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THRESHOLD SELECTION BASED ON IMAGE APPROXIMATION

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The problem of automatic threshold selection is considered. After a brief review of the available techniques, a novel method is proposed. It is based on an image approximation and can be computed using only information from the gray-level histogram. A threshold is selected corresponding to the gray value of the highest peak from an adequate histogram modification. The effectiveness of the method is illustrated in some practical examples.

1. Introduction. The selective isolation of interesting objects from a digital gray-scale picture represents a common problem in digital picture analysis. This task is part of the more comprehensive field of picture segmentation where for each pixel, it has to be decided to what class of picture points it belongs. In the simplest case of objects isolation, just two classes are required : the class of objects and the class of background (non-object) points. Objects are compact regions corresponding to physical entities from a picture. These regions are often characterized by well-defined borders and the contrast of the interior texture with a surrounding texture. Not all scenes conform to this model, but it is applicable to a variety of imaging environments including thermal imagery analysis, chromosome classification, military target detection, industrial parts inspection. In automatic microscope picture analysis for the investigation of cytological preparations three pixel classes are required for the decision between nucleus, cytoplasm and background. Such a decision must be realized by consideration of gray level values alone. The most currently used segmentation criterion is based on the determination of gray-level thresholds. Generally, thresholds are associated to extrema of a first-order gray-level statistics which is the gray-level histogram. Many different techniques have been used to select good thresholds in a picture. Some of them are described in Weszka's [13] survey article. Several threshold selection methods have been proposed over the last years [1, 2, 3, 5]. Threshold selection involves the choice of a gray level t such that all gray levels greater than t are mapped into the object label, while the other levels are mapped into the background label. Ideally, this thresholds will be the mean gray values of the border points of the objects. Thus, the threshold is chosen usually as a lowest point between two major peaks on the gray-level histogram [7], or as a highest peak in the gradient histo gram [11]. An alternative method of threshold selection is the approximation of the given gray-level histogram by a linear combination of standard probability densities, e.g. Gaussian, and the choice of the minimum error threshold defined by these Gaussians. This fitting procedure is difficult and time-consuming.

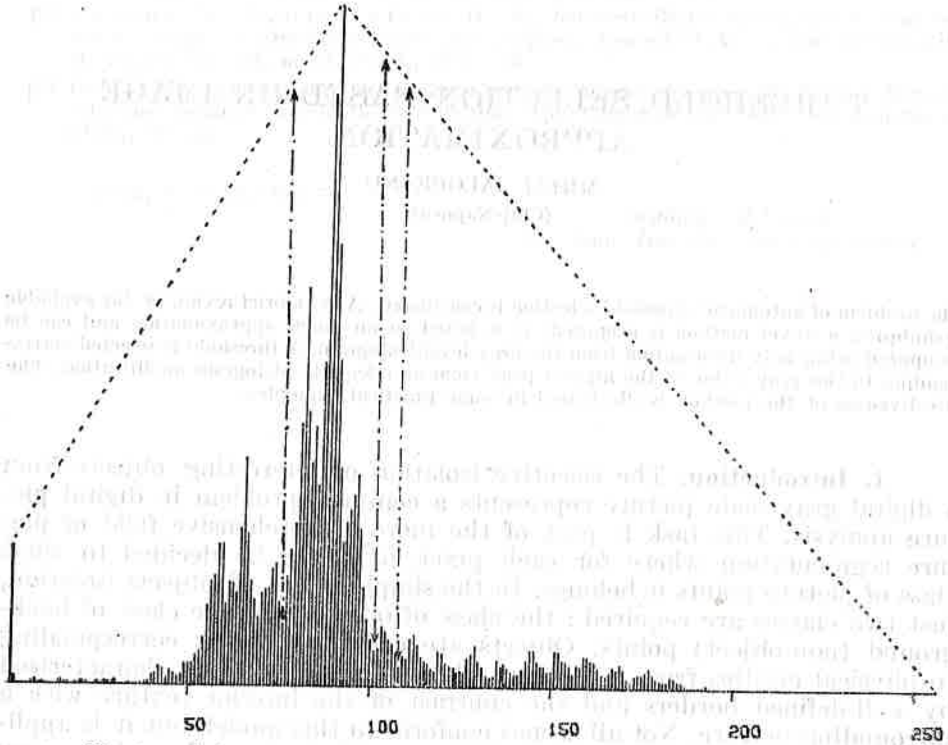

Fig. 1. — A histogram, the convex hull and candidate threshold value.

Histogram valleys, which are usually good locations for threshold, are associated with concavities on the histogram. Thus, constructing the convex hull of the histogram, the threshold can be located as the deepest concavity point, i.e. the maximum of the vertical difference between the convex hull and histogram [1,8].

