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GENERALIZED TRANSFORMATIONS ON RATIOS OF FIBONACCI AND LUCAS NUMBERS

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1. INTRODUCTION

Not only do the ratios of the consecutive Fibonacci numbers converge to φ (the golden number), but they are the "best" rational approximation to φ [2, p. 151]. We give a family of transformations, which include those of Newton, Halley, modified Newton, Schröder which produces some ratios of generalized Fibonacci and Lucas numbers.

Let p and q be two nonzero real numbers. Define the generalized Fibonacci sequence

(1) $$u_0 = 0,\ u_1 = 1,\ u_{n+1} = pu_n - qu_{n-1},\ n \geq 1,$$

and the generalized Lucas sequence

(2) $$v_0 = 2,\ v_1 = p,\ v_{n+1} = pv_n - qv_{n-1},\ n \geq 1.$$

Let d be a natural number. If $u_n \neq 0$, define the ratio

(3) $$r_n = \frac{u_{n+d}}{u_n},$$

and if $v_n \neq 0$, we also define the ratio

(4) $$R_n = \frac{v_{n+d}}{v_n},$$

The characteristic equation related to the recurrence relations appearing in (1) and (2) is

(5) $$x^2 - px + q = 0.$$

If α and β are the roots of (5), then they satisfy ([3])

(6) $$\alpha + \beta = p,\ \alpha\beta = q,\ (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = p^2 - 4q.$$

If $\alpha = \beta$ then

(7) $$2\alpha = p,\ \alpha^2 = q = (p/2)^2,\ p^2 - 4q = 4\alpha^2 - 4\alpha^2 = 0.$$

LEMMA 1. [3] *If* α *and* β *are the distinct roots of* (5) *and* $n \geq 0$, *then*

$$u_n = \frac{\alpha^n - \beta^n}{\alpha - \beta},\ \textit{and}\ v_n = \alpha^n + \beta^n.$$

LEMMA 2. [5] *If* α *is the double root of* (5) *and* $n \geq 0$, *then* $u_n = n(\frac{p}{2})^{n-1}$ *and* $v_n = 2(\frac{p}{2})^n$.

If $d \geq 1$ and the roots of (5) are real, then the sequences of ratios $\{r_n\}$ and $\{R_n\}$ will converge to the d-th power of a root of (5). If equation (5) has distinct roots, then the sequences $\{r_n\}$ and $\{R_n\}$ converge to the root of largest modulus, and if the roots are equal they converge to that root. In other words, the sequences of ratios $\{r_n\}$ and $\{R_n\}$ converge to a root of

(8) $$x^2 - (\alpha^d + \beta^d)x + (\alpha\beta)^d = x^2 - v_d x + q^d = 0,$$

by Lemmas 1 and 2 and (6) and (7).

A necessary condition for a generalized Fibonacci or Lucas sequence to have nonzero members is that equation (5) have complex roots ([4]).

In the following we consider some transformations for approximating the roots of a single nonlinear equation $f(x) = 0$:

1. Newton transformation, $N(x) = x - \dfrac{f(x)}{f'(x)}$,

2. Halley transformation, $H(x) = x - \dfrac{f(x)f'(x)}{(f'(x))^2 - \frac{1}{2}f(x)f''(x)}$,

3. Newton modified transformation, $M(x) = x - \dfrac{f(x)f'(x)}{[f'(x)]^2 - f(x)f''(x)}$,

4. Schröder transformation, $S(x) = x - \dfrac{f(x)[f'(x)]^2 - [f(x)]^2 f''(x)}{[f'(x)]^3 - \frac{3}{2}f(x)f'(x)f''(x)}$.

Related to these transformations the following references were proved [1], [5], [6]: If $f(x) = x^2 - v_d x + q^d$, as long as division by zero does not occur for the integers n,d, then

1. $N\left(\dfrac{u_{n+d}}{u_n}\right) = \dfrac{u_{2n+d}}{u_{2n}}$,

2. $H\left(\dfrac{u_{n+d}}{u_n}\right) = \dfrac{u_{3n+d}}{u_{3n}}$,

3. $M\left(\dfrac{u_{n+d}}{u_n}\right) = \dfrac{v_{2n+d}}{v_{2n}}$,

4. $S\left(\dfrac{u_{n+d}}{u_n}\right) = \dfrac{v_{3n+d}}{v_{3n}}$,

We shall give an infinite family of transformations which includes all the above transformations. This family of transformations also has similar properties related to the ratios of generalized Fibonacci and Lucas numbers.

