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ON A MEASURE — THEORETICAL CONCEPT OF CONVEXITY

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1. Let X be a compact Hausdorff space, $M(X)$ the set of all Radon measures on X, $M_+(X)$ the set of all positive Radon measures on X and $M_+^1(X)$ the set of all μ in $M_+(X)$ for which $\mu(1) = 1$.

DEFINITION 1. *We say that the convex cone $S \subset C(X)$ is separable if there exists a countable set $H \subset S$ such that $S \subset \overline{H}$, the closure being considered in the uniform norm.*

EXAMPLES. a) If $M \subset C(X)$ is a countable set, then the convex cone generated by M is separable.

b) If X is metrizable, then every convex cone $S \subset C(X)$ is separable.

Let $S \subset C(X)$ be a convex cone. If μ, ν are in $M(X)$ write $\mu \leqslant_S \nu$ if $\mu(s) \leqslant \nu(s)$ for each s in S. This relation is clearly transitive and reflexive.

For x in X let ε_x be the evaluation functional at x. Let $\hat{S}$ be the set of all lower semicontinuous functions $f: X \to (-\infty, +\infty]$ for which

$$x \in X,\ \mu \in M_+(X),\ \mu \leqslant_S \varepsilon_x \Rightarrow \mu(f) \leqslant f(x) \tag{1}$$

(Concerning these functions see [3], [4]).

Lest $\check{S}$ be the set of all functions in $\hat{S}$ for which

$$x \in X,\ \mu \in M_+(X),\ \mu \leqslant_S \varepsilon_x,\ \mu|_S \neq \varepsilon_x|_S \Rightarrow \mu(f) < f(x) \tag{2}$$

EXAMPLES. a) Let X be a metrizable compact convex subset of a locally convex space E, and let

$$S = \{\min(h_1, \ldots, h_n) : n \in \mathbf{N},\ h_1, \ldots, h_n \in (E^* + R)|_X\}.$$

Then $\hat{S} = \{f : X \to (-\infty, +\infty] : f$ a l.s.c. concave function$\}$ (see [3]), and $\check{S} = \{f : X \to (-\infty, +\infty] : f$ a l.s.c. strictly concave function$\}$ (see the lemma in [7]).

b) Let n be a nonnegative integer and let $S \subset C[a, b]$ be the cone of all continuous nonconvex functions of order n. If $g \in C[a, b]$ is a concave function of order n, then g is in $\check{S}$ (see [8], theorem 1).

2. In the example a) above S and $\check{S}$ are disjoint.

PROPOSITION 1. *If $S \subset C(X)$ is a convex separable cone, then $\bar{S} \cap \check{S}$ is nonvoid.*

Proof (see [3], [4]). Let s_n be in S, $s_n \neq 0$, such that $S \subset cl\{s_n : n$ in $N\}$. Then the function $f = \sum_{n=1}^{\infty} \frac{1}{2^n} \frac{s_n}{\|s_n\|}$ is in $\bar{S} \cap \check{S}$.

Let $S \subset C(X)$ be a convex cone which contains a function $s > 0$, and let μ be in $M_+(X)$. A map $x \to T_x$ of X into $M_+(X)$ is said to be a S — dilation with respect to μ if for every f in $C(X)$ the function $x \to T_x(f)$ is μ-measurable and for every s in S the relation $T_x(s) \leq s(x)$ holds a.e.μ. (see [3]). Let μT be defined by

$$\mu T(f) = \int_X T_x(f) d\mu(x), \qquad f \text{ in } C(X). \tag{3}$$

LEMMA 1 ([3]). *Let S be a convex cone which contains a function $s > 0$, and suppose that $\bar{S}$ is min-stable. If μ and ν are in $M_+(X)$, $\nu \leq_S \mu$, then there exists a S-dilation T with respect to μ such that $\nu = \mu T$.*

