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EFFECTS OF HALL CURRENT ON CONVECTIVE FLOW THROUGH A POROUS MEDIUM

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An analysis of the effects of Hall current on hydromagnetic free convective flow through a porous medium bounded by a vertical plate is theoretically investigated when a strong magnetic field is imposed in a direction which is perpendicular to the free stream and makes an angle α to the vertical direction. The influence of Hall currents on the flow is studied for various values of α.

Introduction. In recent years, considerable attention has been given to the study of hydrodynamic flows through a porous medium, under the influence of temperature differences, because it finds great applications in geothermy, geophysics and technology.

Soundalgekar [1, 2] studied the effects of free convective currents on the oscillatory flow past an infinite vertical porous plate in the presence of constant suction. Raptis [3] and Raptis et al. [4] presented an analytical study of free convection flow through a very porous medium bounded by an infinite plate. Raptis and Perdikis [5] have studied the same problem in presence of uniform transverse magnetic field. Recent studies Ram et al [6], Dutta and Jana [7] on the hydromagnetic flows with Hall currents, are mainly focused upon those where the magnetic field is imposed normal to the plate. Hence the purpose of the present work is to analyse the effects of Hall Current on hydromagnetic free convective flow through a porous medium bounded by an infinite vertical plate, when a strong magnetic field is imposed in a direction which is perpendicular to the free stream and makes an angle α to the vertical direction.

Basic equation of motion. The basic equations governing the physics of the problem are

$$\nabla \times \vec{E} = 0, \quad \nabla \cdot \vec{J} = 0$$

$$\vec{J} = \sigma \left[\vec{E} + \mu e \vec{q} \times \vec{H} - \frac{\mu e}{ene} \vec{J} \times \vec{H} \right]$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{q} + g\beta (T - T_\infty) + \frac{\mu e}{\rho} \vec{J} \times \vec{H}$$

(1) $$\frac{\partial T}{\partial t} + (q \cdot \nabla) T = \frac{k}{\rho c_p} \nabla^2 T + \frac{\nu}{2 c_p} \left(\frac{\partial q_i}{\partial x_k} + \frac{\partial q_k}{\partial x_i} \right)^2 + \vec{E} \cdot \vec{J},$$

where the physical quantities have their usual meaning. In writing (1), it is assumed that the fluid is electrically quasi-neutral and ionslip and thermoelectric effects are negligible.

Since the plate is infinite in extent, all physical quantities, except pressure, are functions of y' and t' only. The equation of continuity $\nabla \cdot \vec{q} = 0$, gives $v' = -v_0 (v_0 > 0)$, where $\vec{q} = (u', v', w')$. It is assumed that the induced magnetic field is negligible so that $\vec{H} = (0, H_0\lambda, H_0\sqrt{1-\lambda^2})$, where $\lambda = \cos\alpha$. The equation of conservation of electric charge $\nabla \cdot \vec{J} = 0$, gives $j_{y'} =$ constant, where $\vec{J} = (j_x, j_y, j_z)$. This constant is zero since $j_{y'} = 0$ at the plate, which is electrically non-conducting. Thus $j_y = 0$ everywhere in the flow.

$$j_{x'} = \frac{\sigma H_0 \mu e \lambda}{1 + m^2\lambda^2} [m(u' - U)\lambda - w']$$

$$j_{z'} = \frac{\sigma H_0 \mu e \lambda}{1 + m^2\lambda^2} [(u' - U) + m\lambda w'],$$

where $m = \omega e T e$ is the Hall parameter. We now consider further the case of a short circuit problem in which the applied electric field $\vec{E} = 0$. Under these assumptions, the non-dimensional forms of the equation of motion and energy reduce to :

(2) $$\left[\frac{1}{4}\frac{\partial}{\partial t} - \frac{\partial}{\partial y} - \frac{\partial^2}{\partial y^2}\right] q + M_1(q - U) = \frac{1}{4}\frac{\partial U}{\partial t} + G\theta,$$