2. PROPERTIES OF GENERALIZED FIBONACCI AND LUCAS NUMBERS

For $n > 0$ define $v_{-n} = \alpha^{-n} + \beta^{-n}$. Then by (6) and Lemma 1

(9) $$q^n v_{-n} = (\alpha\beta)^n v_{-n} = \beta^n + \alpha^n = v_n$$

Similarly, if equation (5) has distinct roots, define $u_{-n} = \dfrac{\alpha^{-n} - \beta^{-n}}{\alpha - \beta}$.

By (6) and Lemma 1

(10) $$q^n u_{-n} = (\alpha\beta)^n u_{-n} = \frac{\beta^n - \alpha^n}{\alpha - \beta} = -u_n.$$

If equation (5) has a double root, u_{-n} is defined by $-n(\frac{p}{2})^{n-1}$ then formula (10) also hold. Relations (1) and (2) are also valid for negative subscripts.

In the following we shall list some elementary relationships about the sequences given by (1) and (2).

LEMMA 3. [3] *If* n *is an integer then* $u_{2n} = u_n v_n$.

LEMMA 4. [3] *If* n, m, *and* e *are integers, then*

(a) $u_{n+e}u_{n-e} - u_n^2 = -q^{n-e}u_e^2$,

(b) $u_{n+e}u_m - u_n u_{m+e} = -q^m u_e u_{n-m}$,

(c) $u_{n+e}u_{m+e} - q^e u_n u_m = u_e u_{n+m+e}$,

(d) $u_{n+e} - q^e u_{n-e} = v_e u_e$,

(e) $u_{n+e} - q^e u_n = -q^e u_{n-e}$.

LEMMA 5. [3] *If n, m, and e are integers, then,*

(a) $v_{n+e} v_{n-e} - v_n^2 = q^{n-e}(p^2 - 4q) u_e^2$,

(b) $v_{n+e} v_m - v_n v_{m+e} = q^m (p^2 - 4q) u_e u_{n-m}$,

(c) $v_{n+e} v_{m+e} - q^e v_n v_m = (p^2 - 4q) u_e u_{n+m+e}$,

(d) $v_{n+e} - q^e v_{n-e} = (p^2 - 4q) u_n u_e$,

(e) $v_{n+e} - v_e v_n = -q^e v_{n-e}$.

If (5) has a double root ($p^2 - 4q = 0$), the left side of relations (a)–(d) vanishes.

LEMMA 6. [3] *If n, m, and e are integers, then* $u_{n+e} v_m - u_n v_{m+e} = q^m u_e v_{n-m}$.

LEMMA 7. [3] *If n is an integer then* $u_n (v_n^2 - q^n) = u_{3n}$.

Now we can state some properties useful for the next section.

Remarks: Let $h \in N^*$, n integers, $e = (h+1)n$, and assume that division by zero does not occur in the following relations. Then:

1. By Lemma 4(d) and (10) it follows

$$(11)\qquad \frac{u_{(h+1)n}}{u_{(h+2)n}} = \frac{1}{v_n - q^n \dfrac{u_{hn}}{u_{(h+1)n}}}$$

2. By Lemma 5(e) and (9) it follows

$$(12)\qquad \frac{v_{(h+1)n}}{v_{(h+2)n}} = \frac{1}{v_n - q^n \dfrac{v_{hn}}{v_{(h+1)n}}}$$

