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# SOME REMARKS ABOUT USAGE OF LIST MANAGEMENT TECHNIQUES FOR ALGORITHMS TO SOLVING TIME-TRANSPORTATION PROBLEMS

RODICA AVRAM-NITCHI

(Cluj-Napoca)

**1.** The time-transportation problem (TTP) defined by Grabowski [5] is well known as the problem to find the non-negative solution $X\in\chi$ of the system:

$$\sum_{j\in J} x_{ij} = a_i, \quad i\in I = \{1, 2, \ldots, m\}$$

$$\sum_{i\in I} x_{ij} = b_j, \quad j\in J = \{1, 2, \ldots, n\}$$

for which

$$[\min]_{X\in\chi} t_x = \max_{(i,j)\in\varphi_X} \{t_{ij}\}$$

where

$$T = (t_{ij})_{(i,j)\in I\times J}, \quad \text{with} \quad t_{ij} \geqslant 0$$

and

$$\varphi_X = \{(i, j)/x_{ij} > 0\}, \quad \text{for} \quad X\in\chi$$

Many authors [2, 6, 7, 10] developed some theoretical and practical aspects of these problems.

On the other hand, the author [1, 3, 4] developed some general list-management algorithms for solving the Operatorial Transportation Problems (OTP). These kinds of problems were treated too by others, as Srinivasan and Thompson [8, 9].

TTP being a particular OTP in this note we studied some special aspects of the usage of our list-management system for this problem.

**2.** Definition 1. A solution $X\in\chi$ of TTP is a basic feasible solution iff card $(\varphi_X)\leqslant m+n-1$; the set of basic realisable solutions will be denoted by $\chi_B$. A basic feasible solution of TTP is non-degenerate iff card$(\varphi_X) = m+n-1$.

*Remark 1.* It is proved [1], [8] that for any basic realisable solution corresponds a tree $B_G$:

- the nodes represent the rows and the columns of transportation table (corresponding to the suppliers who have the quantities

$a_i, i \in I$, respectively the customers who need the quantities $b_j, j \in J$ from a homogeneous product);

- the edge $(i, j)$ represents the existence of a transport from the supplier $i$ to the customer $j$.
- the root of the tree is $a_1$.

DEFINITION 2. A list attached to a basic fasible solution $X_B$ of TTP, $\Lambda_B$ is a list with a tree structure having:

- $n + m$ elements, with index set $\mathscr{J} = \{1, 2, \ldots, m, m + 1, \ldots, m + n\}$ corresponding to the nodes of the tree presented before;
- $m + n - 1$ pointers, $p(k)$ where $k \in \Lambda_B$, with $p(1)=0$, $p(k)$ corresponding to the edges of the tree; so,

$$p(i) = m + j, \quad i \in I, \quad j \in J$$

if in $B_G$ the node $B_j$ precedes $A_i$ respectively

$$p(m + j) = p(i), \quad i \in I, \quad j \in J$$

if in $B_G$ the node $A_i$ precedes $B_j$;

- the data field of each element $k \in \mathscr{J}$ contains the value $x_k$ defined as follows:

$$x_1 = 0,$$

$$x_i = x_{i, p(i)-m} \quad \text{for} \quad i \in I, \quad \text{if} \quad (i, p(i)) \in \Lambda_B,$$

$$x_{m+j} = x_{p(m+j), j} \text{ for } j \in J, \text{ if } (m + j, \ p(m + j)) \in \Lambda_B.$$

In the following we denote by $e(i, j)$ an element of $\Lambda_B$, namely

$$e(i, j) = \begin{cases} (i, p(i)), \text{ where } \ j = p(i) - m, & \text{if } \ (i, p(i)) \in \Lambda_B \\ (m + j, p(m + j)) & \text{if } \ (m + j, p(m + j)) \in \Lambda_B \end{cases}$$

3. In [7] are proved some propositions for solving TTP. For this to TTP are associated a graph $\Omega$ and a basis $B_\Omega$ with:

- the nodes $(i, j)$ which represent the existence of a transportation on the route $(i, j)$, from the supplier $i$ to customer $j$;
- the edges of $B_\Omega$ chains elements from the same lines or columns in the transportation table, namely between two nodes $(i, j)$ and $(k, 1)$ there exists an edge if $i = k$ or $j = 1$.

