is a complete topological convexity space locally convex and Hausdorff, K C X a convex full cone which doesn't contain linear sets (except the singleton sets) and $x, y \subset X$ with $x \leq_K y$ than there exist two neighborhoods $U \in \mathcal{O}(x)$ and $V \in \mathcal{O}(y)$ such that if $a \subset U$ and $b \subset V$ than either $a \leq_K b$ or a and b are not comparable by \leq_K .

Proof. Let us suppose that $x, y \subset K$. Then let $K' \subset K$ a convex full cone such that $y \subset K'$, $x \not\subset \hat{K}'$ (see [2]) and $x \leq_{K'} y$. Since τ is Hausdorff and X is locally convex there exist a convex and open neighborhood $U \in \mathcal{O}(x)$ such that $U \cap K' = \emptyset$. Than, the theorem of Eidelheit type (see [1]) shows us that there exist a hyperplane H which separates the sets U and K'. Let $V \in \mathcal{O}(y) \cap \tau$, $a \subset U$ and $b \subset V$. Let $H = \widehat{C} \cap \widehat{D}$ where \widehat{C} and \widehat{D} are the closed half-spaces associated with H (see [3]) and let us suppose that $K' \subset \widehat{C}$. Than $b/a \subset \widehat{C}$ and we have either $b/a \subset K' \subset$ $\subset K$, $b/a \subset K$ or there exist a $z \subset b/a$ such that $z/a \subset \widehat{C} \setminus K$. Hence we have either $a \leq_{\kappa} b$ or a and b are not comparable $by \leq_{\kappa} c$.

If $x \subset X \setminus K$, $y \subset K$ we take K' = K and we use the same reasoning. If $x \subset X \setminus K$, $y \subset X \setminus K$ we take instead of K' the set $K'' = [y \cup K]$ and we use the same separating theorem and the same argument. The lemma is now completly prooved.

THEOREM 4. If (X, \cdot, τ) is a complete topological convexity space, locally convex, Hausdorff, K C X a convex cone which doesn't contain linear sets (except the singleton sets), $(x_n)_{n=1}^{\infty} \subset X$, $(y_n)_{n=1}^{\infty} \subset X$ with $x_n \leq_K \leq_K y_n$, $n=1, 2, \ldots$ and if $x_n \to x$ and $y_n \to y$ $(n \to \infty)$ than either $x \leq_K y$ or x and y are not comparable by \leq_{κ} .

Proof. Let us suppose that the assertion $x \leq_K y$ is false. Than either $y \leq_K x$ or x and y are not comparable by the order relation induced by K. Since the topology τ is Hausdorff and X locally convex than there exist $U \in \mathcal{O}(x)$, $V \in \mathcal{O}(y)$ convex and open such that $U \cap V = \emptyset$. Than, from [1] it follows that the sets U and V can be separated by a hyperplane H. Let $a \subset xy \cap H$. By the hypothesis it follows that there exist the convex and open neighborhoods $U' \in \mathcal{O}(x)$, $V' \in \mathcal{O}(y)$, $A \in \mathcal{O}(a)$ such that Lemma 1 and (iit) take place. This means that for any $u \subset U'$ and $v \subset V'$ we have either $v \leq_{K} u$ or v and u are not comparable $by \leq_{K}$. But since $x_n \to x$ and $y_n \to y$ when $n \to \infty$ it follows that there exist a $n_0 \subset N$ such that $x_n \subset U'$ and $y_n \subset V'$ for $n \ge n_0$. Hence for $n \ge n_0$ we have either $y_n \leq_K x_n$ or y_n and x_n are not comparable by \leq_K , which is a contradiction with the hypothesis $x_n \leq_K y$ for every $n \subset N$.

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NECESSARY OPTIMALITY CRITERIA IN NONLINEAR PROGRAMMING IN COMPLEX SPACE WITH DIFFERENTIABILITY

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DOREL I. DUCA (Cluj-Napoca)

In this paper we consider the problem

(P) Minimize $\operatorname{Re} f(\mathbf{z}, \overline{\mathbf{z}})$ subject to $\mathbf{z} \in X$, $\mathbf{g}(\mathbf{z}, \overline{\mathbf{z}}) \in S$,

where X is a nonempty open set in \mathbb{C}^n , S is a polyhedral cone in \mathbb{C}^m , $f: X \times \overline{X} \to C$ and $g: X \times \overline{X} \to \mathbb{C}^m$.

The paper is devided into four sections. In Section 1 notation is introduced and some preliminary results are given. In Section 2 we establish a necessary condition of the Fritz John type for Problem (P). In Section 3 seven kinds of complex constraint qualification (CCQ) are given and relations between them are established. In Section 4 we prove a Kuhn-Tucker type necessary condition for Problem (P).

1. Notation and Preliminary Results

Let \mathbf{C}^n (\mathbf{R}^n) denote the *n*-dimensional complex (real) vector space with Hermitian (Euclidean) norm $||\cdot||$, $\mathbf{R}^n_+ = \{\mathbf{x}/\mathbf{x} = (x_j) \in \mathbf{R}^n, \ x_j \ge 0, \ j = 1, \ldots, n\}$ the non-negative orthant of \mathbf{R}^n , and $\mathbf{C}^{m \times n}$ the set of $m \times n$ complex matrices.

If **A** is a matrix or vector, then \mathbf{A}^T , $\overline{\mathbf{A}}$, \mathbf{A}^H denote its transpose, complex conjugate and conjugate transpose respectively. For $\mathbf{z} = (z_j)$, $\mathbf{w} = (w_j) \in \mathbb{C}^n$; $\langle \mathbf{z}, \mathbf{w} \rangle = \mathbf{w}^H \mathbf{z}$ denotes the inner product of \mathbf{z} and \mathbf{w} and $\operatorname{Re} \mathbf{z} = (\operatorname{Re} z_j) \in \mathbb{R}^n$ denotes the real part of \mathbf{z} .