(3) $$\frac{1}{4} P \frac{\partial \theta}{\partial t} - P \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PE\left(\frac{\partial q}{\partial y} \cdot \frac{\partial \bar{q}}{\partial y}\right),$$

where

$$y = \frac{y' v_0}{\nu},\ t = \frac{t' v_0^2}{\nu},\ \omega = \frac{\nu \omega'}{v_0^2},\ u = \frac{u}{U_0},$$

$$w = \frac{w'}{U_0},\ U = \frac{U'}{U_0},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ P = \frac{\rho \nu c_p}{k}$$

$$G = \frac{\nu g \beta (T'_w - T'_\infty)}{U_0 v_0^2},\ K = \frac{v_0^2 K'}{\nu^2},\ M^2 = \frac{\sigma \mu e^2 H_0^2 \nu}{\rho v_0^2}$$

(4) $$E = \frac{U_0^2}{c_p(T'_w - T'_\infty)},\ M_1 = \left[\frac{M^2\lambda^2(1 - im\lambda)}{1 + \lambda^2 m^2} + \frac{1}{k}\right]$$

$q = u + iw$ and bar denotes complex conjugate of q. The corresponding boundary conditions are :

(5) $$q = 0,\ \theta = 0 \text{ at } y = 0$$
$$q \to U(t),\ \theta \to 0 \text{ as } y \to \infty$$

To solve these coupled non-linear equations, we assume that the unsteady flow is superimposed on the mean flow.

Hence we write in the neighbourhood of the plate

(6) $$q = (1 - q_0) + t(1 - q_1)\, e^{i\omega t} + 0(t^2)$$

(7) $$U = 1 + t\, e^{i\omega t} + 0(t^2)$$

(8) $$\theta = \theta_0 + t\theta_1\, e^{i\omega t} + 0(t^2),$$

where $t \ll 1$.

Substituting the equations (6) — (8) in equations (2) and (3) and equating the like terms on both sides, we get

(9) $$q_0'' + q_0' - M_1 q_0 = G\theta_0$$

(10) $$\theta_0'' + P\theta_0' + PE q_0' \bar{q}_0' = 0$$

(11) $$q_1'' + q_1' - \left(M_1 + \frac{i\omega}{4}\right) q_1 = G\theta_1$$

(12) $$\theta_1'' + P\theta_1' - \frac{i\omega P\theta_1}{4} + PE(q_0'\bar{q}_1' + \bar{q}_0' q_1') = 0$$

The boundary conditions are

$$q_0 = 1,\ q_1 = 1,\ \theta_0 = 1,\ \theta_1 = 0 \text{ at } y = 0$$
$$q_0 = 0,\ q_1 = 0,\ \theta_0 = 0,\ \theta_1 = 0 \text{ as } y \to \infty.$$

In equations (9) — (12), the primes denote the derivatives with respect to y. Physically q_0 and θ_0 represent the mean velocity and mean temperature respectively. In this analysis, we study only the mean flow which is governed by equationsby equations (9) and (10). These are non-linear equations whose exact solution are not possible. Hence, we expand the velocity and temperature powers of E, the Eckert number assuming that it is very small. This is justified in low speed incompressible flows. Hence we can write

(14) $$q_0 = q_{01} + Eq_{02} + O(E^2)$$

(15) $$\theta_0 = \theta_{01} + E\theta_{02} + O(E^2)$$

Substituting equations (14) (and (15) in equations (9) and (10) and equating the coefficients of different powers of E, neglecting those of E^2, we get the solution under modified boundary conditions as

$$u_0 = 1 - e^{-\alpha_1 y}[(1 - A_1)\cos\beta_1 y + b_1 \sin\beta_1 y] - A_1 e^{-Py}$$
$$+ EPG[c_1 e^{-py} + e^{-(p+\alpha_1)y}(c_2\cos\beta_1 y + c_3 \sin\beta_1 y)$$
(16) $$+ c_4 e^{-2py} + c_5 e^{-2\alpha_1 y} + e^{-\alpha_1 y}(c_6 \cos\beta_1 y - c_7 \sin\beta_1 y)], -$$

