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NUMERICAL METHODS IN GAME THEORY

(ADDENDUM)

GENERALIZED COOPERATIVE GAME

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1. Mathematical Foundation. The cooperative game may be modelled at different levels of generality. We start from the concept of cooperative game (c.g.), as considered in [1], § 12. All the concepts related to it will be used in the sense defined in [1, 2]. In this paper we shall introduce the concept of *generalized cooperative game.*

A close connection exists between the payoff production and its distribution. Frequently, the intercoalitional transfer of payoff is not possible. Under these circumstances, one supposes that the payoff achieved by each coalition is distributed among its members. But the extent of the payoff depends on the mixed strategies applied by the coalitions. The fundamental notions connected to the problem of choosing the strategies are: admitted intercoalition and assured minimum payoff. By defining the concepts of admitted intercoalition, assured minimum payoff, c.g. becomes a new mathematical object, called *generalized cooperative game* (g.c.g.).

Admitted Intercoalition. One calls intercoalition any subsystem of a solutional complete system of coalitions (s.c.s.c.). As a consequence, the intercoalition is at the same time a coalition, too, but generally impossible. However, it is a union of possible coalitions. The intercoalition is called *admitted intercoalition* when it is declared as such. By declaring an intercoalition as admitted, it does not become a possible coalition, but, unlike unadmitted (forbidden) intercoalitions, it will apply coordinated mixed strategies, i.e. of the corresponding coalition. There are two particular extreme cases of this intercoalition: 1) when the intercoalition is reduced to the possible coalition and then the g.c.g. is identified with the c.g.; 2) when the intercoalition is identified with the coalition of all the players I. In the latter case, g.c.g. is also called *arbitration game.*

Assured Minimum Payoff. Should the set of players I be decomposed in any system of admitted intercoalitions, the payoff achieved by every component coalition is not smaller than the payoff that might be assured by each coalition playing antagonistically with the anticoalition, identical with the intercoalition of the other coalitions. This payoff is called *minimum payoff*. The minimum payoff is called *assured minimum payoff* when the intercoalition is a possible coalition component of the s.c.s.c. . As a consequence, in the case of c.g., the minimum payoff is an assured payoff, and in the case of a proper g.c.g. the minimum payoff is only *warranted minimum payoff*. For the component coalitions of the veritable intercoalitions the minimum payoff is warranted. The extreme case in the one of the intercoalition I consisting of all the players when I is a veritable intercoalition, and all the component coalitions of the s.c.s.c. have only warranted payoff. Intermediate arbitration games lie between the two extreme cases. The c.g. is changed into g.c.g. of different generality levels by enlarging the domain of definition of the probability vector P in the definition D12.2 [1]. The product space is replaced by product spaces in which the components are no more possible coalitions. Of course, the payoff achieved in addition is distributed among the players of the coalitions so that the stability principle, both of the intracoalitional stability and also of the intercoalitional one, persists. In this way, the g.c.g. is compatible with Nash's arbitration scheme [1, 3, 5]. Depending on the domain of definition (bargaining domain) of the probability vector P, there are several types of arbitration games, among which we discuss two in detail.

A first type of *arbitration game* is obtained when the bargaining domain of the probability vector $P(t)$, associated with the s.c.s.c. S_t, $S_t = \bigcup K_h$, $h \in M_t$, $t = \overline{1, u}$, is given through the following system of restrictions :

$$(*) \begin{cases} P(t)J^{\tau} = 1 \\ P(t)H_{K_h} \neq V_{K_h}, \quad h \in M_t, \end{cases}$$

where : J is a line matrix consisting of elements equal to 1 ; H_{K_h} is the payoff matrix of the coalition K_h, $H_{K_h} = \sum H_i$, $i \in K_h$, written as a column matrix with elements linearly ordered according to the lexicographical order of situations s, $s = (s_1, \ldots, s_n)$: V_{K_h} is the value of the characteristic function corresponding to the coalition K_h. Solving the system of restrictions (*) as a linear programming problem, one obtains the basic solutions $P(t, j)$, $j = \overline{1, j_t}$. With their help, the probability vectors $P(t)$, $t = \overline{1, u}$, and P are determined as in the case of the c.g. from D 12.2 [1].