LEMMA 8. *Let n be an integer, and assume that division by zero does not occur for all $h \in N^*$. Then*

$$(13)\qquad \frac{u_{hn}}{u_{(h+1)n}} = \frac{\sum_{k=0}^{\left[\frac{h-1}{2}\right]} (-1)^k q^{nk} C_{h-(k+1)}^k v_n^{h-(2k+1)}}{\sum_{k=0}^{\left[\frac{h}{2}\right]} (-1)^k q^{nk} C_{h-k}^k v_n^{h-2k}}, \quad \forall\ h \in N^*.$$

Proof. We shall prove the above relations by induction. For $h = 1$, by Lemma 3 we get $\dfrac{u_n}{u_{2n}} = \dfrac{1}{v_n}$. For $h = 2$, by Lemma 3 and Lemma 7 we also get $\dfrac{u_{2n}}{u_{3n}} = \dfrac{v_n}{v_n^2 - q^n}$. We suppose that (13) hold for h. By (11) and (13) it follows

$$(14)\qquad \frac{u_{(h+1)n}}{u_{(h+2)n}} = \frac{\sum_{k=0}^{\left[\frac{h}{2}\right]} (-1)^k q^{nk} C_{h-k}^k v_n^{h-2k}}{v_n \sum_{k=0}^{\left[\frac{h}{2}\right]} (-1)^k q^{nk} C_{h-k}^k v_n^{h-2k} - q^n \sum_{k=0}^{\left[\frac{h-1}{2}\right]} (-1)^k q^{nk} C_{h-(k+1)}^k v_n^{h-(2k+1)}}.$$

We distinguish two cases:

1. If h is even, it follows that $\left[\frac{h-1}{2}\right] = \left[\frac{h}{2}\right] - 1$, and since $C_{h-(k+1)}^{k+1} + C_{h-(k+1)}^k = C_{h-k}^{k+1}$, we obtain

$$v_n \sum_{k=0}^{\left[\frac{h}{2}\right]} (-1)^k q^{nk} C_{h-k}^k v_n^{h-2k} - q^n \sum_{k=0}^{\left[\frac{h-1}{2}\right]} (-1)^k q^{nk} C_{h-(k+1)}^k v_n^{h-(2k+1)} =$$

$$= (-1)^0 q^{n0} C_{h-0}^0 v_n^{h+1} + \sum_{k=0}^{\left[\frac{h-1}{2}\right]} (-1)^{k+1} q^{n(k+1)} C_{h-k}^{k+1} v_n^{h+1-(2k+1)} =$$

$$= (-1)^0 q^{n0} C_{h-0}^0 v_n^{h+1} + \sum_{l=1}^{\left[\frac{h+1}{2}\right]} (-1)^l q^{nl} C_{h+1-l}^l v_n^{h+1-2l} = (-1)^0 q^{n0} C_{h+1-0}^0 v_n^{h+1} +$$

$$+ \sum_{l=1}^{\left[\frac{h+1}{2}\right]} (-1)^l q^{nl} C_{h+1-l}^l v_n^{h+1-2l} = \sum_{l=0}^{\left[\frac{h+1}{2}\right]} (-1)^l q^{nl} C_{h+1-l}^l v_n^{h+1-2l},$$

and from (14)

$$\frac{u_{(h+1)n}}{u_{(h+2)n}} = \frac{\sum_{k=0}^{\left[\frac{h}{2}\right]} (-1)^k q^{nk} C_{h-k}^k v_n^{h-2k}}{\sum_{k=0}^{\left[\frac{h+1}{2}\right]} (-1)^k q^{nk} C_{h+1-k}^k v_n^{h+1-2k}},$$

that is, (13) hold for $h + 1$.

2. If h is odd, then $\left[\frac{h-1}{2}\right]=\left[\frac{h}{2}\right]$, and since $C_{h-\left[\frac{h+1}{2}\right]}^{\left[\frac{h-1}{2}\right]}=C_{h+1-\left[\frac{h+1}{2}\right]}^{\left[\frac{h+1}{2}\right]}$, it follows that

$$v_n\sum_{k=0}^{\left[\frac{h}{2}\right]}(-1)^k q^{nk} C_{h-k}^k v_n^{h-2k}-q^n\sum_{k=0}^{\left[\frac{h-1}{2}\right]}(-1)^k q^{nk} C_{h-(k+1)}^k v_n^{h-(2k+1)}=$$

