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ON A NUMERICAL COMPUTATION IN THE THEORY OF PLANE SUBSONIC GAS JETS

by

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In this note the results concerning the examination of the numerical computations done by S. K. ASLANOV and V.A. LEGKOVA [1] for the problem of a gas jet going out from a container of finite width are presented. The general solution of this problem has been done by S. V. FAL'KOVICH [2] As a result of our computations we obtained a five decimal table. These numbers give the dependence of the coefficient of contraction of the jet on the relative width of the orifice. The entries of the four decimal table from [1, 4] are pointed out where the values of the global characteristic elements of the flow are wrong.

1. Formulation of the problem

Consider a plane irrotational subsonic flow of a jet gas which goes out from a symmetric container of finite width, represented in fig. 1. Let us denote by $\mathbf{v}_1$ the velocity at the infinity inside the container; $\mathbf{v}_2$ stands for the velocity on the boundary of the jet; L is the width of the container; $\mathbf{l}$ is the width of the orifice; b stand for the width of the jet at the infinity. With these notations the ratio $\mathbf{l}/L$ is called the relative width of the orifice and $b/\mathbf{l} = \mathbf{k}$ is said the coefficient of contraction of the jet. The solution of the problem has the form [2]

$$(1) \qquad \frac{\mathbf{l}}{L} = \frac{1}{\mathbf{k}} \frac{v_1}{\mathbf{v}_2} \left(\frac{1 - \tau_1}{1 - \tau_2} \right)^{\beta}$$

$$(2) \qquad \frac{1}{\mathbf{k}} = 1 + \frac{8}{\pi} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n \mathbf{x}_n(\tau_2)}{4n^2 - 1} + \frac{8}{\pi} \left(\frac{1 - \tau_2}{1 - \tau_1} \right)^{\beta} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n^2 - 1} \frac{\tau_1 \mathbf{Z}'_n(\tau_1)}{\mathbf{Z}_n(\tau_2)}$$

where $\tau_i = v_i^2/v_{max}^2$ ($i = 1, 2$; v_{max} is the maximum possible velocity of the flow from the container); $\beta = 1/(\chi - 1)$ where χ is the ratio between the specific temperatures (for the air $\beta = 2.5$); $\mathbf{Z}_n(\tau)$ is the Chaplygin function [3]; $\mathbf{x}_n(\tau) = \frac{\tau}{n} \mathbf{Z}_n'(\tau)/\mathbf{Z}_n(\tau)$ is the logarithmic derivative of the Chaplygin function.

The fourdecimal table of the values of $\mathbf{l}/L$ and $\mathbf{k}$ as functions of τ_1 and τ_2 has been given in [1] and, without any changes, it was reproduced in M.I. Gurevich monograph [4], page 425.

By verifying the computations from [1] we found that at some points (τ_1, τ_2) the third significant decimal of the values of $\mathbf{l}/L$ and $\mathbf{k}$ are wrong and at (0.04; 0.08) these values are not the true ones.

In the table reproduced below we retained only those points (τ_1, τ_2) which exist in [1, 4], where the absolute error of the computations from [1] overpass $5 \cdot 10^{-4}$. The corresponding values τ_1 from table are underlined. The case of the flow from the pipe, when $\tau_1 = \tau_2$, is not included in table because in this case obviously $\mathbf{l}/L = 1$ and $\mathbf{k} = 1$.

In the paper [1] the values of the functions $\mathbf{Z}_n(\tau)$ and $\mathbf{Z}_n'(\tau)$ for $n \leqslant 7$ were taken from FERGUSSON and LIGTHILL's tables [5] and for $n > 7$ asymptotic formulae were used.

2. Results of the improved computations

We performed new calculations for the given problem using the formulae (1) and (2); the results of these calculations are represented in the table.

In order to perform the calculation of the functions $\mathbf{Z}_n(\tau)$ and $\mathbf{Z}_n'(\tau)$ we used a specially elaborated algorithm which guaranteed and accuracy of the calculation of these functions not smaller than 8 true significant decimals. On the basis of this method we gave the tables of Chaplygin functions [6] which considerably extent and complete the well-known tables [5].

In order to calculate the coefficient of contraction $\mathbf{k}$ we used 10 terms of the series (2) and we performed a transformation which increased the convergence of this series. The increase of the convergence was obtained by means of two different methods and the results of these computations have been compared.

The first method. The increase of the convergence of the series by means of Shanks formula [7]

$$e(S_n) = \frac{S_{n-1}S_{n+1} - S_n^2}{S_{n-1} + S_{n+1} - 2S_n} \tag{3}$$

This nonlinear transformation were applied first to the sequence of the partial sums of the series (2) $\{S_1, \ldots, S_{10}\}$ and the sequence $\{e(S_2), \ldots, e(S_9)\}$ and so on until two close values of $\mathbf{k}$ were reached.