A second type of arbitration game, also called *arbitration game of minimax type*, is obtained from the first type, replacing the free member V_{K_h} by the function $V + V_{K_h}$. The maximization of the function V is required. The term "minimax" is motivated by the fact that the payoff, achieved over the assured payoff with the minimum value of some coalitions K_h, has the maximum possible value.

In the case of both types of arbitration game, the following properties hold :

The probability vector $P(t)$ is determined in such a way that the effective achieved payoff $P(t)H_{K_h}$ is greater than or equal to the minimum payoff V_{K_h}, assured in the case of c.g., but not assured in the case of the arbitration game, only warranted by an "arbiter".

Usually, certain coalitions cannot be formed because of the incommensurability of the payoffs, these ones being expressed in different unknown measures. Thus, when no intracoalitional cooperation is possible, an intercoalitional one still remains possible.

2. Adaptation of the Program JØC002. An adaptation is required to solve the arbitration game with the program JØC002 [1].

Data Organization. Entry data of type A undergo the modifications given in Table 1.

Table 1

ENTRY DATA OF TYPE A

(Modified Part)

SYMBOL	SIGNIFICANCE
IWV(10)	Admitted intercoalitions : 0 — possible coalitions, components of the solutional complete system of coalitions ≠ 0 — coalition I 1 — case of the arbitration game 2 — case of the arbitration game of minimax type

Program Structure. The structure of the program JØC002, given by Table 45 in chapter VI of the paper [1], undergoes the alterations given in Table 2.

The main functions of the new program units, included in Appendix B [1] are the following :

GENPRØB. By calling certain subroutines, it solves the arbitration game in both versions.

MATRCØMP. It achieves the matrix of the bargaining domain of the arbitration game ;

CALC1. By calling certain subroutines, it determines the basic solutions of the *intercoalitional cooperation strategies* on the bargaining domain and it calculates the effective corresponding payoffs ; among the basic solutions, it retains only those corresponding to the domination criterion ;

REEV1. It revalues the players' payoff of every coalition of the s.c.s.c. on the basis of the basic solutions of the intercoalitional cooperation strategies ;

PØZSTRX. It associates the strategies belonging to the elements of the (n-dimensional) matrix of the matrix game to the arcs of the ordered tree of the game in extensive form ;

ZIN4. It determines certain codes for the subroutine SIMP ;

Table 2

INITIAL VERTEX	FINAL VERTEX
PØZ3	. . . NUMGEN1 GENPRØB VEVE . . .
GENPRØB	WPI MATRCØMP DØM CALC1
MATRCØMP	WTI WTR BIN ZIN4 WTD
CALC1	SIMP(SIMP1) TRANSX REEV1 PØZSTRX DRUMREDO EDIT3 WPI WTI WTD
REEV1	WTD BIN PRØDSCJ
PØZSTRX	WTD CANT SUBARB DRUMRED1 DIVARC
DØM	WTD REDMATR SIMP(SIMP1)
REDMATR	WTI WTD
TRANSX	WTD

