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AN IMPROVED CVBEM FOR PLANE HYDRODYNAMICS

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The aim of the present work is to give the foundations of a new numerical technique — a Complex Variable Boundary Element Method — in determining holomorphic functions fulfilling some given conditions related to the concrete problems of plane hydrodynamics.

It is well known that the main step of a BEM consists in the construction of an integral representation joint to the boundary problem and of the corresponding integral equation on the boundary. But if the boundary problem is formulated in the language of an unknown holomorphic function, the direct use of the Cauchy formula gives immediately an integral representation attached to the considered boundary problem which, in addition, leads automatically to an integral equation — with Cauchy singularity — on the boundary.

Moreover, using a certain system of interpolating functions of the unknown function, the solving of the boundary integral equation could be performed without any approximation of the boundary or any numerical quadrature.

Let $f(z)$ be an holomorphic function into the simple connected domain D the outside of a rectifiable Jordan curve C. It is assumed that $f(z)$ is continuous on C where it is known either its real or its imaginary part or a combination of the two.

Let $f(z) = a_0 + \frac{a_1}{z} + \ldots + \frac{a_n}{z^n} + \ldots$ be the development of the considered function in the neighbourhood of infinity. Then the Cauchy formula for $f(z)$ and the domain $D \cup C$ could be written as is known,

$$f(z) = \frac{1}{2\pi i} \int_{\overset{\curvearrowleft}{C}} \frac{f(\zeta)}{\zeta - z}\, d\zeta + a_0, \quad z \in D.$$

This formula, which is in fact the integral representation attached to the proposed boundary problem, allows us to determine $f(z)$ once the values of $f(\zeta)$ on the boundary C are known. But these values of $f(\zeta)$ on C also satisfy a singular integral equation obtained — performing $z \to \zeta^* \in C$ in the above formula — and which represents the boundary integral equation of the procedure (CVBEM). Unfortunately, the problem of solving this integral equation even numerically is usually a very difficult one. In what follows, we shall try to overpass the shortcomings connected with this equation and to include all the data of our problem in the algorithm of calculus i.e. the values of $\operatorname{Im} f(\zeta)$ or of $\operatorname{Re} f(\zeta)$ or their combination.

Let us consider a set of nodal points $z_0, z_1, z_2, \ldots, z_n$ $(z_0 \equiv z_n)$ on C, disposed counterclockwise separating the curve C into boundary elements C_j $(j = \overline{1, n})$, where C_j is the simple arc linking the points z_{j-1} and z_j. Let now the following approximation $\tilde{f}(\zeta)$ of the unknown function $f(\zeta)$ be defined by $\tilde{f}(\zeta) = \sum_{j=1}^{n} f_j L_j(\zeta)$, where $f_j = f(z_j)$ and the functions $L_j(\zeta)$ are the interpolating Lagrange functions constructed on each arc respectively, i.e. ([2], [3]),

$$L_j(\zeta) = \begin{cases} \dfrac{\zeta - z_{j-1}}{z_j - z_{j-1}} & \text{for } \zeta \in C_j \\ \dfrac{\zeta - z_{j+1}}{z_j - z_{j+1}} & \text{for } \zeta \in C_{j+1} \\ 0 & \text{otherwise} \end{cases}$$

We then get for the above Cauchy integral — up to the additional constant a_0 — the approximation $f^*(z) = \sum_{j=1}^{n} f_j \tilde{L}_j(z)$, where

$$\tilde{L}_j(z) = \frac{1}{2\pi i} \int_C \frac{L_j(\zeta)}{\zeta - z}\, d\zeta =$$

$$= \frac{1}{2\pi i}\left(\frac{z - z_{j-1}}{z_j - z_{j-1}} \ln \frac{z - z_j}{z - z_{j-1}} + \frac{z - z_{j+1}}{z_j - z_{j+1}} \ln \frac{z - z_{j+1}}{z_j - z_{j+1}} \right)$$

and where one chooses the principal determination for the complex logarithm.