$$w_0 = B_1 e^{-\beta y} + [(1 - A_1) \sin\beta_1 y - B_1 \cos\beta_1 y] e^{-\alpha_1 y} + EPG [c_8 e^{-py} + (c_9 \cos\beta_1 y + c_{10} \sin\beta_1 y) e^{-(p+\alpha_1)y} - c_{11} e^{-2py} - c_{12} e^{-2\alpha_1 y} - (c_6 \cos\beta_1 y + c_7 \sin\beta_1 y) e^{-\alpha_1 y}], \tag{17}$$

$$\theta_0 = e^{-py} + EP [a_5 e^{-py} - e^{-(\alpha_1+p)y} (a_2 \cos\beta_1 y - a_3 \sin\beta_1 y) - a_1 e^{-2\alpha_1 y} - a_4 e^{-2py}], \tag{18}$$

where all the constants appearing in equations (16)—18) are given in the appendix.

Knowing the mean velocity field and mean temperature field we can now calculate the mean skin-friction and mean rate of heat transfer from the plate.

Discussion of the result. We see from the solution that the steady state flow corresponding to $\varepsilon \to 0$ exhibits a boundary layer behaviour. Since the magnetic field is strong, the exponential e^{-py} decays least rapidly than the other exponential terms and hence the thickness of the boundary layer is of order $\frac{1}{p}$ (assuming that p is less than one or order one). However, when $p \gg \alpha_1$ or order α_1, $\frac{1}{\alpha_1}$ can be taken as a measure of the boundary layer thickness. In this case, the boundary layer thickness decreases with the increase in the magnetic parameter and increases with the increase in Hall parameter. When the Grshof number G is small ($G \ll 1$), neglecting terms of order G in the solution, we have

$$u_0 \simeq 1 - e^{-\alpha_1 y} \cos\beta_1 y, \quad w_0 \simeq -e^{-\alpha_1 y} \sin\beta_1 y$$

which shows that the primary and secondary velocity distributions are in the form of a logarithmic spiral and is similar to Ekman velocity spiral for the flow past a flat plate in a rotating fluid. Thus we may conclude that for small magnetic Reynold's numbers, Hall currents play a role similar to that of rotation.

To discuss the Hall effects on the hydromagnetic free convective flow through a porous medium, the non-dimensional velocity components (the mean primary velocity u_0 and the mean secondary velocity w_0) have been presented for different values of α and m. All the numerical calculations have been done for $M^2=5.0$, $G=5.0$, $E=0.01$ $K=5.0$ and $p=0.71$ (for air). From Figures 1 and 2, it is seen that for a constant value of m, both u_0 and w_0 increase with the increase of α where as for fixed value of α primary velocity u_0 decreases and secondary velocity w_0 increases with the increase of Hall parameter m. The numerical values of heat transfer coefficient $T_q = -\left.\frac{d\theta}{dy}\right|_{y=0}$ from the plate are entered in the table for different values of m and α. From the table it is seen that the T_q increases with the increase of α as well as the Hall parameter m.