$$=(-1)^0 q^{n0}C_{h-0}^0 v_n^{h+1}+\sum_{k=0}^{\left[\frac{h-1}{2}\right]}(-1)^{k+1}q^{n(k+1)}C_{h-k}^{k+1}v_n^{h+1-(2k+1)}+$$

$$+(-1)^{\left[\frac{h-1}{2}\right]+1}q^{n\left(\left[\frac{h-1}{2}\right]+1\right)}C_{h-\left(\left[\frac{h-1}{2}\right]+1\right)}^{\left[\frac{h-1}{2}\right]}v_n^{h-\left(2\left[\frac{h-1}{2}\right]+1\right)}=$$

$$=(-1)^0 q^{n0}C_{h-0}^0 v_n^{h+1}+\sum_{l=1}^{\left[\frac{h+1}{2}\right]-1}(-1)^l q^{nl}C_{h+1-l}^l v_n^{h+1-2l}+$$

$$+(-1)^{\left[\frac{h+1}{2}\right]}q^{n\left[\frac{h+1}{2}\right]}C_{h-\left[\frac{h+1}{2}\right]}^{\left[\frac{h-1}{2}\right]}v_n^{h+2\left[\frac{h+1}{2}\right]}=\sum_{l=0}^{\left[\frac{h+1}{2}\right]}(-1)^l q^{nl}C_{h+1-l}^l v_n^{h+1-2l}.$$

Now from (14)

$$\frac{u_{(h+1)n}}{u_{(h+2)n}}=\frac{\sum_{k=0}^{\left[\frac{h}{2}\right]}(-1)^k q^{nk}C_{h-k}^k v_n^{h-2k}}{\sum_{k=0}^{\left[\frac{h+1}{2}\right]}(-1)^k q^{nk}C_{h+1-k}^k v_n^{h+1-2k}},$$

that is, (13) hold for $h+1$.

LEMMA 9. *Let n be an integer, and assume that division by zero does not occur for all $h\in N^*$. Then*

$$\frac{v_{hn}}{v_{(h+1)n}}=\frac{\sum_{k=0}^{\left[\frac{h}{2}\right]}(-1)^k q^{nk}C_{h+1-k}^k v_n^{h-2k}}{\sum_{k=0}^{\left[\frac{h+1}{2}\right]}(-1)^k q^{nk}C_{h+2-k}^k v_n^{h+1-2k}},\quad\forall\ h\in N^*.$$

Proof. Similarly to Lemma 8

3. TRANSFORMATIONS OF SEQUENCES $\{r_N\}$ AND $\{R_N\}$

Now we can give some generalizations concerning the transformations presented in Introduction. For a real function f, we consider transformations of the form

$$(15)\qquad T_m(x)=x-\frac{f(x)}{f'(x)}\frac{\sum_{k=0}^{m}a_k\left[f'(x)\right]^{2m-2k}\left[f(x)\right]^k\left[f''(x)\right]^k}{\sum_{k=0}^{m}b_k\left[f'(x)\right]^{2m-2k}\left[f(x)\right]^k\left[f''(x)\right]^k},$$

where $m\in N^*$ and a_k, $b_k\in R$.

In the case of equation (8), for $f(x)=x^2-v_d x+q^d$, we get

$$(16)\qquad T_m\left(\frac{u_{n+d}}{u_n}\right)=\frac{u_{n+d}}{u_n}-\frac{q^n u_d\sum_{k=0}^{m}a_k 2^k q^{nk}v_n^{2m-2k}}{u_n v_n\sum_{k=0}^{m}b_k 2^k q^{nk}v_n^{2m-2k}}.$$