Let us now suppose that the function $f^*(z)$ is evaluated in all the nodal points $z_k (k = \overline{1, n})$ i.e. $f^*(z_k) = \sum_{j=1}^{n} f_j \tilde{L}_j(z_k)$, $k = \overline{1, n}$. Considering then an approximation of the equality just written, precisely

$$f_k(u_k + i v_k) = \sum_{j=1}^{n} f_j L_{jk}, \quad k = \overline{1, n},$$

where $L_{jk} = \tilde{L}_j(z_k) = M_{kj} + i N_{kj}$, we are led to the following real system of $2n$ equations in $2n$ unknowns

$$u_k = \sum_{j=1}^{n} M_{kj} u_j - \sum_{j=1}^{n} N_{kj} v_j$$

$$v_k = \sum_{j=1}^{n} M_{kj} v_j + \sum_{j=1}^{n} N_{kj} u_j$$

By solving this system within the data of the boundary problem, we get the looked for approximation $\tilde{f}(\zeta)$ of the function $f(\zeta)$ and, implicitly, *via* Cauchy's formula, the solution of the proposed boundary problem in all the points of the domain D.

Concerning the coefficients L_{kj}, for $k \neq j$ they could be directly calculated from the expression of $\tilde{L}_j(z)$ using the equality $\lim_{z \to z_p} (z - z_p) \times \ln(z - z_p) = 0$ in the case of $k = j - 1$ or $k = j + 1$. For $k = j$, as we have [2], [3]

$$\tilde{L}_j(z) = \frac{1}{2\pi i}\left\{ \frac{z - z_j}{z_j - z_{j-1}} \ln \frac{z - z_j}{z - z_{j+1}} + \right.$$

$$\left. + \frac{z - z_j}{z_j - z_{j+1}} \ln \frac{z - z_{j+1}}{z - z_j} + \ln \frac{z - z_{j+1}}{z - z_{j-1}} \right\},$$

we get immediately $L_{jj} = \dfrac{1}{2\pi i} \ln \left(\dfrac{z_j - z_{j+1}}{z_j - z_{j-1}} \right)$, where one takes the same principal determination for logarithm.

We note that the solving of the above problem by BEM with complex variables (CVBEM) did not need any approximation of the curve C or any numerical quadrature formula. The only approximation used was that connected with the interpolation of the function in the points of the boundary.

Let us now suppose that the Jordan rectifiable curve C has in $z_p \in C$ an angular point, precisely the counterclockwise oriented angle of semi-tangents, in this point being $\pi - \mu\pi$ with $-1 \leqslant \mu < 0$. It is known that the Cauchy integral still exists in that case and the behaviour of the function $f(z)$ in the neighbourhood of the angular point z_p will be given by $f(z) - f(z_p) = (z - z_p)^{\frac{1}{1-\mu}} h(z)$, where $h(z_p) \neq 0$, which implies for the derivative $\dfrac{df}{dz}$ a behaviour of the type $0\ [(z - z_p)^{\frac{\mu}{1-\mu}}]$, i.e. this derivative becomes unbounded in z_p for $-1 \leqslant \mu < 0$.

In the event of using the BEM in the variant CV one must take into account the existence of this singularity in z_p. More precisely, the "piecewise" interpolation has to match with the behaviour in $V(z_p)$ of the function $f(z)$. To put it briefly, in the neighbourhood of a nodal singular point z_p which is considered to belong to both C_p and C_{p+1}, we shall interpolate the function $f(\zeta)$ by [3]

$$f(\zeta) = \begin{cases} f_p + (f_{p-1} - f_p) \left(\dfrac{\zeta - z_p}{z_{p-1} - z_p} \right)^{\frac{1}{1-\mu}}, & \text{for } \zeta \in C_p, \\ f_p + (f_{p+1} - f_p) \left(\dfrac{\zeta - z_p}{z_{p+1} - z_p} \right)^{\frac{1}{1-\mu}}, & \text{for } \zeta \in C_{p+1}. \end{cases}$$