Another approach to threshold selection is the solution of the equation

$$(1)\qquad 2t = \sum_{k=0}^{t-1} kH(f)_k \Big/ \sum_{k=0}^{t-1} H(f)_k + \sum_{k=t}^{P-1} kH(f)_k \Big/ \sum_{k=t}^{P-1} H(f)_k$$

where $H(f)_k$ is the number of pixels with the gray level k in the picture f, for $k = 0,1,\ldots,P-1$ [2,6, 9, 5].

Otsu suggested the calculation of the between-class separation (BCS) of the gray-level distribution [4] as a suitable measure for the candidate threshold value. The best threshold is that for which BCS is a maximum. Otsu has motivated this by considering the theory of discriminant analysis but he also showed that this method is equivalent to minimizing the

mean square error between the gray-level picture and its binary representation obtained by thresholding at gray level t. Reddy et al. [5] have recently suggested the equivalence of the Otsu BCS condition for threshold selection with solving equation (1). They generalize the method to the case of two thresholds selection and construct an iterative algorithm which can be used to select one or two stable thresholds. They did not observe that the algorithm is the same with that formulated in Trussell's comment on Ridler's iterative method [6, 9]. We have extended and used this schema [2] to select six threshold values for reproduction purposes of gray-level pictures to the graphic printer.

Weszka, Nagel and Rosenfeld [14] suggest the determination of a histogram for only those pixels having high Laplacian magnitude. In the new histogram, the threshold is selected by the valley seeking method. Weszka and Rosenfeld [15] suggests the choice of a threshold value as that point near the histogram valley between the two peaks which minimizes the business. For a given threshold, business is the percentage of pixels having a neighbour whose thresholded value is different from their own thresholded value. Watanabe suggests the choice of a threshold value which maximizes the sum of gradients taken over all pixels whose gray level equals the threshold value. Kohler and Wang, and Haralick [11] suggest different modification of Watanabe's idea.

This paper presents a new method for threshold selection based on gray-level histogram analysis. This method is motivated by considering an adequate representation of the picture obtained by thresholding at gray level t, and minimizing the mismatch measure between the original picture and this representation. Corresponding to each of the two mismatch measures, namely the Euclidian distance and city block distance between two pictures, two thresholds are obtained.

2. Preliminaries. Generally, the selection of a threshold value is based upon an analysis of the gray-level histogram of the given image. Gradient approach or other histogram midification scheme are time-consuming procedures. Unfortunately, the simplest procedures like valley seeking or histogram concavity analysis cannot be used in situations where there are no significant peaks in the histogram or there exist many small peaks in the histogram, etc. Also, the multithreshold procedure cannot be derived by this single threshold procedure.

Our attempt is to formulate new methods of threshold selection, minimizing the error between the picture f, and correspondingly defined thresholding at t. We shall use the following mismatch measures [7, vol. 2, p. 37] between two pictures f and g,

$$\sum_{i=1}^{D}\sum_{j=1}^{D} |f_{ij} - g_{ij}|, \quad \sum_{i=1}^{D}\sum_{j=1}^{D} (f_{ij} - g_{ij})^2$$

called the city block distance and Euclidean distance respectively.

Let $\mathrm{Im}(D, P)$ denote the class of gray-scale pictures of dimension D and with the gray-level range $0,1,\ldots,P-1$. Thus, $f \in \mathrm{Im}(D, P)$ means that f is a gray-scale picture with D^2 pixels in the range $0 \leqslant f_{ij} \leqslant P-1$, $i = 1, 2, \ldots, D$; $j = 1, 2 \ldots, D$.

If the picture $f \in \mathrm{Im}(D, P)$ then, for comparison purpose, f_t the thresholding of f at t must be a picture in the same class $f_t \in \mathrm{Im}(D, P)$, although it is a bi-level picture. If f_t is defined by

$$f_t(i, j) = \begin{cases} P-1 & \text{if the pixel } f_{ij} \geqslant t \\ 0 & \text{otherwise} \end{cases}$$

then, like in exercise 10.4 [7 (vol. 2, p. 66)], the threshold value minimizing one of the above-defined distances between f and f_t, is the middle of the gray range, namely $t = (P-1)/2$, independently of the structure of the picture f.