The second method. The increase of the convergence of the series (2) was obtained by the method of arithmetic averages. To the sequence $\{S_1, \ldots, S_{10}\}$ we applied the transformation

$$u(S_n) = \frac{S_n + S_{n+1}}{2} \quad (n = 1, \ldots, 9). \tag{4}$$

Then, we applied the same transformation to the sequence of arithmetic averages $\{u(S_1), \ldots, u(S_9)\}$ and so on until one value which gives with a high accuracy the value of the sum of the series (2) was reached.

The above-mentioned calculations were performed at the highspeed omputer FELIX-C 256 of the Center for Computation of the University of Bucharest.

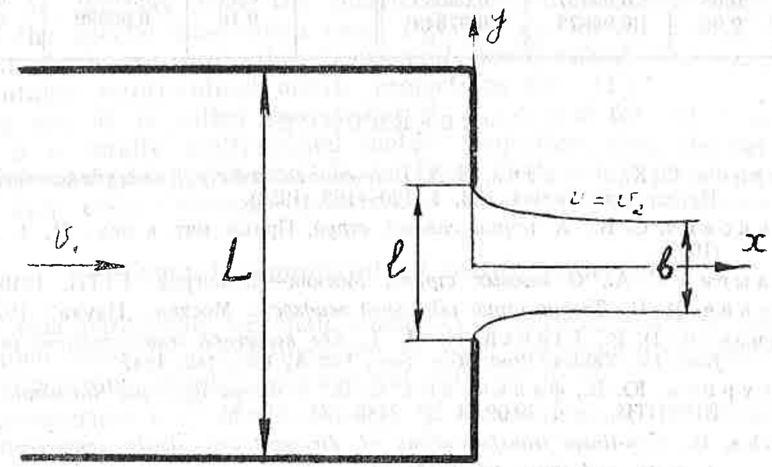

Fig. 1

TABLE

τ_2	τ_1	$1/L$	k
0,02	*0,0025*	0,55582	0,66487
	0,005	0,73415	0,70742
	0,01	0,91036	0,79670
	0,015	0,98171	0,89346
0,04	0,005	0,56914	0,67938
	0,01	0,74719	0,72269
	0,015	0,85187	0,76657
	0,025	0,95917	0,85680
	0,03	0,98379	0,90340
	0,035	0,99635	0,95112
0,06	0,005	0,48927	0,68013
	0,015	0,76047	0,73902
	0,03	0,92522	0,82671
	0,05	0,99423	0,94279
0,08	0,005	0,44160	0,68865
	0,015	0,69955	0,73418
	0,03	0,87329	0,80042
	0,04	0,93271	0,84323
	0,05	0,96809	0,88484
	0,07	0,99741	0,96353
0,1	0,005	0,40997	0,70094
	0,015	0,65692	0,73878
	0,03	0,83284	0,79309
	0,05	0,94025	0,86088
	0,07	0,98439	0,92253
	0,08	0,99407	0,95059
	0,09	0,99873	0,97649

τ_2	τ_1	$1/L$	k
0,12	0,005	0,38767	0,71578
	0,015	0,62595	0,74868
	0,03	0,80178	0,79550
	0,05	0,91615	0,85316
	0,07	0,96920	0,90478
	0,09	0,99201	0,94932
	0,10	0,99700	0,96853
	0,11	0,99937	0,98548
0,14	0,005	0,37136	0,73271
	0,01	0,50825	0,74765
	0,03	0,77790	0,80400
	0,05	0,89651	0,85493
	0,07	0 95539	0 90005
	0,09	0,98388	0,93858
	0,10	0,99141	0,95509
	0,12	0,99857	0,98200
	0,13	0,99972	0,99216
1/6	0,005	0,35580	0,75833
	0,015	0,58062	0,78483
	0,03	0,75444	0,82205
	0,05	0,87658	0,86703
	0,07	0,94067	0,90646
	0,10	0,98418	0,95403
	0,12	0,99518	0,97707
	0,14	0,99919	0,99241
	0,15	0,99981	0,99702
	0,16	0,99999	0,99952

REFERENCES

1. Асланов, С. К., Легкова, В. А., *Истечение газовой струи изсосуда конечной ширины*, Прикл. мат. и мех., 23, I, 190—193 (1959).
2. фалькович, С. В., *К теории газовых струй*, Прикл. мат. и мех., 21, 4, 459—464, (1957).
3. Чаплыгин, С. А., *О газовых струях*, Москва—Ленинград, ГТТИ, 1949, 144 стр.
4. Гуревич, М. И., *Теория струй идеальной жидкости*, Москва, ,,Наука'', 1979, с. 425.
5 Fergusson, D. F., Lighthill, M. J., *The hodograph transformations in transonic flow*, IV, Tables, Proc. Roy. Soc., **192 A**, 135—142, 1947.
6. Сунгурцев, Ю. В., фалькович, С. В., *Таблицы функций Чаплыгина*, Москва, ВИНИТИ, деп. 19.09.74 № 2489—74, 51 стр.
7 Shanks, D., *Non-linear transformations of divergent and slowly convergent sequences*, J. Math. and Phys., **34**, 1955.

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