PROPOSITION 2. *Let $S \subset C(X)$ be a convex separable cone, μ, ν in $M_+(X)$, $\nu \leq_S \mu$, f in $\bar{S} \cap \check{S}$ such that $\nu(f) = \mu(f)$. If T is a S-dilation with respect to μ, for which $\mu T = \nu$, then there exists $B \subset X$, $\mu(B) = 0$, such that $T_x|_S = \varepsilon_x|_S$ for each x in $X \setminus B$.*

Proof. Let s_n be in S such that $S \subset cl\{s_n : n$ in $\mathbf{N}\}$, and let t_n be in S such that $t_n \to f$. Then there exists $A \subset X$, $\mu(A) = 0$ such that for every x in $X \setminus A$ and every n in N:

$$T_x(t_n) \leq t_n(x) \tag{4}$$

$$T_x(s_n) \leq s_n(x) \tag{5}$$

From (4) it follows

$$T_x(f) \leq f(x), \qquad x \text{ in } X \setminus A. \tag{6}$$

Since $\mu(f) = \nu(f) = \mu T(f) = \int_X T_x(f) d\mu(x)$, we have

$$\int_X T_x(f) d\mu(x) = \int_X f(x) d\mu(x) \tag{7}$$

From (6) and (7) we deduce

$$T_x(f) = f(x) \qquad \text{a.e.}\mu. \tag{8}$$

From (5) it follows $T_x(s) \leq s(x)$ for each s in S and x in $X \setminus A$. Thus,

$$T_x \leq_S \varepsilon_x \text{ for each } x \text{ in } X \setminus A. \tag{9}$$

From (8) and (9) it follows that there exists $B \subset X$, $\mu(B) = 0$, such that for every x in $X \setminus B$ we have $T_x(f) = f(x)$ and $T_x \leq_S \varepsilon_x$. But f is in $\check{S}$, thus $T_x|_S = \varepsilon_x|_S$ for each x in $X \setminus B$, q.e.d.

THEOREM 1. *Let S be a convex separable cone wich contains a function $s > 0$. Suppose that $\bar{S}$ is min-stable. If μ is in $M(X)$, $0 \leq_S \mu$, $\mu|_S \neq 0$, then $\mu(f) > 0$ for every f in $\bar{S} \cap \check{S}$.*

Proof. Since f is in $\bar{S}$, it follows $\mu(f) \geq 0$. Let $\mu(f) = 0$. We have $\mu = \mu^+ - \mu^-$ where μ^+, μ^- are in $M_+(X)$ and $\mu^- \leq_S \mu^+$. Then there exists a S-dilation T with respect to μ^+ such that $\mu^- = \mu^+ T$ (see lemma 1). Since $\mu^-(f) = \mu^+(f)$, from proposition 2 we deduce

$$T_x|_S = \varepsilon_x|_S \quad \text{a.e.}\mu^+. \text{ If } s \text{ is in } S, \text{ then}$$

$$\mu^-(s) = \mu^+ T(s) = \int_X T_x(s) d\mu^+(x) = \int_X \varepsilon_x(s) d\mu^+(x) = \mu^+(s).$$

Thus $\mu(s) = 0$ for each s in S. So $\mu|_S = 0$, a contradiction.

COROLLARY 1. *Let X be a metrizable compact convex subset of a locally convex space, and let S be the cone of all continuous concave functions on X. If λ, ν are in $M(X)$, $\lambda \leq_S \nu$, $\lambda \neq \nu$, and if f is a strictly concave continuous function on X, then $\lambda(f) < \nu(f)$.*

Proof. It suffices to apply theorem 1 for $\mu = \nu - \lambda$.

COROLLARY 2. *Let $S \subset C(X)$ be a convex separable cone, μ in $M_+(X)$ and T a S-dilation with respect to μ. If $\mu T|_S = \mu|_S$ then there exists $B \subset X$, $\mu(B) = 0$ such that $T_x|_S = \varepsilon_x|_S$ for each x in $X \setminus B$.*

Proof. Let f be in $\bar{S} \cap \check{S}$. Then $\mu T(f) = \mu(f)$, and we can apply proposition 2.

