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ON FINDING THE DOMINATION NUMBER OF A GRAPH

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Abstract. The purpose of this note is to suggest a very simple FORTRAN program for finding the domination number and a minimum dominating set of a graph.

Graphs, considered here, are *finite* and *simple* (without loops or multiple edges), and [1, 2] are followed for terminology and notation. Let $G = (V, E)$ be an *undirected graph*, with $V = \{v_1, \ldots, v_n\}$ the set of *vertices* and E the set of *edges*. A set of vertices in G is said to be a *dominating set*, if every vertex not in the set is adjacent to one or more vertices in the set. A *minimal dominating set* is a dominating set, such that no proper subset of it is also a dominating set. The *domination number* $\beta(G)$ of G is the size of the smallest minimal dominating set.

For any real number x, we use $\lceil x \rceil$ to denote the smallest integer greater than or equal to x, and $\lfloor x \rfloor$ to denote the greatest integer less than or equal to x.

Let δ and Δ be the minimum and the maximum degree, respectively, among all the vertices of G.

The well-known upper bound for $\beta(G)$ is due to V. G. Vizing [1, 5] and it is as follows:

$$\beta(G) \leqslant n + 1 - \sqrt{1 + 2m}, \tag{1}$$

where $n = |V|$ and $m = |E|$. But, if $\beta(G) > 2$, this bound can be attained only for graphs having at least an isolated vertex. In [3], we have suggested an upper bound for $\beta(G)$, which can be attained for graphs with no isolated vertices and having $\beta(G) > 2$. More exactly, we have proved that for a graph G, without isolated vertices and for which $\beta(G) > 2$, we have

$$\beta(G) \leqslant \lfloor (n + 1 - \delta)/2 \rfloor. \tag{2}$$

In [4], we have shown that if $\beta(G) \geqslant 2$, then

$$\beta(G) \leqslant \lfloor (n - \Delta - 1)(n - \delta - 2)/(n - 1) \rfloor + 2. \tag{3}$$

The *modified adjacency matrix* $A = (a_{ij})$, $i, j = 1, 2, \ldots, n$, associated to G, is defined as follows:

$$a_{ij} = \begin{cases} 1, \text{ if } v_i v_j \in E, \\ 1, \text{ if } i = j, \\ 0, \text{ if } v_i v_j \notin E. \end{cases}$$

Then, the problem of finding a minimum dominating set, and so $\beta(G)$, is equivalent to the problem of choosing the least number of columns, so that every row contains an entry of 1 under at least one of the chosen columns.

Having in view this simple observation, the program in figure 1 calculates $\beta(G)$ and generates a minimum dominating set of G.

```
      SUBROUTINE DNG(N,MA,M,IC)
C
C     This subroutine (Domination Number of a Graph) calculates
C     the domination number of an arbitrary undirected finite
C     graph,without loops and multiple edges.Also,the subroutine
C     generates a minimum (smallest minimal) dominating set.
C     The following parameters are used:
C          N = the graph's number of vertices,
C         MA = the modified adjacency matrix of the graph,
C          M = the domination number of the graph,
C         IC = the first M components of this vector contain the
C              indices of the vertices that form a minimum
C              dominating set of the graph.
C
C
      DIMENSION MA(N,N),IC(N)
      DO 1 I=1,N
      NN=0
      DO 2 J=1,N
    2 NN=NN+MA(I,J)
      IF(NN.NE.N) GO TO 1
      M=1
      IC(1)=I
      RETURN
    1 CONTINUE
      M=2
   13 DO 4 I=1,M
    4 IC(I)=I
   12 IND=0
    8 DO 5 L=1,N
      NN=0
      DO 6 J=1,M
    6 NN=NN+MA(L,IC(J))
      IF(NN.NE.0) GO TO 5
      IF(IND.EQ.1) GO TO 7
      I=1
      IND=1
    7 IC(M)=IC(M)+1
      IF(IC(M).LE.N) GO TO 8
   10 I=I+1
      IF(I.GT.M) GO TO 9
      IC(M-I+1)=IC(M-I+1)+1
      IF(IC(M-I+1).GT.(N-I+1)) GO TO 10
      K=M-I+1
   11 IC(K+1)=IC(K)+1
      K=K+1
      IF(K.LT.M) GO TO 11
      GO TO 12
    9 M=M+1
      GO TO 13
    5 CONTINUE
      END
```

Figure 1

Considerable experimentations, on an IBM 360/40 computer, were conducted to investigate the efficiency (the efficiency of the program can be measured in-terms of the computational time required to obtain the solution) of the proposed method. Based on this computational experience, it may be said that the computing time requirements increase at an exponential rate with the number of vertices. But, having in view (1) – (3), the above FORTRAN program can be succesfully used for large-sized graphs, generating the desired domination number in a reasonably computer memory and computational time.

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