LEMMA 10. *Let $s\in N^*-\{1\}$, n be an integer and assume that division by zero does not occur. If $m\in N^*$ and y_k, $z_k\in R$, $k=0,\dots,m$, is such that*

$$(17)\qquad \frac{u_{(s-1)n}}{u_{sn}}=\frac{\sum_{k=0}^{m}y_k 2^k q^{nk}v_n^{2m-2k}}{v_n\sum_{k=0}^{m}z_k 2^k q^{nk}v_n^{2m-2k}},$$

then, for $a_k=y_k$ and $b_k=z_k$, $k=0,\dots,m$,

$$T_m\left(\frac{u_{n+d}}{u_n}\right)=\frac{u_{sn+d}}{u_{sn}}.$$

Proof. By (10) and Lemma 4(b)

$$q^n u_d u_{(s-1)n}=-q^n q^{(s-1)n}u_d u_{-(s-1)n}=-q^{sn}u_d u_{-(s-1)n}=$$

$$=-q^{sn}u_d u_{n-sn}=u_{n+d}u_{sn}-u_n u_{sn+d},$$

and by (17)

$$\frac{u_{n+d}\,u_{sn}-u_n\,u_{sn+d}}{u_n\,u_{sn}}=\frac{q^n u_d\,u_{(s-1)n}}{u_n\,u_{sn}}=\frac{q^n u_d\sum_{k=0}^{m} y_k 2^k q^{nk} v_n^{2m-2k}}{u_n v_n\sum_{k=0}^{m} z_k 2^k q^{nk} v_n^{2m-2k}},$$

that is,

$$\frac{u_{n+d}}{u_n}-\frac{q^n u_d\sum_{k=0}^{m} y_k 2^k q^{nk} v_n^{2m-2k}}{u_n v_n\sum_{k=0}^{m} z_k 2^k q^{nk} v_n^{2m-2k}}=\frac{u_{sn+d}}{u_{sn}}.$$

From the above relation and (16) we obtain

$$T_m\left(\frac{u_{n+d}}{u_n}\right)=\frac{u_{sn+d}}{u_{sn}}.$$

THEOREM 11. *Let $m\in N^*$ and assume that division by zero does not occur. If*

$$a_k=\frac{(-1)^k C^k_{2m-(k+2)}}{2^k},\ k=0,\dots,m-1,\ \text{and}\ a_m=0,$$

$$b_k=\frac{(-1)^k C^k_{2m-(k+1)}}{2^k},\ k=0,\dots,m-1,\ \text{and}\ b_m=0,$$

then

$$T_m\left(\frac{u_{n+d}}{u_n}\right)=\frac{u_{2mn+d}}{u_{2mn}}.$$

Proof. By Lemma 8

$$\frac{u_{(2m-1)n}}{u_{2mn}}=\frac{\sum_{k=0}^{m-1}(-1)^k q^{nk} C^k_{2(m-1)-k} v_n^{2(m-1)-2k}}{\sum_{k=0}^{m-1}(-1)^k q^{nk} C^k_{2m-1-k} v_n^{2m-1-2k}}=\frac{\sum_{k=0}^{m-1}(-1)^k q^{nk} C^k_{2m-(k+2)} v_n^{2m-2k}}{v_n\sum_{k=0}^{m-1}(-1)^k q^{nk} C^k_{2m-(k+1)} v_n^{2m-2k}}=$$

$$=\frac{\sum_{k=0}^{m-1} c_k 2^k q^{nk} v_n^{2m-2k}}{v_n\sum_{k=0}^{m-1} d_k 2^k q^{nk} v_n^{2m-2k}},$$

where

$$c_k=\frac{(-1)^k C^k_{2m-(k+2)}}{2^k},\ d_k=\frac{(-1)^k C^k_{2m-(k+1)}}{2^k},\ \text{for}\ k=0,\dots,m-1,$$

and

$$\frac{u_{(2m-1)n}}{u_{2mn}}=\frac{\sum_{k=0}^{m} c_k 2^k q^{nk} v_n^{2m-2k}}{v_n\sum_{k=0}^{m} d_k 2^k q^{nk} v_n^{2m-2k}},\ \text{for}\ c_m=d_m=0.$$

By Lemma 10,

$$T_m\left(\frac{u_{n+d}}{u_n}\right)=\frac{u_{2mn+d}}{u_{2mn}}.$$

Similarly, we can prove:

