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CONDITIONS OF EXISTENCE IN THE FUTURE FOR LIÉNARD-RAYLEIGH-TYPE SYSTEMS OF EQUATIONS

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1. Introduction. We shall consider in this paper Liénard-Rayleigh-type systems of differential equations, studying the conditions of existence in the future for the solutions of these systems on the basis of the results obtained by T. Hara, T. Yoneyama and J. Sugie in [1] – [3]. Compared with other papers, the difference consist of the use of two Liapunov functions. We state the main results from [1], which constitutes the basis of our study. Consider the system :

$$\begin{cases} \dot{x} = f(t, x, y) \\ \dot{y} = g(t, x, y) \end{cases} \tag{1}$$

where x and y are n-dimensional and m-dimensional vectors, respectively, while f and g are defined and continuous on $[0, \infty) \times R^n \times R^m$ and ensure the existence and uniqueness of any formulated Cauchy problem.

Definition. A scalar, continuous function $\varphi : [0, \infty) \times R \to R$ is said of class $\mathcal{G}$ if the maximal solution $u(t; t_0, u_0)$ of the scalar equation $\dot{u} = \varphi(t, u)$ exists in future whatever be $t_0 \geqslant 0$ and $u_0 \in R$. Denote $S_K = \{y \in R^m \mid \|y\| \leqslant K,\ K > 0\}$

THEOREM *3.1, 1. Let $V : [0, \infty) \times R^n \times R^m \to R$ be locally Lipschitzian, satisfying*

(i) *$V(t, x, y) \to \infty$ as $\|y\| \to \infty$ uniformly in x for each fixed t, and there exists $\varphi \in \mathcal{G}$ such that*

(ii) $\dot{V}(t, x, y) \leqslant \varphi(t, V(t, x, y))$

Moreover, suppose that for $K > 0$ and $T > 0$, there exists $W : [0, T] \times R^n \times S_K \to R$, locally Lipschitzian, satisfying

(iii) *$W(t, x, y) \to \infty$ as $\|x\| \to \infty$ for each fixed (t, y), and there exists $\psi \in \mathcal{G}$ such that*

(iv) $\dot{W}(t, x, y) \leqslant \psi(t, W(t, x, y))$

Then every solution of (1) *exists in the future.*

2. Liénard-Rayleigh-type systems. Let us now consider a system of differential equations which generalizes the Liénard systems (5.2) from [1]. On the basis of Theorems 3.1 and 5.1 from [1], we shall construct two

Liapunov functions and establish new conditons ensuring the continuability in the future of every solution of the considered system.

Let be the system:

$$(2)\qquad \begin{cases} \dot{x} = h(y) - F(x) \\ \dot{y} = -g(x) + e(t) \end{cases}$$

where the functions $F(x)$, $g(x)$, $h(y)$, $e(t)$ are continuous, $F(x) = \int_0^x f(s)ds$, $G(x) = \int_0^x g(s)ds$, $H(y) = \int_0^x h(s)ds$; suppose that they fulfil all conditions which ensure the existence and uniqueness of every Cauchy problem.

THEOREM. *Let be the system* (2). *If there exist the numbers* $p, q \in R_+$ *such that:*

a) $G(x) \geq -p, \forall x \in R$;

b) $g(x)F(x) \geq -q[G(x) + p + 1]$;

c) $h^2(y) \leq 2qH(y)$, $H(y) \to \infty$ *when* $|y| \to \infty$;

d) $$\int_0^{\infty} \frac{dx}{1 + F_-(x)} = \infty, \quad \int_{-\infty}^{0} \frac{dx}{1 + F_+(x)} = \infty,$$

where $F_-(x) = \max\{0, -F(x)\}$, $F_+(x) = \max\{0, F(x)\}$, *then every solution of the system* (2) *exists in the future.*

Proof. We construct two Liapunov functions:

$$V(x, y) = H(y) + G(x) + p + 1$$

and $V(x, y) \to \infty$ when $|y| \to \infty$; in addition, the derivative by virtue of the system (2) is:

$$\dot{V}(t, x, y) = -g(x)F(x) + h(y)e(t) \leq qV(x, y) + \frac{1}{2}e^2(t).$$

Hence there exists $\varphi \in \mathcal{G}$ such that $\dot{V}(t, x, y) \leq \varphi(t, V(x, y))$. Moreover, we have $W(x) = |x|$, $W(x) \to \infty$ when $|x| \to \infty$. The derivative of W by virtue of the system (2) is $\dot{W} = h(y) - F(x) \leq K + F_-(W)$ for $x \geq 0$, and $\dot{W} \leq K + F_+(-W)$ for $x \leq 0$. Hence there exists $\psi \in \mathcal{G}$ such that $\dot{W}(t, x, y) \leq \psi(t, W(t, x, y))$. Thus all conditions of Theorem 3.1 from [1] are fulfilled, therefore all solutions of the system (2) exist in the future.

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