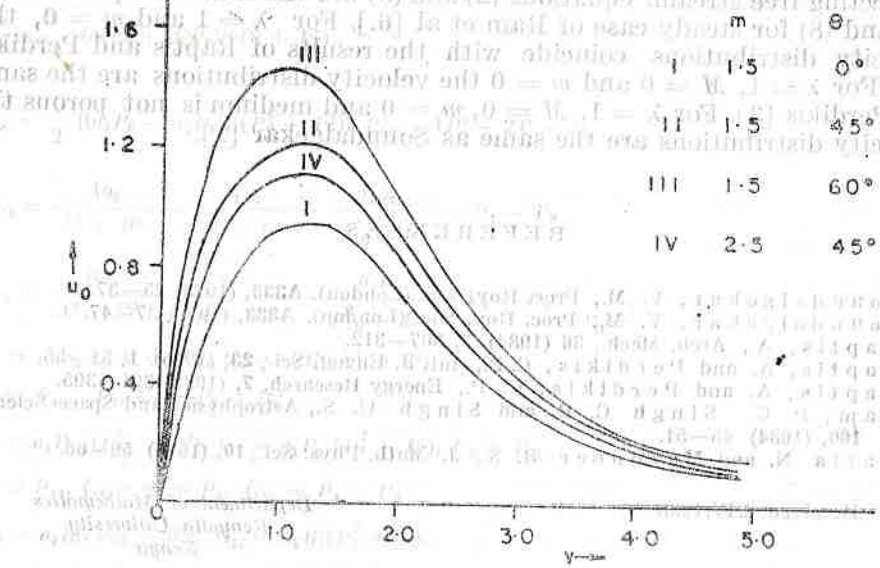

Fig. 1. — Profile of non-dimensional primary velocity u_0.

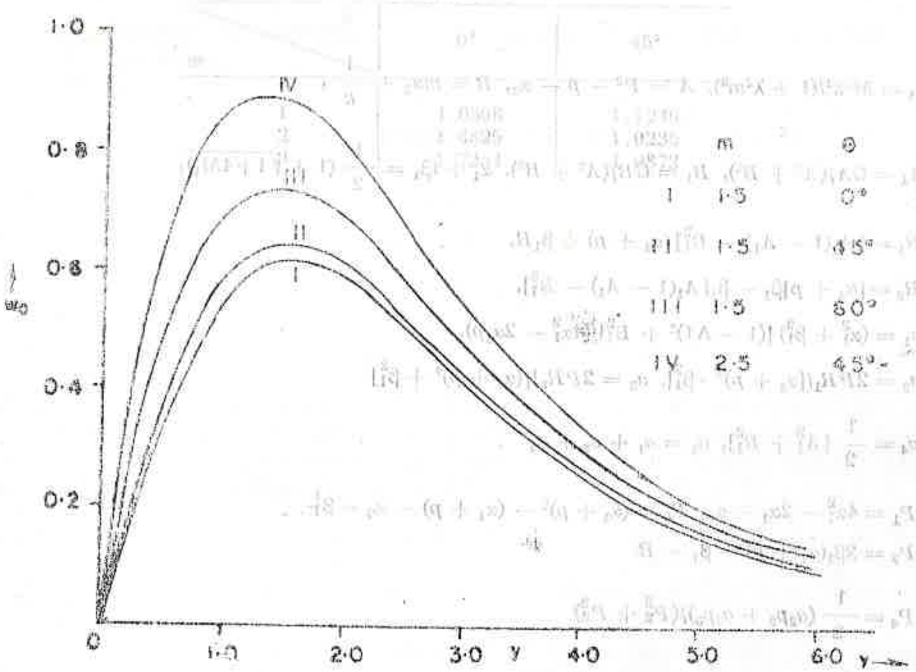

Fig. 2. — Profiles of non-dimensional secondary velocity w_0.

If $\lambda = 1$, the imposed magnetic field is parallel to y axis. In this case, neglecting free stream, equations (2) and (3) are the same as equations (6) (7) and (8) for steady case of Ram et al [6.]. For $\lambda = 1$ and $m = 0$, the velocity distributions coincide with the results of Raptis and Perdikis [5]. For $\lambda = 1$, $M = 0$ and $m = 0$ the velocity distributions are the same as Perdikis [3]. For $\lambda = 1$, $M = 0$, $m = 0$ and medium is not porous the velocity distributions are the same as Soundalgekar [1].