Consequently, in the approximation on the whole C, $f(\zeta) = \sum_{j=1}^{n} f_j L_j$, the expressions of the polynomials $L_j(\zeta)$ are identical to those already written above for $j \neq p-1,\ p,\ p+1$ while for the cases $j = p-1$, p, $p+1$ we shall now have

$$L_{p-1}(\zeta) = \begin{cases} \dfrac{\zeta - z_{k-2}}{z_{k-1} - z_{k-2}}, & \text{for } \zeta \in C_{p-1}, \\ \left(\dfrac{\zeta - z_k}{z_{k-1} - z_k}\right)^{\frac{1}{1-\mu}}, & \text{for } \zeta \in C_p, \\ 0 & \text{otherwise} \end{cases}$$

$$L_p(\zeta) = \begin{cases} 1 - \left(\dfrac{\zeta - z_p}{z_{p-1} - z_x}\right)^{\frac{1}{1-\mu}}, & \text{for } \zeta \in C_p, \\ 1 - \left(\dfrac{\zeta - z_p}{z_{p-1} - z_p}\right)^{\frac{1}{1-\mu}}, & \text{for } \zeta \in C_{p+1}, \\ 0 & \text{otherwise} \end{cases}$$

$$L_{p+1}(\zeta) = \begin{cases} \dfrac{\zeta - z_{p+2}}{z_{p+1} - z_{p+2}}, & \text{for } \zeta \in C_{p+2}, \\ \left(\dfrac{\zeta - z_p}{z_{p+1} - z_p}\right)^{\frac{1}{1-\mu}}, & \text{for } \zeta \in C_{p+1}, \\ 0, & \text{otherwise} \end{cases}$$

At once we also obtain

$$\tilde{L}_{p-1}(z) = \frac{1}{2\pi i}\left\{\frac{z - z_{p-2}}{z_{p-1} - z_{p-2}} \ln \frac{z - z_{p-1}}{z - z_{p-2}} + 1 - F_{1/1-\mu}\left(\frac{z - z_p}{z_{p-1} - z_p}\right)\right\},$$

$$\tilde{L}_p(z) = \frac{1}{2\pi i}\left\{\ln \frac{z - z_{p+1}}{z - z_{p-1}} + F_{1/1-\mu}\left(\frac{z - z_p}{z_{p-1} - z_p}\right) - F_{1/1-\mu}\left(\frac{z - z_p}{z_{p+1} - z_p}\right)\right\},$$

$$\tilde{L}_1(z) = \frac{1}{2\pi i}\left\{\frac{z - z_{p+2}}{z_{p+1} - z_{p+2}} \ln \frac{z - z_{p+2}}{z - z_{p+1}} - 1 + F_{1/1-\mu}\left(\frac{z - z_p}{z_{p+1} - z_p}\right)\right\},$$

where $F_\alpha(z) = \int_0^1 \frac{t^\alpha}{t - \alpha}\, dt^*$ while for the others $\tilde{L}_j(z)$ $(j \neq p,\ p-1,\ p+1)$ the already established expressions are still valid.

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* This integral could be analytically performed if $\alpha = m/n$ (a rational number with $m < n$) [3].

2. Let us now consider a plane incompressible potential inviscid fluid flow. It is well known that it is allways possible to join to such a flow an analytic function $f(z)$ — called the complex potential of the flow — whose knowledge is entirely equivalent with the complete determination of the flow.

Conversely, any holomorphic function in a given domain could be interpreted as a complex potential of a plane incompressible potential inviscid flow pending addition of some logarithmic terms (multiform functions) in the case of multiply-connected domains.

If we consider only a simple connected domain like the outside of an obstacle (C) — the complex potential of a fluid flow with the qualities mentioned above, around the obstacle (C) — this will be an analytical function in every finite point, having in the neighbourhood of infinity the development

$$f(z, t) = w_\infty z + \frac{\Gamma}{2\pi i} \ln z + a_0 + \frac{a_1}{z} + \frac{a_2}{z^2} + \ldots$$

We denoted here by $w_\infty = \lim_{|z| \to \infty} \frac{df}{dz}$ the complex velocity of the fluid at great distances, by Γ a real function of time (which could be a constant or even zero) called the circulation of the flow and which represents the multiformity period of the real part of the complex potential f, and by t, the time which could explicitly appear, the flow being then a nonstationary one.