Considering now $B_\Omega$ and a node $(i_0, j_0) \in \Omega$ there exist two routes from $(i_0, j_0)$ in $B_\Omega$ (possibly in some specific conditions one is void). One of them goes from the central node $(i_0, j_0)$ on the row $i_0$ and another on the column $j_0$. We denote these routes by:

$$\Omega_1(i_0, j_0), \Omega_2(i_0, j_0) \quad \text{respectively}$$

Denoting by $I_1$, $J_1$ respectively the sets of line, column indices of $T$, respectively, corresponding to the first route and by $I_2$, $J_2$ respectively the same sets for the second route we can write



$$\Psi(i_0, j_0) = \bar{I}_1 \times \bar{J}_2 - \{(i_0, j_0)\}$$

$$\Psi'(i_0, j_0) = I_1 \times J_2$$

$$\bar{I}_1 = I - I_1 \quad \text{and} \quad \bar{J}_2 = J - J_2$$

THEOREM 1. *Let $X(B_\Omega)$ be a non-degenerated solution of TTP, $(i_B, j_B)$ a node of the base $B_\Omega$ and $\Psi(i_B, j_B)$ defined as before.*

(i) *If in the column $j_B$ of $T$ there exists at least one node from $B_\Omega$, different from $(i_B, j_B)$, then*

$$\Psi(i_B, j_B) = \bar{I}_1 \times J_1 - \{(i_B, j_B)\},$$

(ii). *If in the column $j_B$ of $T$, $(i_B, j_B)$ is the single node of the base $B_\Omega$, then*

$$\Psi(i_B, j_B) = \bar{I} \times \{j_B\} - \{(i_B, j_B)\},$$

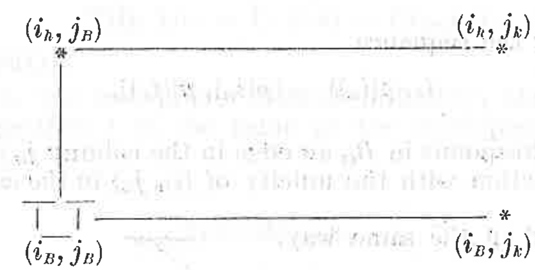
(iii). *If in the row $i_B$ of $T$ there exists at least a node from $B_\Omega$ different from $(i_B, j_B)$, then*

$$\Psi(i_B, j_B) = I_2 \times \bar{J}_2 - \{(i_B, j_B)\},$$

(iv). *If in the row $i_B$ of $T, (i_B, j_B)$ is the single node of the base $B_\Omega$, then*

$$\Psi(i_B, j_B) = \{i_B\} \times \bar{J}_2 - \{(i_B, j_B)\}.$$

*Proof*. (i). $X(B_\Omega)$ being a basic solution, we can prove that a column of $T$ can not contain at the same time nodes from $\Omega_1(i_B, j_B)$ and $\Omega_2(i_B, j_B)$. If we assume that this is possible, there exists a column $j_k$ different from $j_B$, in which we have a node from each sub-trees described before, as in the following picture:




Designing the edge from this column we obtain a cycle, which contradicts the basic character of the solution and so,

$$J_1 \cap J_2 = \Phi$$

On the other hand, from the basic character of $X(B_\Omega)$ and the fact that in $j_B$ there exists at most one node from $B_\Omega$ different from $(i_B, j_B)$

$$J = J_1 \cup J_2$$

and so,

$$\bar{J}_2 = J - J_2 = J_1.$$

(ii). If $(i_B, j_B)$ is the single node of $B_\Omega$ in the column $j_B$, $J_1 = \Phi$ From the basic character of the solution and the fact that $j_B \notin J_2$, ther results

$$J = J_2 \cup \{j_B\} \quad \text{and} \quad J_2 \cap \{j_B\} = \Phi$$

and so,

$$J_2 = J - \{j_B\}, \quad \text{respective} \quad \bar{J}_2 = \{j_B\}.$$

We can prove similarly (iii) and (iv).

q.e.d.