Let $S \subset C(X)$ be a convex cone. We say that μ in $M_+(X)$ is a S-minimal measure if

$$\nu \in M_+(X), \qquad \nu \leq_S \mu \Rightarrow \nu|_S = \mu|_S. \tag{10}$$

Let μ be in $M_+(X)$ and $f : X \to R$. Denote $Q_\mu(f) = \inf\{\mu(s) : f \leq s \in S\}$.

G. MOKOBODZKI [6] has obtained the following characterization of the S-minimal measures:

PROPOSITION 3. *Let $S \subset C(X)$ be a convex cone which contains a function $s > 0$. Then μ in $M_+(X)$ is a S-minimal measure if and only if $Q_\mu(t) = \mu(t)$ for each t in $-S$.*

From theorem 1 we will obtain:

COROLLARY 3. *Let $S \subset C(X)$ be a convex cone which contains a function $s > 0$, and suppose that S is min-stable. Let μ be in $M_+(X)$ and t in $-(S \cap \check{S})$. Then μ is a S-minimal measure if and only if $Q_\mu(t) = \mu(t)$.*

Proof. There exists ν in $M_+(X)$, $\nu \leqslant_S \mu$ such that $Q_\mu(t) = \nu(t)$ (see [3], lemma 1.3). If μ is a S-minimal measure, then $\nu|_S = \mu|_S$; it follows $\mu|_S = \nu|_S$ (see [4], prop. 1.7). Thus $\mu(t) = \nu(t) = Q_\mu(t)$.

Let now $Q_\mu(t) = \mu(t)$. By Zorn's lemma, there exists a S-minimal measure λ in $M_+(X)$ such that $\lambda \leqslant_S \mu$. Then $\lambda(t) \geqslant \mu(t)$ (see [4], prop. 1.7). It follows $Q_\mu(t) = \mu(t) \leqslant \lambda(t) \leqslant Q_\mu(t)$. Hence $\mu(t) = \lambda(t)$. By theorem 1, $(\mu - \lambda)|_S = 0$. Thus $\mu|_S = \lambda|_S$ and we deduce immediately that μ is a S-minimal measure.

3. Let Q be a metrizable compact space. Let $T: C(Q) \to C(Q)$ be a positive linear operator such that $T^2 = T$ and $T1 = 1$. Denote $H = T(C(Q))$. Then the subspace H contains the constant functions and $H = \{h \in C(Q) : Th = h\}$. Suppose that H separates the points of Q. Denote $\Gamma = \{f \in C(Q) : f \leqslant Tf\}$.

Then Γ is a closed convex cone in $C(Q)$, and Γ is max-stable. Moreover, $H = \Gamma \cap (-\Gamma)$, and $\Gamma - \Gamma$ is a dense subspace of $C(Q)$ (see [9]).

Let T^* be the adjoint of T. If we denote by ∂H the Choquet boundary of H, we have (see [9]):

$$\partial H = \bigcap_{f \in \Gamma} \{x \in Q : \varepsilon_x(f) = T^*\varepsilon_x(f)\}. \tag{11}$$

F. ALTOMARE [2] has proved:

PROPOSITION 4. a) *There exists a function φ in Γ such that:*

$$H = \{x \in Q : \varepsilon_x(\varphi) = T^*\varepsilon_x(\varphi)\}. \tag{12}$$

b) *Let $H(\varphi)$ be the linear subspace of $C(Q)$ generated by H and φ. If μ, ν are in $M_+(Q)$, $\nu|_{H(\varphi)} = \mu|_{H(\varphi)}$, and ν is maximal with respect to $\leqslant_\Gamma$, then $\nu = \mu$.*

Let now S be the closed, convex, min-stable cone generated by H in $C(Q)$. Then S contains the constant functions and separates the points of Q; $S - S$ is a dense subspace of $C(Q)$. Q being metrizable, S is separable. Then $S \cap \check{S}$ is nonvoid.