The main idea of this paper is the following definition of the thresholding of the picture f at t

$$f_t(i, j) = t \cdot T(f_{ij}, t) = \begin{cases} t & \text{if } f_{ij} \geqslant t \\ 0 & \text{otherwise} \end{cases}$$

Thus, the bi-level picture f_t for $t = 1, 2, \ldots, P-1$ is directly dependent of t. The notation $t \cdot T(f, t)$ leads to the usual definition of the two-valued picture $T(f, t)$

$$T(f_{ij}, t) = \begin{cases} 1 & \text{if } f_{ij} \geqslant t \\ 0 & \text{otherwise.} \end{cases}$$

We shall call the picture $t \cdot T(f, t)$ the right thresholding of f at t. For a given gray-scale picture $f \in \mathrm{Im}(D, P)$, we have $P-1$ such bilevel pictures $t \cdot T(f, t)$; $t = 1, 2, \ldots, P-1$ with the following properties:

1. $T(f, 1) \geqslant 2T(f, 2) \geqslant \ldots \geqslant (P-1)T(f, P-1)$, $f \in \mathrm{Im}(D, P)$;
2. $t \cdot T(f, t) \leqslant f$ for every $t = 1, 2, \ldots, P-1$, $f \in \mathrm{Im}(D, P)$;
3. $f_{ij} = \max\ (t \cdot T(f_{ij}, t))$, $i = 1, 2, \ldots, D$; $j = 1, 2, \ldots, D$, $f \in \mathrm{Im}(D, P)$;
4. $\sum_{i=1}^{D} \sum_{j=1}^{D} t \cdot T(f_{ij}, t) = t \cdot \sum_{k=t}^{P-1} H(f)_k$.

According to these results, the city block distance between the gray-scale picture $f \in \mathrm{Im}(D, P)$ and the right thresholding of f at t is

$$d(f, t \cdot T(f, t)) = \sum_{i=1}^{D} \sum_{j=1}^{D} |f_{ij} - t \cdot T(f_{ij}, t)| = \sum_{k=0}^{P-1} kH(f)_k - t \cdot \sum_{k=t}^{P-1} H(f)_k$$

Also, the Euclidean distance between the same pictures,

$$\sum_{i=1}^{D} \sum_{j=1}^{D} (f_{ij} - t \cdot T(f_{ij}, t))^2 = \sum_{k=0}^{P-1} k^2 H(f)_k - 2t \sum_{k=t}^{P-1} kH(f)_k + t \sum_{k=t}^{P-1} H(f)_k$$

3. A new thresholding method. Definition. *The gray value t minimizing one of the distances between the picture $f \in \mathrm{Im}(D, P)$ and the right thresholding of f at t will be called an optimum right threshold in the sense of the corresponding distance.*

The minimization of the city block distance between the picture f and the right thresholding of f at t is equivalent with the maximization of the expression

$$E_1(f, t) = t \cdot \sum_{k=t}^{P-1} H(f)_k$$

the term $\sum_{k=0}^{P-1} kH(f)_k$ being a constant associated to f.

In the same way, the minimization of the Euclidean distance is equivalent with the maximization of the expression

$$E_2(f, t) = t\left(2 \cdot \sum_{k=t}^{P-1} kH(f)_k - t \sum_{k=t}^{P-1} H(f)_k\right),$$

$\sum_{k=0}^{P-1} k^2 H(f)_k$ being also a constant associated to f.

Let us denote $R_t(f) = \sum_{k=t}^{P-1} H(f)_k$, and $S_t(f) = \sum_{k=t}^{P-1} kH(f)_k$ irrespective of what the gray-level histogram of f is, we have the chains

$$\sum_{k=0}^{P-1} H(f)_k = R_0(f) \geqslant R_1(f) \geqslant \ldots \geqslant R_{P-1}(f) = H(f)_{P-1} \geqslant 0, \ f \in \mathrm{Im}(D, P)$$

$$\sum_{i=1}^{D} \sum_{j=1}^{D} f_{ij} = S_0(f) \geqslant S_1(f) \geqslant \ldots \geqslant S_{P-1}(f) = (P-1)\ H(f)_{P-1} \geqslant 0.$$

The expression $E_2(f, t)$ can be successively written as

$$E_2(f, t) = t\left(2 \sum_{k=t}^{P-1} kH(f)_k - t \sum_{k=t}^{P-1} H(f)_k\right) = t\left(S_k(f) + \sum_{k=t}^{P-1} (k-t)\, H(f)_k\right) =$$

$= t\left(S_k(f) + \sum_{h=k}^{P-1} R_h(f)\right)$, and the expression $W_k(f) = S_k(f) + \sum_{h=k+1}^{P-1} R_h(f)$ generates also a nonincreasing chain

$$W_0(f) \geqslant W_1(f) \geqslant \ldots \geqslant W_{P-1}(f) = S_{P-1}(f) \geqslant 0.$$