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* *  SEGMENT A1
*          SUBRØUTINE GENPRØB
*        -(II,I5,MH2,XX,NRV,MP,LP,KNN,LNN,NA,XNØRM2,
*        -PFIN,IPU,NUM1,IZ,MX,MO,N1,N2,N3,N4,N5,TE)
*          INTEGER Z
*          DØUBLE PRECISIØN A,WS,X,D,WV
*          DØUBLE PRECISIØN XX,XNØRM2,PFIN
*          CØMMØN /A1/Z(16),A(32,32),WS(16),X(50),D,
*        -NAT,IND(35),INDX(64),WV(8),IWV(16)
*          DIMENSIØN XX(I5,MH2),XNØRM2(MH2),PFIN(NA)
*          DIMENSIØN NUM1(IPU),TE(MO,N1,N2,N3,N4,N5)
*          DIMENSIØN MBO(32),MEM1(5)
*          DØ 10 I=1,MH2
*      10  MBO(I)=IWV(I)
*          CALL WPI(1,MBO(1),LNN,II,KNN,IZ,MX)
*          NO=N1*N2*N3*N4*N5
*          CALL MATRCØMP
*        -(IZ,NO,IPU,NUM1,,MBO,I5,MS,MEM1,
*        -MO,N1,N2,N3,N4,N5,TE)
*          CALL DØM(IZ,MX,NO,MS,LNN,KNN,IPU,MBO)
*          MP=O
*          CALL CALC1
*        -(LNN,II,KNN,IZ,MX,LP,MO,NO,
*        -I5,MH2,NRV,XX,XNØRM2,
*        -NA,PFIN,IPU,MBO,MS,MEM1)
*          Z(7)=O
*          CALL WPI(1,MBO(1),LNN,II,KNN,IZ,MX)
*          RETURN
*          END
*
* *  SEGMENT AO,A1
*          SUBRØUTINE MATRCØMP
*        -(IZ,NO,IPU,NUM1,MBO,I5,MS,MEM1,
*        -MO,N1,N2,N3,N4,N5,TE)
*          INTEGER Z
*          DØUBLE PRECISIØN FC,CT,VV,ZZ,FCO,XNØRM
*          DØUBLE PRECISIØN A,WS,X,D,WV
*          DØUBLE PRECISIØN S,WO
*          CØMMØN /A0/ISD(16),FC(32),CT,IFC(32),
*        -VV(5,32),ZZ(5,32),FCO(5),INDD(32),INDE(32),
*        -INNM(32),MEØI(32),MEØJ(32),XNØRM(32)
*          CØMMØN /A1/Z(16),A(32,32),WS(16),X(50),D,
*        -NAT,IND(35),INDX(64),WV(8),IT,IWV(15)
*          DIMENSIØN NUM1(IPU),MBO(IPU)
*          DIMENSIØN MEM1(I5)
*          DIMENSIØN TE(MO,N1,N2,N3,N4,N5)
*          DIMENSIØN NM(5)
*          DATA ME,WO/32,1.D-6/
*          CALL WTI(2,MBO(2),IPU,1,IPU,1,NUM1)
*          CALL WTR(3,MBO(3),MO,NO,MO,NO,TE)
*          DØ 10 I=1,I5
*      10  MEM1(I)=0
*          DØ 20 J=1,ME
*          DØ 20 I=1,ME
*      20  A(I,J)=O
*          N=O
*          DØ 50 J1=1,N1
*          DØ 50 J2=1,N2
*          DØ 50 J3=1,N3
*          DØ 50 J4=1,N4
*          DØ 50 J5=1,N5
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          N=N+1
          M=1
          A(N,M)=1
          DØ 40 K=1,IPU
          L=NUM1(K)
          IF(L.EQ.O)GØ TØ 40
          MEM1(M)=K
          M=M+1
          CALL BIN(K,MO,NM)
          A(N,M)=O
          S=FC(K)-WO
          DØ 30 J=1,MO
          IF(NM(J).EQ.O)GØ TØ 30
          IF(INDD(K).NE.1.ØR.ISD(12).EQ.1)S=S-FCO(J)
          A(N,M)=A(N,M)+TE(J,J1,J2,J3,J4,J5)
     30 CØNTINUE
          A(NO+1,M)=S
     40 CØNTINUE
     50 CØNTINUE
          A(NO+1,1)=1
          MS=M-1
          NOO=NO
          NOOO=NOO+1
          MOO=MS+1
          MOOO=MOO+1
          IF(IZ.EQ.1)GØ TØ 70
          NOO=NO+1
          NOOO=NOO+1
          A(NOO,1)=O
          A(NOOO,1)=1
          DØ 60 M=2,MOO
          A(NOOO,M)=A(NOO,M)
     60 A(NOO,M)=-1
          A(NOO,MOOO)=1
     70 CALL ZIN4(MS,NOO)
          CALL WTI(4,MBO(4),MS,1,MS,1,MEM1)
          CALL WTD(5,MBO(5),ME,ME,NOOO,MOOO,A)
          RETURN
          END