PROPOSITION 5. *Let φ be in $-(S \cap \check{S})$; then $\varphi \in \Gamma$ and $\partial H = \{x \in Q : \varepsilon_x(\varphi) = T^*\varepsilon_x(\varphi)\}$. If μ, ν are in $M_+(Q)$, $\nu \leqslant_S \mu$ and $\nu(\varphi) = \mu(\varphi)$, then $\nu = \mu$.*

Proof. Clearly $S \subset -\Gamma$, hence φ is in Γ. From (11), $\partial H \subset \{x \in Q : \varepsilon_x(\varphi) = T^*\varepsilon_x(\varphi)\}$. Let now x be in Q such that $\varepsilon_x(\varphi) = T^*\varepsilon_x(\varphi)$. It is known (see [9]) that $T^*\varepsilon_x$ is the unique maximal (with respect to $\leqslant_\Gamma$) measure which majorizes ε_x. From $\varepsilon_x \leqslant_\Gamma T^*\varepsilon_x$ it follows $T^*\varepsilon_x \leqslant_S \varepsilon_x$. But φ is in $\check{S}$, and therefore $T^*\varepsilon_x|_S = \varepsilon_x|_S$.

Since $\overline{S - S} = C(Q)$, we deduce $T^*\varepsilon_x = \varepsilon_x$. It follows that x is in the Choquet boundary of the cone Γ, which coincides with ∂H (see [9], corollary 31). Thus x is in ∂H.

The last statement of proposition 5 is a consequence of theorem 1.

REMARK. If f is in Γ and $\partial H = \{x \in Q : \varepsilon_x(f) = T^*\varepsilon_x(f)\}$, it does not follow that f is in $-\check{S}$. For example, let $Q = [0, 1]$, $T : C[0, 1] \to C[0, 1]$

$$Tg(x) = (1 - x)\, g(0) + x\, g(1).$$

Then H is the space of all affine continuous functions on $[0, 1]$ and $\partial H = \{0, 1\}$. S is the set of all continuous concave functions on $[0, 1]$, and $\check{S}$ is the set of all continuous strictly concave functions on $[0, 1]$. Let f be defined by

$$f(x) = \begin{cases} 1 - 3x, & x \text{ in } [0, 1/3] \\ 0 \quad , & x \text{ in } (1/3, 2/3) \\ 3x - 2, & x \text{ in } [2/3, 1] \end{cases}$$

Then $f \leqslant Tf$, hence f if in Γ. Moreover, $\partial H = \{0, 1\} = \{x \in [0, 1] : \varepsilon_x(f) = T^*\varepsilon_x(f)\}$. Clearly f is not in $-\check{S}$.

4. Next we use the following notations (X being a compact Hausdorff space and $S \subset C(X)$ a convex cone):

$$\partial_S H = \{x \in X : \mu \in M_+(X),\ \mu \leqslant_S \varepsilon_x \Rightarrow \mu = \varepsilon_x\}$$

$$x \in X,\ [x] = \{y \in X : \varepsilon_y|_S = \varepsilon_x|_S\}$$

$$\Delta_S X = \{x \in X : \mu \in M_+(X),\ \mu \leqslant_S \varepsilon_x \Rightarrow \operatorname{supp} \mu \subset [x]\}$$

$$d_S X = \{x \in X : \mu \in M_+(X),\ \mu \leqslant_S \varepsilon_x \Rightarrow \mu|_S = \varepsilon_x|_S\}$$

$$s \in \check{S},\ M(s) = \{x \in X : s(x) = \min_X s\}$$

$$M_S X = \bigcup_{s \in S} M(s)$$

DEFINITION 2. *Let T be a set of lower semicontinuous functions from X into $(-\infty, +\infty]$. We say that a subset F of X is a boundary for T if for each t in T there exists x in F such that $t(x) = \min_X t$.*

B. FUCHSSTEINER [5] has proved that $\Delta_T X$ is a boundary for T. If the convex cone $S \subset C(X)$ contains the constant functions and separates the points of X, then $\partial_S X$ is a boundary for S (Bauer's Minimum Principle, see [1]).