So, we can formulate our main result:

THEOREM. *The optimum right threshold value for a given picture $f \in \mathrm{Im}\,(D, P)$ in the sense of the city block distance is that gray level t maximizing the product $t \cdot R_t(f)$, for $R_t(f) = \sum_{k=t}^{P-1} H(f)_k$. The optimum right threshold value for $f \in \mathrm{Im}(D, P)$ in the sense of the Euclidean distance is that gray level t maximizing the product $t \cdot W_t(f)$, for $W_t(f) = \sum_{k=t}^{P-1} (2k - t) H(f)_k$.*

We shall extend the chain $\{R_k(f)$; $k = 0, 1, \ldots, P-1\}$ to a step real function $R(x, f) = R_{\mathrm{int}(x)}(f)$, for $x \in (0, P)$; respectively, the chain $\{W_k(f)$; $k = 0, 1, \ldots, P-1)\}$ to the step function $W(x, f) = W_{\mathrm{int}(x)}(f)$, where $\mathrm{int}(x)$

is the integer part of the real number $x \in (0,P)$. Both these functions are nonincreasing. Considering the graphic representations of these functions, we obtain the following geometric interpretation of the theorem.

COROLLARY. *The selection of the optimum right threshold value in the sense of the city block distance (or Euclidean distance) means the selection of the maximum-area rectangle inscribed in the surface delimited by the function $R(x, f)$ (respectively by $W(x, f)$) and the coordinate axis.*

This geometric interpretation illustrates the simplicity of the procedure. Incidentally, we can obtain two or more maximum-area rectangles, with the same area, of course. To avoid this situation, we shall take the smallest right threshold value t maximizing the area of the rectangle (fig. 2).

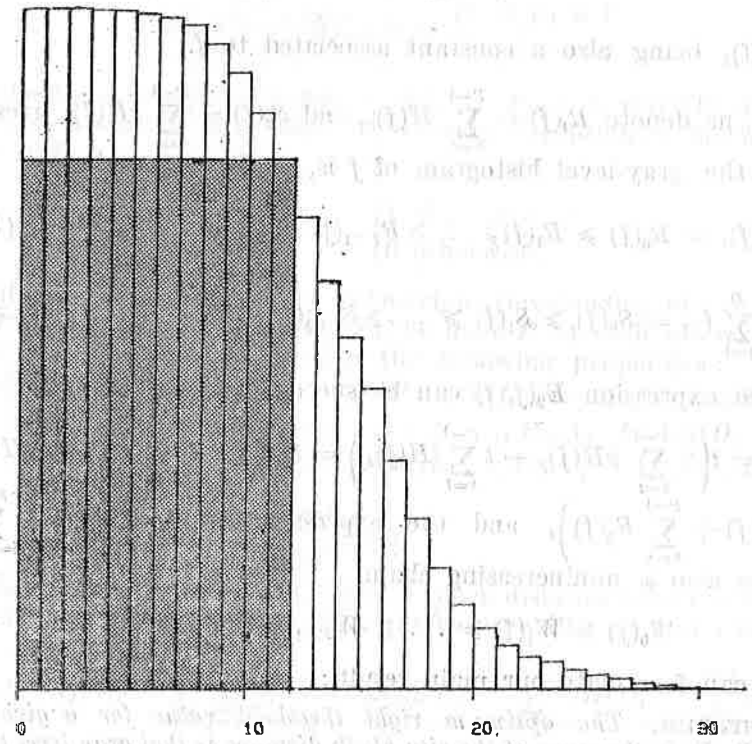

Fig. 2. — Geometric interpretation of the method.

4. Discussion. The threshold selection method proposed herein is as complex as the simple histogram analysis methods [1, 2, 5, 8, 10], but we hope that selected threshold is better. In many cases, the two optimum right threshold values (depending on the distance type) are almost equal. When they exhibit a significant difference, the single thresholding of the picture cannot be used.

Further importance of the method derives from its possible generalization to multiple threshold values selection. In a geometrical interpretation, for a fixed number r of thresholds, we can obtain the best approximation of the area $R(x, f)$, respectively $W(x, f)$, by a set of r inscribed rectangles.