* SEGMENT AO,A1,A2
          SUBRØUTINE CALC1
        -(LNN,II,KNN,IZ,MX,LP,MO,NO,
        -I5,MH2,NRV,XX,XNØRM2,
        -NAH,PFIN,IPU,MBO,MS,MEM1)
          INTEGER B,Z
          DØUBLE PRECISIØN FC,CT,VV,ZZ,FCO,XNØRM
          DØUBLE PRECISIØN A,WS,X,D,WV
          DØUBLE PRECISIØN P,WET,XMNO,XT
          DØUBLE PRECISIØN XX,WO,R,S,Q
          DØUBLE PRECISIØN PA,XNØRM2,PFIN
          CØMMØN /AO/ISD(16),FC(32),CT,IFC(32),
        -VV(5,32),ZZ(5,32),FCO(5),INDD(32),INDE(32),
        -INNM(32),MEØI(32),MEØJ(32),XNØRM(32)
          CØMMØN /A1/Z(16),A(32,32),WS(16),X(50),D,
        -NAT,IND(35),INDX(64),WV(8),IT,JWV(15)
          CØMMØN /A2/JP,ICØD,NJ,M,NO1(5),NO2(5),
        -NN(32),MM(32),P(100),B(200),
        -M1(100),M2(100),M3(100),LI(100),NA,KO,
        -NDE(32),NNM(32),NXL,IPØZ3,IO,IØ,
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        -TDN(50,32),HO(5,100),WET,XMNO(5),XT(5,32)
          DIMENSIØN XX(I5,MH2),XNØRM2(MH2)
          DIMENSIØN PFIN(NAH)
          DIMENSIØN MBO(IPU)
          DIMENSIØN MEM1(MS)
          DIMENSIØN Q(100),IR(100),MØMØ(32),PA(100)
          N=NO+MS
          IF(IZ.EQ.2)N=N+1
          WO=WS(12)
          NR=O
          CALL SIMP(&20)
          GØ TØ 150
     20 FICTIVE=1
          CALL TRANSX(IZ,MX,NO,MS,IPU,MBO)
          IF(KNN.EQ.2.AND.NR.GE.NRV)GØ TØ 150
          IF(KNN.EQ.2.AND.IND(14).EQ.MØMØ(NR+1))
        -GØ TØ 90
          IF(KNN.EQ.2.AND.IND(14).NE.MØMØ(NR+1))
        -GØ TØ 140
          NRO=NR+1
          IF(NR.GE.MH2)WRITE(108,160)
          IF(NR.GE.MH2)GØ TØ 150
          CALL REEV1
        -(IZ,MO,N,X,I5,XX(1,NRO),XT(1,II),MS,MEM1,
        -IPU,MBO)
          IF(NR.EQ.O)GØ TØ 90
          DØ 40 JU=1,NR
          DØ 30 I=1,MO
          IF(XX(I,NRO).GT.XX(I,JU)+WO)GØ TØ 40
     30 CØNTINUE
          GØ TØ 140
     40 CØNTINUE
          JU=O
     50 JU=JU+1
          DØ 60 I=1,MO
          IF(XX(I,JU).GT.XX(I,NRO)+WO)GØ TØ 80
     60 CØNTINUE
          DØ 70 JJ=JU,NR
          MØMØ(JJ)=MØMØ(JJ+1)
          DØ 70 I=1,MO
     70 XX(I,JJ)=XX(I,JJ+1)
          NR=NR-1
     80 IF(JU.LT.NR)GØ TØ 50
     90 NR=NR+1
          MØMØ(NR)=IND(14)
          IF(LNN.EQ.1.AND.LP.EQ.1)GØ TØ 140
          IF(KNN.EQ.1)GØ TØ 140
          IF(XNØRM2(NR).EQ.O)GØ TØ 140
          CALL PØZSTRX
        -(NO,X,KO,B,NA,M1,M2,M3,IR,Q,
        -M,NN,MM,LI,NJ,NO1,NO2,IPU,MBO,
        -ISD(4),ISD(5),ISD(6),ISD(7),ISD(8))
          DØ 110 L=1,NA
          PA(L)=O
          IF(M1(L).EQ.O)PA(L)=P(L)
    110 CØNTINUE
          DØ 120 L=1,NA
    120 PA(L)=PA(L)+Q(L)
          CALL DRUMREDO(KO,NA,B,PA)
          DØ 130 K=1,NA
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      PA(K)=PA(K) *XNØRM(II) *XNØRM2(NR)
  130 PFIN(K)=PFIN(K)+PA(K)
      IED=3
      CALL EDIT3
     -(KO,NA,NJ,IED,I1,I5,NR,B,M1,PA,HO,
     -1,1,1,1,1,3,1,1,1)
  140 CALL SIMP1(&20)
  150 FICTIVE=1
      NRV=NR
      CALL WP1(6,MBO(6),LNN,I1,KNN,NRV,1)
      CALL WTD(7,MBO(7),NXL,1,NXL,1,XNØRM)
      CALL WTD(8,MBO(8),MO,1,MO,1,XT(1,I1))
      CALL WTD(9,MBO(9),I5,NRV,MO,NRV,XX)
      CALL WTD(10,MBO(10),NRV,1,NRV,1,XNØRM2)
      CALL WTD(11,MBO(11),NA,1,NA,1,PFIN)
      CALL WT1(12,MBO(12),NRV,1,NRV,1,MØMØ)
      RETURN
  160 FØRMAT(10X,'ØWERFLØW IN CALC1')
      END