Additionally, the imaginary part of the values of the function $f(z, t)$ (i.e. the stream function ψ) are given along the contour C. Supposing that the obstacle (C) is performing a general rototranslation in the mass of the fluid then, if $l(t)$, $m(t)$ are the components of the translation velocity in a point $z_A \in (C)$ — evaluated in a mobile system of coordinates Oxy centered in $z_A \equiv 0$ and ω the instantaneous rotation of the profile, the boundary condition for the function ψ in the points of C is $\psi|_C = ly - mx + \frac{\omega}{2}(x^2 + y^2) + \text{arbitrary function of time}|_C$. We remark that if instead of the complex potential $f(z, t)$ we would construct the complex velocity $w(z, t) = \frac{df}{dz}$, this will be a holomorphic function in the whole outside of the profile (C) which also includes the point at infinity. In the neighbourhood of this point the function $w(z; t)$ has a development of the type

$$w(z; t) = w_\infty + \frac{\Gamma}{2\pi i} \frac{1}{z} + \frac{b_2}{z^2} + \frac{b_3}{z^3} + \ldots$$

It is just this regularity of the complex velocity that determines us to use the above developed CVBEM for this function $w(z; t)$ and not for the complex potential as we would have been tempted to.

Concerning the boundary conditions in the points of the contour C it will be written for the function $w(z; t)$ under the form :

There is a real function $V(\beta)$ so that for every $\beta \in [0, 2\pi)$ we have $w(\zeta(\beta)) = V(\beta)\dfrac{\dot\zeta(\beta)}{|\dot\zeta(\beta)|} + 1 + \mathrm{i}\, m + \mathrm{i}\omega\,[\zeta(\beta) - z_A]$, where $\zeta = \zeta(\beta)$ the parametrical equation of the Jordan rectifiable curve C, is a 2π — periodical function, bounded and derivable in $[0, 2\pi)$ so that $\dot\zeta(\beta) \neq 0$ and $\dot\zeta(\beta) < M$ when M is a finite constant.

Finally, the possible multiformity of the function f leads to the fulfilment of the equality

$$\int_C w(z, t)dz = \Gamma(t),$$

where $\Gamma(t)$ is the "a priori" given circulation. In the case of the profiles with an angular point in $z = z_p \in C$, where the semi-tangents angle is equal to $\pi - \mu\pi$ $(-1 \leqslant \mu < 0)$, the behaviour of the complex velocity in this point, i.e. $w(z; t) = 0$ $[(z - z_p)^{\frac{\mu}{1-\mu}}]$ requires — to avoid the unboundness of w in z_p — to choose the circulation such that [4] $\Gamma = L \cdot l + M \cdot m + N \cdot \omega$, where the uniquely determined coefficients L, M, N, depend upon the considered profile (C).

In what follows, we want to illustrate how CVBEM works for determining the fluid flow induced by a displacement (rototranslation) in the mass of the fluid, of a profile (C), the fluid, having already a given basic flow of complex velocity $w_B(z)$ and which superposes over the first flow.

For more generality, we shall suppose that the profile (C) has an angular point and the basic flow presents some given singularities (vortices, sources, etc.). Obviously, the envisaged problem contains also the particular case of a flow past a fixed profile (C), the condition with an "a priori" given circulation becoming the famous Jukovski condition. Additionally, the same method could be used for an arbitrary system of profiles performing independent displacements in the mass of the fluid, in the possible presence of some walls, i.e. practically for the majority of the models of plane hydrodynamics.

Retaking, for the sake of simplicity, the case of only one profile (C), the proposed problem can be formulated as follows.