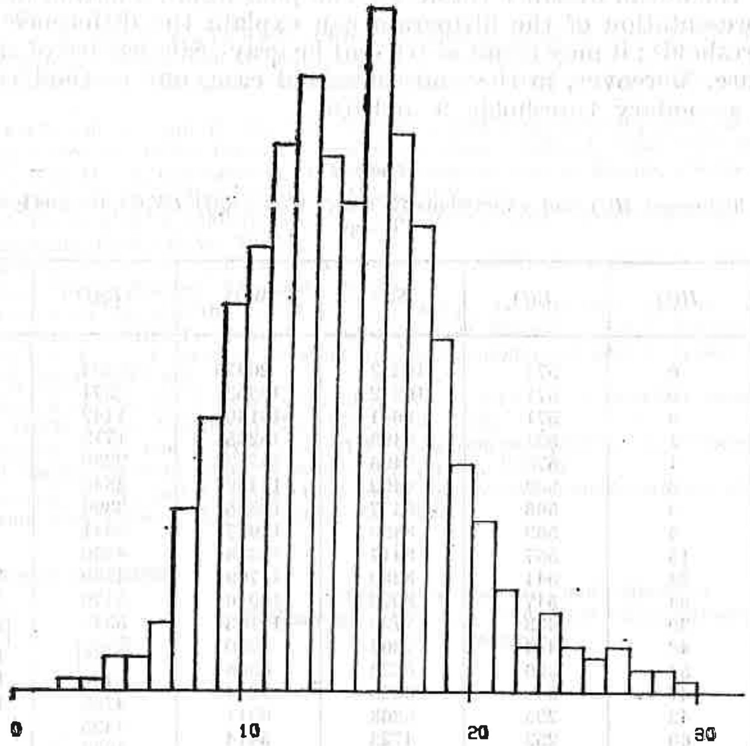

Fig. 3. — A histogram of a Landsat image [11], and threshold candidates.

As an illustration of the quality of our threshold selection procedure, let us consider the histogram [11] (fig. 3). This is a histogram of a Landsat image segmented by Wang and Haralick. By the concavity analysis method [1,8], the gray level 23 is selected as optimum threshold (also, values 7 and 15 are selected as possible candidates). The iterative method [2] based on equation 1 solving [4, 5, 9], selects 16 as stable thresholds. This would be selected as threshold by Watanabe's method, also. However, Wang and Haralick [12] use a very sophisticated method to select the gray level 12 as a good threshold. By a recursive multithreshold selection technique, they select also as thresholds the values 20 and 26, but then never select 16 because it is not compatible with the others in getting pure bright or dark regions, as they proved. It must be underlined that the method of Wang and Haralick [11] is based on the decomposition of the edge pixels histogram in two other histograms where peaks are detected. A special rule selects the threshold between candidate peaks using the probabilities of getting dark, respectively bright, regions by thresholding at t. Like other methods based on edge detection, this method is time-consumming and computationally expensive.

By our method, the gray level 12 is selected as an optimum right threshold in the sense of city-block distance, and 11 is selected in the sense of Euclidean distance (table 1). The poor approximation in the graphic representation of the histogram can explain the difference between these thresholds; it may result also fromt he gray-scale nature of the analysed picture. Moreover, in the multithreshold case, our method can select the two secondary thresholds 9 and 16.

Table 1

Numerical histogram $H(f)$ and expressions $R(f)$, $S(f)$ $W(f)$, $tR_t(f)$, $tW_t(f)$, for analysed Landsat image

Gray level t	$H(f)$	$R(f)$	$S(f)$	$W(f)$	$tR_t(f)$	$tW_t(f)$
0	0	571	10212	20424	20424	0
1	0	571	10212	19853	571	19853
2	0	571	9641	18140	1142	36280
3	1	571	8499	15285	1713	45855
4	1	570	8496	14712	2280	58848
5	3	569	8492	14139	2845	70695
6	3	566	8477	13558	3396	81349
7	6	563	8459	12977	3941	90839
8	16	557	8417	12378	4456	99024
9	24	541	8289	11709	4869	105381
10	34	517	8073	10976	5170	109760
11	39	483	7733	10153	5313	$\boxed{111683}$
12	48	444	7304	9280	$\boxed{5328}$	111360
13	54	396	6728	8308	5148	108004
14	47	342	6026	7264	4788	101696
15	43	295	5368	6311	4425	94665
16	60	252	4723	5414	4032	86624
17	49	192	3763	4262	3264	72454
18	41	143	2930	3266	2574	58788
19	31	102	2192	2446	1938	46474
22	20	71	1603	1786	1420	35720
21	15	51	1203	1335	1071	28035
22	9	36	888	984	792	21648
23	4	27	690	759	621	17457
24	7	23	598	644	552	15456
25	4	16	430	460	400	11500
26	3	12	330	348	312	9048
27	4	9	252	261	243	7047
28	2	5	144	148	140	4144
29	2	3	88	89	87	2581
30	1	1	30	30	30	900
31	0	0	0	0	0	0

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