      SUBRØUTINE REEV1
     -(IZ,MO,N,X,I5,XX,Z,MS,MEM1,IPU,MBO)
      DØUBLE PRECISIØN X,XX,Z,S
      DIMENSIØN X(N),XX(I5),Z(I5)
      DIMENSIØN MEM1(MS)
      DIMENSIØN MBO(IPU)
      DIMENSIØN NM(5)
      CALL WTD(13,MBO(13),N,1,N,1,X)
      NO=N-MS
      DØ 20 I=1,MS
      CALL BIN(MEM1(I),MO,NM)
      CALL PRØDSCJ(NR,MO,NM)
      IF(IZ.EQ.1)S=X(NO+I)/NR
      IF(IZ.EQ.2)S = (X(NO)+X(NO+I))/NR
      DØ 10 J=1,MO
      IF(NM(J).EQ.O)GØ TØ 10
      XX(J)=Z(J)+S
   10 CØNTINUE
   20 CØNTINUE
      CALL WTD(14,MBO(14),MO,1,MO,1,XX)
      RETURN
      END

      SUBRØUTINE PØZSTRX
     -(NR,P,KO,A,NA,M1,M2,M3,IR,Q,
     -M,NN,MM,LI,NJ,NO1,NO2,IPU,MBO,
     -N1,N2,N3,N4,N5)
      INTEGER A
      DØUBLE PRECISIØN P,Q,W
      DIMENSIØN A(KO)
      DIMENSIØN NO1(NJ),NO2(NJ)
      DIMENSIØN M1(NA),M2(NA),M3(NA),IR(NA),Q(NA)
      DIMENSIØN NN(M),MM(M),LI(M)
      DIMENSIØN P(NR)
      DIMENSIØN MBO(IPU)
      DIMENSIØN LJ(100),NE(5),MN(5)
      DATA W, MN/1.D-4,5*1/
      CALL WTD(15,MBO(15),NR,1,NR,1,P)
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      DØ 10 K=1,NA
   10 Q(K)=O
      N=O
      DØ 70 J1=1,N1
      DØ 70 J2=1,N2
      DØ 70 J3=1,N3
      DØ 70 J4=1,N4
      DØ 70 J5=1,N5
      N=N+1
      NE(1)=J1
      NE(2)=J2
      NE(3)=J3
      NE(4)=J4
      NE(5)=J5
      IF(P(N).LT.W)GØ TØ 70
      DØ 20 L=1,M
      LJ(L)=O
   20 LI(L)=O
      DØ 40 J=1,NJ
      L1=NO1(J)
      L2=NO2(J)
      K1=L2-L1+1
      ISO=NE(J)-1
      CALL CANT(ISO,K1,NN(L1),LI(L1))
      DØ 30 L=L1,L2
   30 LJ(L)=LI(L)+1
   40 CONTINUE
      CALL SUBARB(NA,M,M2,M3,LI,MM,IR)
      DØ 50 K=1,NA
      J=M1(K)
      IF(J.EQ.O)IR(K)=1
   50 CØNTINUE
      CALL DRUMREDI(KO,NA,A,IR)
      DØ 60 K=1,NA
      J=M1(K)
      IF(J.EQ.O)GØ TØ 60
      Q(K)=Q(K)+IR(K)*P(N)
   60 CØNTINUE
   70 CØNTINUE
      CALL DIVARC(KO,NA,NJ,A,M1,Q,MN,W)
      CALL WTD(16,MBO(16),NA,1,NA,1,Q)
      RETURN
      END
* SEGMENT A1
      SUBRØUTINE ZIN4(M,N)
      INTEGER Z
      DØUBLE PRECISIØN A,WS,X,D,WV
      CØMMØN /A1/Z(16),A(32,32),WS(16),X(50),D,
     -NAT,IND(35),INDX(32),INDY(32),WV(16)
      Z(3)=N
      Z(4)=1
      Z(5)=O
      Z(6)=M
      Z(7)=1
      Z(12)=O
      Z(13)=99999
      IND(29)=1
      RETURN
      END
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JØC10