DEFINITION 3. *A closed subset A of X is called S-absorbent if*

$$x \in A,\ \mu \in M_+(X),\ \mu \leqslant_S \varepsilon_x \Rightarrow \mu(X \setminus A) = 0 \tag{13}$$

It is easy to verify that the intersection of an arbitrary family of S-absorbent sets is also a S-absorbent set. For x in X we denote by A_x the smallest S-absorbent set which contains x.

LEMMA 2. *Let x be in $\Delta_S X$. If there exist s_1, s_2 in S such that $s_1 > 0$, $s_2(x) < 0$, then A_x is a minimal (with respect to the inclusion) S-absorbent set, and $A_x \subset d_S X$.*

Proof. We use the following notations (see [4]):

$$(14)\qquad T_x = \bigcup \{\text{supp } \mu : \mu \in M_+(X),\ \mu \leqslant_S \varepsilon_x\}$$

$$(15)\qquad N_x = \{y \in X : \text{there exist } a,\ b > 0 \text{ such that } a\varepsilon_y \leqslant_S \varepsilon_x,\ b\varepsilon_x \leqslant_S \varepsilon_y\}.$$

It is easy to verify that $N_x \subset T_x \subset A_x$. Moreover, the following assertions are equivalent (see [4]):

(i) A_x is a minimal S-absorbent set

(ii) $s(y) = s(x)s_1(y)$ for any s in S and any y in A_x

(iii) $N_x = T_x$

Clearly $[x] \subset N_x \subset T_x$. If μ is in $M_+(X)$ and $\mu \leqslant_S \varepsilon_x$, then supp $\mu \subset [x]$; it follows that $T_x \subset [x]$, i.e. $N_x = T_x$. Hence A_x is a minimal S-absorbent set, and from (ii):

$$(16)\qquad \varepsilon_y|_S = s_1(y)\varepsilon_x|_S \text{ for each } y \text{ in } A_x.$$

Let us show that $A_x \subset d_S X$. Let y be in A_x. If μ is in $M_+(X)$ such that $\mu \leqslant_S \varepsilon_x$, then from (16) we deduce $\mu \leqslant_S s_1(y)\varepsilon_x$, hence $\mu/s_1(y) \leqslant_S \varepsilon_x$. It is easy to verify that $\Delta_S X \subset d_S X$, therefore x is in $d_S X$. This implies:

$$(17)\qquad \left.\frac{\mu}{s_1(y)}\right|_S = \varepsilon_x\Big|_S$$

So we obtain $\mu|_S = s_1(y)\varepsilon_x|_S = \varepsilon_y|_S$, and therefore y is in $d_S X$ q.e.d.

THEOREM 2. *Let $S \subset C(X)$ be a convex separable cone which contains the constant functions. Then:*

a) $\partial_S X \subset \Delta_S X \subset M_S X \subset d_S X$, and $M_S X$ is a boundary for $\hat{S}$.

b) if min $(s, 0)$ is in S for any s in S, then $\Delta_S X = M_S X = d_S S$.

c) if min $(s, 0)$ is in S for any s in S and if S separates the points of X, then $\partial_S X = \Delta_S X = M_S X = d_S X$.

Proof. a) Clearly $\partial_S X \subset \Delta_S X$. Let x be in $\Delta_S X$. By lemma 2, $A_x \subset d_S X$. Let g be in $\bar{S} \cap \check{S}$ (see prop. 1). Then there exists $a > 0$ such that $f = 1 + ag > 0$; clearly f is in $\check{S}$. Define the function s by

$$(18)\qquad s(y) = \begin{cases} 0, & y \text{ in } A_x \\ f(y), & y \text{ in } X \setminus A_x. \end{cases}$$