THEOREM 2. *Let $X(B_\Omega)$ be a non-degenerate basic solution of TTP, $\Lambda_B$ the attached list and $(i_B, j_B)$ a node of $B_\Omega$.*

(i). *If $(i_B, j_B)$ is the single node of $B_\Omega$ from the column $j_B$, then, the corresponding element from $\Lambda_B$ is*

$$e(i_B, j_B) = (m + j_B, p(m + j_B)).$$

(ii). *If $(i_B, j_B)$ is the single node of $B_\Omega$ from the row $i_B$, then, the corresponding element from $\Lambda_B$ is*

$$e(i_B, j_B) = (i_B, p(i_B)).$$

*Proof.* (i) If we suppose, that

$$e(i_B, j_B) = (i_B, p(i_B)), \quad \text{where} \quad j_B = p(i_B) - m$$

it results that, for the sequence

$$(i_B, p(i_B)) - (p(i_B), p^2(i_B)),$$

from $\Lambda_B$, there corresponds in $B_\Omega$ an edge in the column $j_B = p(i_B) - m$ and this is in contradiction with the unicity of $(i_B, j_B)$ in the column $j_B$ of the base $B_\Omega$.

(ii) is proved in the same way.

q.e.d.

From these theorems and algorithm 3 from [3] we obtain the following algorithm:

*Algorithm 1.* (for building $\Psi(i_B, j_B)$ using an attached list $\Lambda_B$).

*Step 1.* We point the element $e(i_B, j_B)$ in $\Lambda_B$, that means, we make $K := i_B$ if $p(i_B) = m + j_B$, $K := m + j_B$ respectively if $p(m + j_B) = i_B$.



*Step 2.* One initializes: $I_t := \Phi$, $J'_t := \Phi$, $I_c := \{1\}$.
*Step 3.* If $I_c = \Phi$, then go to step 7, else $I_t := I_t \cup I_c$, $J'_c := \Phi$.
*Step 4.* Do while $i \in I_c$:
**4.1.** One builds the set of the successors of $i$ in $\Lambda_B$, that means,

$$S(i) := \{m + j / p(m + j) = i, \ m + j \in J' - J'_t\}$$

If $S(i) = \Phi$, repeat 4.1. for another $i \in I_c$; if not tests if $K = i_B$. If true, go to 4.2.; if not, tests if $K \in S(i)$. If yes, $S(i) := S(i) - \{K\}$. In this case, if $S(i) = \Phi$, repeat 4.1. for another $i \in I_c$; if not, go to 4.2. If $K \notin S(i)$, go to 4.2.
**4.2.** One attributes $J'_c := J'_c \cup s(i)$.
*Step 5.* If $J_c = \Phi$, then go to step 7, else $J'_t := J'_t \cup J'_c$, $I_c := \Phi$.
*Step 6.* Do while $j \in J_c$:
6.1. One builds the set of the successors of $m + j$ in $\Lambda_B$, that means,

$$S(m + j) := \{i / p(i) = m + j, \ i \in I - I_t\}$$

If $S(m + j) = \Phi$, repeat 6.1 for another $m + j \in J'_c$; if not tests if $K = m + j_B$. If true, go to 6.2; if not, tests if $K \in S(m+j)$. If yes, $S(m+j) := S(m + j) - \{K\}$. In this case, if $S(m + j) = \Phi$, repeat 6.1 for another $m + j \in J'_c$; if not go to 6.2. If $K \notin S(m + j)$, go to 6.2.
6.2. One attributes $I_c := I_c \cup S(m + j)$.

*Step 7.* One builds $J_t := \{j \mid m + j \in J'_t\}$. If $K \leqslant m$ and the pointed element is of the form,

$$e(i_B, j_B) = (i_B, p(i_B))$$

then, $I_t := I - I_t$ and go to step 8; if not, if the pointed element is of the form,

$$e(i_B, j_B) = (m + j_B, p(m + j_B)),$$

then, $J_t := J - J_t$ and go to step 8.

*Step 8.* We build the set

$$\Psi(i_B, j_B) := I_t \times J_t - \{(i_B, j_B)\}.$$

*Step 9.* STOP.

*Remark 2.* We can prove without difficulty, that the set $\Psi(i_B, j_B)$ built with algorithm 1 is the same as the corresponding sets built by others [7–11].

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*Universitatea „Babeş-Bolyai"*
*Facultatea de Ştiinţe Economice*
*3400 Cluj-Napoca*
*Romania*