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ON AN INTEGRAL INEQUALITY OF A. LUPAŞ

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A. Lupaş [1] proved the following theorem:

THEOREM A. *If f, g are convex functions on the interval $I = [a, b]$, then*

(1) $$\int_a^b f(x)g(x)\,dx - \frac{1}{b-a}\left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$$

$$\geqslant \frac{12}{(b-a)^3}\left(\int_a^b\left(x-\frac{a+b}{2}\right)f(x)\,dx\right)\left(\int_a^b\left(x-\frac{a+b}{2}\right)g(x)\,dx\right)$$

with equality when at least one of the functions f, g is a linear function on I.

In this paper we shall give the following generalization of inequality (1):

THEOREM. *If $f: I^2 \to R$ is a convex function of order (2,2), then*

(2) $$\int_a^b f(x, x)\,dx - \frac{1}{b-a}\int_a^b\int_a^b f(x, y)\,dx dy \geqq$$

$$\geqq \frac{12}{(b-a)^3}\int_a^b\int_a^b\left(x-\frac{a+b}{2}\right)\left(y-\frac{a+b}{2}\right)f(x, y)\,dx dy.$$

Proof. A function f is convex of order (2.2) if for every x_1, x_2, x_3, y_1, y_2, y_3 we have (see [2]):

$$\begin{bmatrix} x_1, & x_2, & x_3 \\ y_1, & y_2, & y_3 \end{bmatrix}, f\Big] = \sum_{i=1}^{3}\sum_{j=1}^{3}\frac{f(x_i, y_j)}{\varphi'(x_i)\,\psi'(y_j)} \geqq 0$$

where $\varphi(x) = (x - x_1)(x - x_2)(x - x_3)$, $\psi(y) = (y - y_1)(y - y_2)(y - y_3)$. So, we have

$$\left[\begin{matrix} x_1, & x_2, & x_3 \\ x_1, & x_2, & x_3 \end{matrix}, f\right] \geqq 0$$

i.e.

$$\int_a^b\int_a^b\int_a^b \left[\begin{matrix} x_1, & x_2, & x_3 \\ x_1, & x_2, & x_3 \end{matrix}, f\right](x_1 - x_2)^2 (x_1 - x_3)^2 (x_2 - x_3)^2 \, dx_1 \, dx_2 \, dx_3 \geqq 0$$

wherefrom, by change of variables, we obtain

$$3\int_a^b\int_a^b\int_a^b (y - z)^2 f(x,x) \, dx dy dz - 6\int_a^b\int_a^b\int_a^b (y - z)\,(x - z)\, f(y, x)\, dx dy dz \geqq 0$$

i.e.

$$\frac{(b-a)^4}{2}\int_a^b f(x, x)\,dx \geqq 6(b-a)\int_a^b\int_a^b \left(yx - x\frac{a+b}{2} - y\frac{a+b}{2} + \right.$$

$$\left. + \frac{a^2 + ab + b^2}{3}\right) f(x, y) dx dy \text{ which is equivalent with (2).}$$

Remark. On some other results concerning Lupaş' inequality see [3] *and* [4].

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