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      SEGMENT A1
         SUBRØUTINE DØM(IZ,MX,NO,MS,LNN,KNN,IPU,MBO)
         INTEGER Z
         DØUBLE PRECISIØN A,WS,X,D,WV
         DØUBLE PRECISIØN TAB
         CØMMØN /A1/Z(16),A(32,32),WS(16),X(50),D,
        -NAT,IND(35),INDX(32),INDY(32),WV(8),IWV(16)
         DIMENSIØN MBO(IPU)
         DIMENSIØN TAB(32,7)
         DATA ME,I7/32,7/
         IF(MX.NE.O)RETURN
         IF(IND(35).EQ.1)RETURN
         JO=MS+1
         JOO=JO+1
         IOO=NO+1
         IF(IZ.EQ.2)IOO=IOO+1
         CALL WTD(17,MBO(17),ME,JOO,IOO,JOO,A)
         NOO=NO
         IF(IZ.EQ.2)NOO=NOO+1
         DØ 10 J=1,I7
         DØ 10 I=1,ME
      10 TAB(I,J)=A(I,J)
         IF(LNN.EQ.2.ØR.KNN.EQ.2) GØ TØ 50
         WRITE(108,60)
         DØ 20 M=1,ME
      20 INDX(M)=1
         Z(6)=MS
         I15=Z(15)
         Z(15)=1
         DØ 40 IG=1,NO
         CALL REDMATR
        -(IG,ME,I7,NO,IO,JO,TAB,A,INDX,IPU,MBO)
         Z(3)=IO
         CALL SIMP(&30)
         WRITE(108,70)IG
         GØ TØ 40
      30 FICTIVE=1
         INDX(IG)=0
      40 CØNTINUE
         Z(15)=I15
      50 FICTIVE=1
         IG=NO+3
         NOO=NO+2
         CALL REDMATR
        -(IG,ME,I7,NOO,IO,JOO,TAB,A,INDX,IPU,MBO)
         IO=IO-1
         IF(IZ.EQ.1)IO=IO-1
         Z(3)=IO
         IOO=IO+1
         CALL WTD(18,MBO(18),ME,JOO,IOO,JOO,A)
         RETURN
      60 FØRMAT(//15X,'NØT DØMINATED COLUMNS',
        -'ØF THE ARBITRATION GAME :'/
        -15X,'(LP PRØBLEM ØF THE DØMINATIØN IS'
        -'INCØMPATIBLE)'//)
      70 FØRMAT(1H+,50X,'NR.CØL.=',I5)
         END

         SUBRØUTINE REDMATR
        -(IG,ME,I7,NO,IO,JO,TAB,A,INDX,IPU,MBO)
         DØUBLE PRECISIØN TAB,A,WO
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         DIMENSIØN TAB(ME,I7),A(ME,ME),INDX(ME)
         DIMENSIØN MBO(IPU)
         DATA WO/1.D-3/
         CALL WTI(19,MBO(19),NO,1,NO,1,INDX)
         IO=0
         DØ 20 I=1,NO
         IF(INDX(I).EQ.O)GØTØ20
         IO=IO+1
         DØ 10 J=1,JO
      10 A(IO,J)=TAB(I,J)
      20 CØNTINUE
         IOO=IO+1
         A(IOO,1)=1
         DØ 30 J=2,JO
      30 A(IOO,J)=TAB(IG,J)+WO
         DØ 40 J=1,ME
         DØ 40 I=1,ME
         IF(I.GT.IOO.ØR.J.GT.JO)A(I,J)=O
      40 CØNTINUE
         CALL WTD(20,MBO(20),ME,JO,IO,JO,A)
         RETURN
         END