Let the function $w_B(z)$ be given, the complex velocity of the basic flow, a function which belongs to a class (a) of functions having the properties :

1 a) they are holomorphic functions in the domain D_1 (the whole plane Oxy, the point of infinity being included) except a bounded number (q) of points z_r placed at a finite distance and which represent singular points for these functions; let D_1^* be the domain D_1 from which one has taken off the singular points $\{z_r\}_{r=\overline{1,q}}$ and let $w_B(\alpha)$ be the value of the limit $\lim\limits_{|z|\to\infty} W_B(z)$ which obviously exists and is finite.

2a) if Γ_B[1] is the circulation of the basic flow, this is equal to $\sum\limits_{r=1}^{q} \Gamma_r$, i.e. with the sum of the circulations of all the given singularities of the flow.

Concerning the unknown function $w(z)$ — the complex velocity of the resultant flow obtained by the above-mentioned superposition — it will be looked for in a class of functions (b) satisfying the properties :

1 b) they are holomorphic functions in the domain $D = D_1 \setminus (\bar{C})$[2] except the same points $\{z_r\}_{r=\overline{1,q}}$ which are the singular points of the same nature as for $w_B(z)$; at infinity, their behaviour is identical with that of $w_B(z)$ i.e. $\lim\limits_{|z|\to\infty} w(z) = w(\infty) = w_B(\infty)$;

2 b) in the neighbourhood of the trailing edge $z_p = \zeta(\beta_0) \in C$, where the semi-tangents angle is $\pi - \mu\pi$, we have

$$w(z) = (z - z_p)^{\frac{\mu}{1-\mu}}\, g(z),\quad g(z_p) \neq 0 ;$$

3 b) in the points of the curve C, the functions $w(\zeta(\beta))$ belong to the class H^* i.e. they are Hölderian functions on C except the angular point $z_p = \zeta(\beta)$ in whose neighbourhood one has

$$w(\zeta(\beta)) = \frac{w^*(\zeta(\beta))}{[\zeta(\beta) - \zeta(\beta_0)]^{\frac{\mu}{\mu-1}}}$$

where $w^* \in H_0$ in the same neighbourhood which means that $w^*(\zeta(\beta))$ is separately Hölderian on the upper side and on the lower side of the profile in the neighbourhood of $z_p = \zeta(\beta_0)$) ;

4 b) in the points of the curve C they satisfy, except the angular point, the following boundary condition :

There is a real continuous fonction $V(\beta)$ such that for every $\beta \in [0, 2\pi) \setminus \{\beta_0\}$ one has

$w(\zeta(\beta)) = V(\beta)\dfrac{\dot\zeta(\beta)}{\dot\zeta(\beta)} + l + \mathrm{i}m + \mathrm{i}\omega[\zeta(\beta) - z_A]$, where $z_A \in (C)$ and $l(t)$, $m(t)$, $\omega(t)$ are the given functions of time determining the rototranslation of the profile (C) ;

5 b) they fulfil the equality $\int_C w(z)dz = \Gamma$[3] where the circulation of the flow Γ is chosen so that one has the boundness of the velocity in z_p, i.e.

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[1] $\Gamma_B = \int_\Delta w_B(z)dz$, being a simple rectifiable curve surrounding all the singularities z_r.

[2] One supposes that during the displacement of (C) we have $(\bar{C}) \subset D_1^*$, i.e. the profile (C) does not cross the points $\{z_r\}_{r=1,q}$ which always belong to the outside of (C).

[3] The singularity in z_p being weak $\left(0 < \dfrac{\mu}{\mu - 1} < 1\right)$ the integral is convergent.

[4], $\Gamma = L \cdot l + M \cdot m + N \cdot n$, where the coefficients L, M, N are given with the obstacle (C).