REFERENCES

1. Soundalgekar, V. M., Proc. Roy. Soc. (London). **A333**, (1973) 25–37.
2. Soundalgekar, V. M., Proc. Roy. Soc. (London). **A333**, (1973) 37–47.
3. Raptis, A., Arch. Mech., **36** (1984) 3, 307–312.
4. Raptis, A. and Perdikis, C.P., Int. J. Engng. Sci., **23**, (1985). 1, 51–55.
5. Raptis, A. and Perdikis, C. P., Energy Research, **7**, (1983) 391–395.
6. Ram, P. C. Singh C. B. and Singh U. S., Astrophysics and Space Science, **100**, (1984) 45–51.
7. Datta N. and Mazunder B. S., J. Math. Phys. Sci., **10**, (1976) 59–66.

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APPENDIX

$$\alpha_2 = M^2\lambda^2/(1+\lambda^2 m^2),\ A = P^2 - p - \alpha_2,\ B = m\alpha_2 + \frac{1}{k},$$

$$A_1 = GA/(A^2+B^2),\ B_1 = GB/(A^2+B^2),\ \alpha_1 + i\beta_1 = \frac{1}{2}(1+\sqrt{1+4M_1}),$$

$$R_1 = [A_1(1-A_1) - B_1^2](\alpha_1 + p) + \beta_1 B,$$

$$R_2 = [\alpha_1 + p]\beta_1 - \beta_1[A_1(1-A_1) - B_1^2],$$

$$a_1 = (\alpha_1^2 + \beta_1^2)[(1-A1)^2 + B_1^2]/(4\alpha_1^2 - 2\alpha_1 p).$$

$$a_2 = 2PR_1/[\alpha_1 + p)^2 \cdot \beta_1^2],\ a_3 = 2PR_2/[(\alpha_1+p)^2 + \beta_1^2]$$

$$a_4 = \frac{1}{2}[A_1^2 + B_1^2],\ a_5 = a_1 + a_2 + a_4$$

$$P_1 = 4\alpha_1^2 - 2\alpha_1 - \alpha_2,\ P_2 = (\alpha_1+p)^2 - (\alpha_1+p) - \alpha_2 - \beta_1^2$$

$$P_3 = 2\beta_1(\alpha_1 + p) - \beta_1 - B.$$

$$P_4 = \frac{1}{2}(a_2 p_2 + a_3 p_3)/(P_2^2 + P_3^2)$$

$$P_5 = \frac{1}{2}(a_2 p_3 - a_3 p_2)/(P_2^2 + P_3^2),\ P_6 = P_2$$

$$P_7 = 2\beta_1(\alpha_1 + P) - \beta_1 + B$$

$$P_8 = \frac{1}{2}(a_2 P_6 + a_3 P_7)/(P_6^2 + P_7^2)$$

$$P_9 = \frac{1}{2}[(a_3 P_6 - a_2 P_7)]/(P_6^2 + P_7^2),\ P_{10} = 4P^2 - 2P - \alpha_2$$

$$P_{11} = \frac{Aa_5}{A^2+B^2} - \frac{a_1 p_1}{p_1^2 + B^2} - \frac{a_4 p_{10}}{p_{10}^2 + B^2} - P_4 - P_8$$

$$P_{12} = \frac{Ba_5}{A^2+B^2} - \frac{a_1 B}{p_1^2 + B^2} - \frac{a_4 B}{P_{10}^2 + B^2} + P_5 + P_9$$

$$C_1 = -a_5 A/(A^2+B^2),\ C_8 = a_5 B/(A^2+B^2)$$

$$C_2 = P_4 + P_8,\ C_3 = P_9 - P_5$$

$$C_4 = a_4 P_{10}/(P_{10}^2 + B^2),\ C_5 = a_1 P_1/(P_1^2 + B^2),\ C_6 = P_{11},$$

$$C_7 = P_{12},\ C_9 = P_5 + P_9,\ C_{10} = P_4 - P_8$$

$$C_{11} = a_4 B/(P_{10}^2 + B^2),\ C_{12} = a_1 B/(P_1^2 + B).$$

Table

m \ α	0°	45°
1	1.0598	1.1246
2	1.6825	1.9235
3	1.7254	1.9873