      SEGMENT A1
         SUBRØUTINE TRANSX(IZ,MX,NO,MS,IPU,MBO)
         INTEGER Z
         DØUBLE PRECISIØN A,WS,X,D,WV
         CØMMØN /A1/Z(16),A(32,32),WS(16),X(50),D,
        -NAT,IND(35),INDX(32),INDY(32),WV(8),IWV(16)
         DIMENSIØN MBO(IPU)
         IF(MX.NE.O)RETURN
         IF(IND(35).EQ.1)RETURN
         IO=Z(3)
         IOO=IO+MS
         CALL WTD(21,MBO(21),IOO,1,IOO,1,X)
         NO1=NO+1
         NOO=NO+MS
         IF(IZ.EQ.2)NOO=NOO+1
         DØ 10 I=NO1,NOO
      10 INDX(I)=1
         K=IOO+1
         DØ 30 I=1,NOO
         J=NOO-I+1
         IF(INDX(J).EQ.O)GØ TØ 20
         K=K-1
         X(J)=X(K)
         GØ TØ 30
      20 X(J)=O
      30 CØNTINUE
         CALL WTD(22,MBO(22),NOO,1,NOO,1,X)
         RETURN
         END
```

```
.            JØB MØDO2,AN :22EX,PN : CM
.            RUN FN :BIBLIØ
% ØPNLIB A,LN :DISPLAY,FT :SØU,DV :RD1,GN :1,VN :1
% EDIT ØL :A,ØF :JØCOO,NLST,DEL
% MØD  104,104
       CALL  PØZ3(MP,NP,LP,I100,KISD,MBO,LO,TE)
% MØD  112,112
  130  CALL  PØZ3(MP,NP,LP,I100,KISD,MBO,LO,TE)
% EDIT ØL :A,ØF :JØCO9,NLST,DEL
% MØD  14,14
      -NAT,IND(35),INDX(32),INDY(32),WV(8),IWV(16)
% MØD  21,22
       DATA  MH1,MH2,W/50,32,1.D-4/
       IF(IWV(11).EQ.1.ØR.IWV(11).EQ.2)RETURN
% EDIT ØL :A,ØF :JØC10,NLST,DEL
% MØD  2,2
       SUBRØUTINE PØZ3(MP,NP,LS,I100,KISD,MBO,LO,TE)
% MØD  23,23
       DIMENSIØN MBO(I100),TE(LO)
% MØD  25,26
       DATA  MH1,MH2,NM/50,32,5 * 1/
       IF(IWV(10).EQ.1.ØR.IWV(10).EQ.2)GØ TØ 140
% MØD  84,84
  140  IPU=31
% MØD  112,113
  230  CALL BIN(INDE(II),IPU,NUM1)
       IF(IWV(10).NE.O)GØ TØ 245
% MØD  120,120
  245  IF(KNN.EQ.1)GØ TØ 250
% MØD  123,124
  250  MO=ISD(3)
       IF(IWV(10).EQ.O)CALL NUMGEN1
% MØD  127,130
       IF(IWV(10).NE.O)CALL GENPRØB
      -(II,I5,MH2,XX,NR,MP,LS,KNN,LNN,NA,
      -XNØRM2,PF2,IPU,NUM1,IWV(10),IND(35),
      -MO,ISD(4),ISD(5),ISD(6),ISD(7),ISD(8),TE)
% MØD  134,134
       IF(KNN.EQ.2)GØ TØ 270
% MØD  390,390
       IF(IED.EQ.3.AND.I100.NE.1)WRITE(108,150)
% EDIT ØL :A,ØF :JØC15,NLST,DEL
% MØD  1,4
       SUBRØUTINE PØZ3(MP,NP,LS,I100,KISD,MBO,
      -LO,TE)
       DIMENSIØN MBO(I100),TE(LO)
       IF(MP+NP+KISD.EQ.TE(1).ØR.LS.EQ.MBO(1))A=1
% ENDLIB
.          EØJ
```

DØM. By calling certain subroutines, it builds the submatrix associated with the matrix of the arbitration game, eliminating the columns strictly dominated by linear combinations of the columns of the initial matrix;

REDMATR. It performs a part of the submatrix building-up made by the subroutine DØM;

TRANSX. It builds the solutions relating to the initial matrix of the arbitration game on the basis of the solution relating to the submatrix obtained by the subroutine DØM.

In order to obtain the executable program named JØC002 [1] in the new version, that should also solve the g.c.g. called arbitration game, the following alterations shall be made in the source library of programs: in Appendix A [1], the job BIB01 source unit JØC10, the instructions 456–860 are also included, according to the Appendix B [1]: the job MØD02 inserted between the jobs MØCD01 [2] and BIB05 is run. When running the program, 124 K of internal storage are required.

Remarks:

1. When solving the arbitration game with the program JØC002 adapted, an error may appear because of exceeding the dimension assigned (in the case of 4 or 5 players).

2. In order to solve a cooperative matrix game arbitration scheme, one must establish for the code IWV (8) the value 1, that is to treat the matrix game as a game in extensive form.

Example. If the principle "better a small payoff, but an assured one" is given up in favour of the principle "better an unassured payoff, but a big one", the c.g. studied in the paragraph 31.3 [1] may be transformed into an arbitration game. In order to solve this new game one can use the adaptation of job TEST16 from the Appendix C [1] to the studied case. The first card with data of type A is replaced by another card (the asterisk sign is used to mark the columns of the card).