Let us now consider the function $w(z) - w_B(z)$. This function known together with $w(z)$ being holomorphic in the outside of (C) the Cauchy formula is valid in D and we immediately have.

$$w(\xi) = w_B(\xi) - \frac{1}{2\pi i}\int_C \frac{w(z)}{z-\xi}dz + \frac{1}{2\pi i}\int_C \frac{w_B(z)}{z-\xi}dz \text{ for } \xi \in D\ ^4.$$

Finally, in order to use the boundary condition on C we perform $\xi \to \zeta = \zeta(\beta^*) \in C \setminus \{z_p\}$ and so we get

$$w(\zeta(\beta^*)) = w_B(\zeta(\beta^*)) - \frac{1}{\pi i}\oint_0^{2\pi} \frac{w(\zeta(\beta)) \cdot \zeta'(\beta)}{\zeta(\beta) - \zeta(\beta^*)}d\beta + \frac{1}{\pi i}\oint_0^{2\pi} \frac{w_B(\zeta(\beta)) \cdot \zeta'(\beta)}{\zeta(\beta) - \zeta(\beta^*)}d\beta\ ^5$$

This is the boundary integral equation which will be used for the effective construction of an approximative solution by CVBEM. Considering then a system of nodal points $z_0, z_1, \ldots, z_{p-1}, z_p, z_{p+1}, \ldots, z_n \equiv z_0$ on the curve C, alltogether with the system of the piecewise interpolating Lagrange functions on each arc C_j (system which takes into account the behaviour in the neighbourhood of z_p) we can write

$\tilde{w}(\zeta(\beta)) - w_B(\zeta(\beta)) + \sum_{j=1}^{n} (w_j - w_{Bj})L_j$, where $L_j(\zeta(\beta))$ for $j \neq p-1$, p, $p+1$ have the expressions specified in the first part of this paper while for $j = p-1$, p, $p+1$ they could be obtained from those previously written by replacing $\frac{1}{1-\mu}$ with $\frac{\mu}{1-\mu}$.

Using then the general calculus already performed for $\tilde{L}_j(z)$ and L_{jk}, if $w(z_k) - w_B(z_k) \equiv u_k - iv_k$ and $L_{jk} \equiv M_{kj} + iN_{kj}$ we are led again to the real algebraic homogeneous system

$$u_k = \sum_{j=1}^{n} M_{kj}u_j + \sum_{j=1}^{n} N_{kj}v_j$$

$$v_k = -\sum_{j=1}^{n} M_{kj}v_j + \sum_{j=1}^{n} N_{kj}u_j,$$

[4] $W(\zeta(\beta)) - w_B(\zeta(\beta)) \in H^*$ and C being a sectionally smooth curve, the integral of Cauchy type exists.

[5] The Plemelj formulas are still valid.

which will be completed in this case by the complex equation

$$\sum_{j=1}^{n} w_j \int_C L_j(\zeta)d\zeta = \Gamma \text{ or, equivalently by}$$

$$\sum_{j=1}^{n} u_j \mathrm{Re}\int_C L_j(\zeta)\,d\zeta + v_i\,\mathrm{Im}\int_C L_j(\zeta)d\zeta = \Gamma$$

$$\sum_{j=1}^{n} u_j \mathrm{Im}\int_C L_j(\zeta)d\zeta = \sum_{j=1}^{n} v_j \mathrm{Re}\int_C L_j(\zeta)d\zeta$$

These last two real equations allow to determine an unique solution of the above homogenous system which includes also the data on C. This unique solution once introduced in the integral representation of the problem (i.e. in our case the Cauchy formula) leads to the complete determination of the complex velocity in every point of the domain of the flow.

The existence and the uniqueness of the solution of the proposed problem (of the function $w(z)$ looked for under the above representation) are not considered here, they being studied earlier [1].

Regarding the singularities $\{z_r\}_{r=\overline{1,q}}$ of the fluid flow admitting that they are vortices (and so $\Gamma_k \neq 0$, $k=\overline{1,q}$) the absence of external forces implies the fulfilling of a so called "freeddom condition" for them [4], i.e.

$$\frac{d\bar{z}_r}{dt} + 1 + im + i\omega\bar{z}_r = \lim_{z \to z_r}\left[w(z) + \frac{i\Gamma_r}{z - z_r}\right],\ r = \overline{1, q}.$$

Under these circonstances, the displacement of the profile C and of the vortices $\{z_r\}_{r=\overline{1,q}}$ with corresponding circulations are correlated by the above additional relations.

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