L'ANALYSE NUMÉRIQUE ET LA THÉORIE DE L'APPROXIMATION Tome 10, N° 2, 1981, pp. 201-204

therefore W. is harmonic function. He will start 8 man of Also, Wellington to confer the will be seen on THEOREM 3 If we is a solution of constant (1) then the function

Vivial Through direct calculation we have

SOME PROPERTIES OF SOLUTIONS OF ANTON S. MURESAN EQUATION $\Delta^4 u = 0$ (II)

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1. In the paper [3] it was studied some properties of the functions which are solutions of equation which are solutions of equation is harmonic, for any farrery within trail in the the meaning lines in the fill meaning shreatened

Properties when the three particular
$$\Delta^4 u = 0$$
, it is the majority of the Association (1) and the state of the state of

where Δ is Laplace's operator.

The purpose of this note is to give another alike properties, as aprioric $W_{\bullet} = W_{\bullet} = W_{\bullet} = W_{\bullet}$ estimates.

The notations and notions are those used in the paper [3].

2. In this section we deduce some triharmonic, biharmonic and harmonic functions in the domain D from the solutions of equation (1).

It holds the following

THEOREM 1. If w = w(x, y, z) is a solution of equation (1) then the tion

$$(2) W_1 = \Delta w_x - \frac{1}{4} (x - a) \Delta^2 w$$

is biharmonic, for any $(a, y, z) \in D$.

Proof. Through direct calculation one finds that

$$\Delta^{2}W_{1} = \Delta^{2} \left(\Delta w_{x} - \frac{1}{4} (x - a) \Delta^{2}w \right) = \Delta^{3}w_{x} - \frac{1}{4} \Delta^{2} [(x - a) \Delta^{2}w] = 0,$$

which shows that W_1 is biharmonic function.

Thus, it holds the following

THEOREM 2. If w is a solution of equation (1) then the function

(3)
$$W_2 = \Delta w_{xx} - \frac{1}{4} \Delta^2 w - \frac{1}{2} (x - a) \Delta^2 w_x + \frac{1}{8} (x - a)^2 \Delta^3 w$$

is harmonic, for any $(a, y, z) \in D$.

Proof. Through direct calculation we have

$$\Delta W_2 = \Delta^2 w_{xx} - \frac{1}{4} \Delta^3 w - \frac{1}{2} \Delta [(x-a) \Delta^2 w_x] + \frac{1}{8} \Delta [(x-a)^2 \Delta^3 w] = 0,$$

therefore W_2 is harmonic function.

Also, we have

THEOREM 3 If w is a solution of equation (1) then the function

$$(4) W_3 = \left| \nabla W_1 - \frac{1}{2} r \cdot \Delta W_1 \right|$$

is subharmonic, for any $(a, y, z) \in D$.

Proof. The Theorem 5 of [1] must be applied to the biharmonic function W_1 .

THEOREM 4 If the function w is solution of equation (1) then the function

(5)
$$W_4 = W_1 - 2r \cdot \nabla W_1 + \frac{1}{2} r^2 \cdot \Delta W_1$$

is harmonic, for any $(a, y, z) \in D$.

Proof. The Theorem 6 of [1] can be used to the biharmonic function W₁.

THEOREM 5. If the function w is solution of equation (1) then the function

(6)
$$W_{5} = \left| \nabla W_{0} - \frac{1}{2} r \cdot \Delta W_{0} \right|$$

is a subharmonic function, for any $(a, y, z) \in D$, where W_0 is the function

(7)
$$W_0 = w_{xx} - \frac{1}{6} \Delta w - \frac{1}{3} (x - a) \Delta W_x + \frac{1}{24} (x - a)^2 \Delta^2 w.$$

Proof. Because W_0 is the biharmonic function of Theorem 2 of [3], we can apply the Theorem 5 of [1] to it.

THEOREM 6. If the function w is solution of equation (1) then the function

(8)
$$W_{6} = W_{0} - 2r \cdot \nabla W_{0} + \frac{1}{2} r^{2} \Delta W_{0}$$

is a harmonic function, for any $(a, y, z) \in D$.

Proof. We can apply the Theorem 6 of [1] to the biharmonic function

3. In this section we give some aprioric estimates for the functions which are solutions of equation (1). which shows that W. is librarmonic function.

Thus, it holds the

THEOREM 7. Let w be a solution of equation (1), which have all partial derivatives until the 6-th order continuous on D and on the boundary Γ of domain D. Then, for any point $(a, b, c) \in D$ the estimate

(9)
$$\Delta w_{xx}(a, b, c) - \frac{1}{4} \Delta^2 w(a, b, c) \leq \max_{\Gamma} W_2$$

is true. 1 an municipal de

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Proof. We can apply the common maximum principle for the harmonic function W_2 of Theorem 2.

Also, it holds the

THEOREM 8. Under the assumtions of Theorem 7 the estimate

$$|\nabla \Delta w_x(a, b, c)| \leq \max_{\Gamma} W_3$$

is true. INTERPOLATING CONVERS PERLANDIALS Proof. We can apply the maximum principle for the subharmonic function $(\Delta u \ge 0)$ in the case of subharmonic function W_3 of the Theorem 3. THEOREM 9. Under the assumptions of Theorem 7 the estimate

(11)
$$\Delta w_x(a, b, c) \leq \max_{\Gamma} W_4$$

is true. The property of a supplication of the property of the Proof. We can apply the common maximum principle in the case of harmonic function W_4 of the Theorem 4. THEOREM 10 Under the assumptions of Theorem 7 the estimate

(12)
$$\left|\nabla\left[w_{xx}(a, b, c) - \frac{1}{6}\Delta w(a, b, c)\right]\right| \leqslant \max_{\Gamma} W_{5}$$

Proof. We can apply the maximum principle for subharmonic functions in the case of subharmonic function W_5 of the Theorem 5.

THEOREM 11 Under the assumptions of Theorem 7 the estimate

(13)
$$w_{xx}(a, b, c) = \frac{1}{6} \Delta w(a, b, c) \leq \max_{\Gamma} W_{6}$$

Proof. We can apply the common maximum principle in the case of harmonic function $W_{\mathfrak{g}}$ of the Theorem 6.

REMARK. The Theorem 11 is just the Theorem 8 of [3] in another words.

REFERENCES

[1] Duffin, R. J., The maximum principle and biharmonic functions, J. Math. Anal. Appl., 3, 399-405 (1961).

[2] Mureşan, A., Application of maximum principle to triharmonic functions, Mathematica, 18 (41), 1, 103-109 (1976). [3] Mureşan, A., Some properties of solution of equation $\Delta^4 u = 0$, Mathematica, L'Analyse

Num. Th. l'Approx., 8, 2, 187-192 (1979).

Received 16.III.1981.

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Proof. We can apply the common maximum principle in the case of

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 $w_{ee}\left(a, b, \frac{1}{a}, \frac{11/a}{a}, \frac{1}{a} \Delta w(a, b), \frac{1}{a} \Delta w(a, b), \chi \leq \min_{\theta} W_{\theta}$

Proof. We can study the common maximum trin (planta the ruse of national lunction if a life theorem is. REMARKS. The Theppenp 111 is juga the Theorem S of 31 in another

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[14] Dulffin, R. L. The maximum principle and Ministrational functions, I. State Condi-Nact. 3, 899-405 (1993).