Then $s \geqslant 0$ and $A_x = \{y \in X : s(y) = 0\}$. A_x being S-absorbent, it is easy to verify that s is in $\check{S}$. Let y be in X and let μ be in $M_+(X)$ such that $\mu \leqslant_S \varepsilon_y$ and $\mu(s) = s(y)$. If y is in A_x, then y si in $d_S X$ i.e. $\mu|_S = \varepsilon_y|_S$. If y is in $X \setminus A_x$, then $f(y) = s(y) = \mu(s) \leqslant \mu(f) \leqslant f(y)$, hence $\mu(f) = f(y)$. Since f is in $\check{S}$, it follows $\mu|_S = \varepsilon_y|_S$. We conclude that s is in $\check{S}$. Furthermore $A_x = M(s) \subset M_S(X)$, and this implies that x is in $M_S X$. So the inclusion $\Delta_S X \subset M_S X$ is proved.

Let now x be in $M_S X$. Then there exists s in $\check{S}$ such that x is in $M(s)$. Let μ be in $M_+(X)$, $\mu \leqslant_S \varepsilon_x$. Then $\mu(1) = 1$. Furthermore $s(x) \geqslant \mu(s) \geqslant \mu(s(x) \cdot 1) = s(x)$. Hence $\mu(s) = s(x)$, so $\mu|_S = \varepsilon_x|_S$. We conclude that x is in $d_S X$, i.e. $M_S X \subset d_S X$.

Let us show that $M_S X$ is a boundary for $\hat{S}$. First we will prove:

$$(19)\qquad s \in \hat{S},\ s|_{M_S X} \geqslant 0 \Rightarrow s \geqslant 0.$$

Let g be in $\bar{S} \cap \check{S}$. Then there exists $a > 0$ such that $f = 1 + ag > 0$. Clearly f is in $\bar{S} \cap \check{S}$. Let s be in $\hat{S}$, $s|_{M_S X} \geqslant 0$. Then $s + cf$ is in $\check{S}$ and $(s + cf)|_{M_S X} > 0$ for any real number $c > 0$. It follows $s + cf > 0$ for any $c > 0$, hence $s \geqslant 0$. So (19) is proved.

Let now s be in $\hat{S}$, $\min_X s = m$. If $s|_{M_X S} > m$, then $s - m$ is in $\hat{S}$ and $(s - m)|_{M_S X} > 0$. From (19) and from [3; theorem 2.5] it follows $s - m > 0$, a contradiction. So $M_S X$ is a boundary for $\hat{S}$.

b) Suppose that min $(s, 0)$ is in S for any s in S. We will prove that $d_S X \subset \Delta_S X$. Let x be in $d_S X$, and μ in $M_+(X)$ such that $\mu \leqslant_S \varepsilon_x$. Let s be in in S. Denote $t = \min(s - s(x), 0)$. Then t is in S and $t \leqslant 0$. Since x is in $d_S X$, it follows $\mu|_S = \varepsilon_x|_S$, hence $\mu(t) = t(x) = 0$. This implies $t|_{\text{supp}\,\mu} = 0$. So we obtain:

$$(20)\qquad s(y) \geqslant s(x) \text{ for each } y \text{ in supp } \mu$$

Furthermore $\mu(1) = 1$, hence $\mu(s - s(x)) = \mu(s) - s(x) = 0$. Together with (20), this implies

$$(21)\qquad s(y) = s(x) \text{ for each } s \text{ in } S \text{ and } y \text{ in supp } \mu.$$

Therefore $\varepsilon_y|_S = \varepsilon_x|_S$ for each y in supp μ. We deduce supp $\mu \subset [x]$, i.e. x is in $\Delta_S X$.

c) Suppose that min $(s, 0)$ is in S for any s in S, and suppose that S separates the points of X. We will prove that $d_S X \subset \partial_S X$. Let x be in $d_S X$, μ in $M_+(X)$, $\mu \leqslant_S \varepsilon_x$. Then $\mu|_S = \varepsilon_x|_S$. If s is in S, then min$(s, 0)$ is in S, hence μ (min $(s, 0)$) = min $(s(x), 0)$. By applying the theorem 2.8 form [3], we deduce that x is in $\partial_S X$, q.e.d.

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