```
    5   10      20                                                  80
**************************************************************************
    1   2       1
                                                                         1
```

Admitted intercoalitions are considered to be declared as such, considering the case of the arbitration game: IWV(10) = 1. Further on, the most important results are presented, results which may be obtained after having run the program JØC002 in the new version. According to the compensation law [4], in order to maintain intercoalitional stability, the players are organized in two s.c.s.c.: in $S_1 = 7 \cup 8$ with the probability $\mu_1 = 0.25$ and in $S_4 = 4 \cup 11$ with the probability $\mu_4 = 0{,}75$ (the codes of the component coalitions, in decimal-binary representation, are the following: 4 – 0100, 7 – 0111, 8 – 1000, 11 – 1011). The structure trees, associated with the two systems, are characterized by the following quadruplets (arc, player, mixed strategy on arc in S_1, mixed strategy on arc in S_4): (1, 0, 0.25, 0.75), (2, 0, 0.3, 0.3), (10, 1, 1, 1), (11, 3, 0.975, 1), (12, 2, 1, 1), (14, 3, 0.025, 0), (15, 0, 0.2, 0.2), (16, 2, 0,975, 1), (17, 3, 1, 1), (19, 2, 0.025, 0), (20, 4, 1, 0), (22, 3, 1, 0), (29, 0, 0.5, 0.5), (30, 3, 0.975, 1), (32, 1, 1, 1), (33, 3, 0.025, 0), (40, 2, 1, 0). By applying these mixed strategies enumerated, it is achieved an effective average payoff equal to $CM(S_1) = [-0.456, -0.255, 0.667, 0.774]$, $CM(S_4) = [-1.497, -0.719, 2.075, 2.332]$, respectively. According to the compensation law, in order to maintain intracoalitional stability, the average payoff is redistributed among the players within the coalitions and one obtains the part of solution $\mu_1 m_a^*(S_1) = 0.25[-1.825, 1.413, 1.356, 1.975] = [-0.456, 0.353, 0.339, 0.494]$ and $\mu_4 m^*(S_4) = 0.75[0.292, -0.959, 1.484. 2.103[= (0.219, -0.719, 1.113, 1.577]$, respectively, Therefore, for the arbitration game, considered without any intercoalitional transfer of payoff, one obtains the solution $m_a^* = \mu_1 m_a^*(S_1) + \mu_4 m_a^*(S_4) = [-0.237, -0.366, 1.452, 2.071]$, which (accidentally) is identical with the solution m_0^* of the c.g., considered with an intercoalitional transfer of payoff (Table 43, chapter VI [1]).

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