THEOREM 12. *Let $m\in N^*$ and assume that division by zero does not occur. If*

$$a_k=\frac{(-1)^k C^k_{2m-(k+1)}}{2^k},\ k=0,\dots,m-1,\ \text{and}\ a_m=0,$$

$$b_k=\frac{(-1)^k C^k_{2m-k}}{2^k},\ k=0,\dots,m,$$

then

$$T_m\left(\frac{u_{n+d}}{u_n}\right)=\frac{u_{(2m+1)n+d}}{u_{(2m+1)n}}.$$

THEOREM 13. *Let $m\in N^*$ and assume that division by zero does not occur. If*

$$a_k=\frac{(-1)^k C^k_{2m-k}}{2^k},\ b_k=\frac{(-1)^k C^k_{2m+1-k}}{2^k},\ k=0,\dots,m,$$

then

$$T_m\left(\frac{u_{n+d}}{u_n}\right)=\frac{u_{(2m+2)n+d}}{u_{(2m+2)n}}.$$

Remarks: 1. For $m=1$ and $a_0=b_0=1$, $a_1=b_1=0$ in (15), we get $T_1(x)=N(x)$, the Newton transformation,

and by Theorem 11

$$T_1\left(\frac{u_{n+d}}{u_n}\right)=\frac{u_{2n+d}}{u_{2n}}.$$

2. For $m=1$ and $a_0=b_0=1$, $a_1=0$, $b_1=-\frac{1}{2}$ in (15), we get

$T_1(x)=H(x)$, the Halley transformation,

and by Theorem 12

$$T_1\left(\frac{u_{n+d}}{u_n}\right)=\frac{u_{3n+d}}{u_{3n}}.$$

3. For $m=1$ and $a_0=b_0=1$, $a_1=-\frac{1}{2}$, $b_1=-1$ in (15), we get

$$T_1(x)=x-\frac{f(x)\left[(f'(x))^2-\frac{1}{2}f(x)f''(x)\right]}{f(x)\left[(f'(x))^2-f(x)f''(x)\right]},$$ a transformation from [7].

and by Theorem 13

$$T_1\left(\frac{u_{n+d}}{u_n}\right)=\frac{u_{4n+d}}{u_{4n}}.$$

Similarly we can choose a_k, b_k in (15) such that,

$$T_m\left(\frac{u_{n+d}}{u_n}\right)=\frac{v_{sn+d}}{v_{sn}}$$

for some values of m and s.

THEOREM 14. *Let $m\in N^*$ and assume that division by zero does not occur. If*

$$a_k=\frac{(-1)^k C_{2m-k}^k}{2^k},\quad k=0,1,\dots,m-1,\ \text{and}\ a_m=0,$$

$$b_k=\frac{(-1)^k C_{2m-(k-1)}^k}{2^k},\quad k=0,1,\dots,m,$$

then

$$T_m\left(\frac{u_{n+d}}{u_n}\right)=\frac{v_{2mn+d}}{v_{2mn}}.$$

Remark. For $m=1$ *and* $a_0=b_0=1$, $a_1=0$, $b_1=-1$ in (15), we get

$T_1(x)=M(x)$, the modified Newton transformation,

and by Theorem 14

$$T_1\left(\frac{u_{n+d}}{u_n}\right)=\frac{v_{2n+d}}{v_{2n}}.$$

THEOREM 15. *Let $m\in N^*$ and assume that division by zero does not occur. If*

$$a_k=\frac{(-1)^k C_{2m-(k-1)}^k}{2^k},\quad k=0,1,\dots,m,$$

$$b_k=\frac{(-1)^k C_{2m-(k-2)}^k}{2^k},\quad k=0,1,\dots,m,$$

then

$$T_m\left(\frac{u_{n+d}}{u_n}\right)=\frac{v_{(2m+1)n+2}}{v_{(2m+1)n}}.$$

Remark. For $m=1$ and $a_0=b_0=1$, $a_1=-1$, $b_1=-\frac{3}{2}$ in (15), we get

$T_1(x)=S(x)$, the Schröder transformation,

and by Theorem 15

$$T_1\left(\frac{u_{n+d}}{u_n}\right)=\frac{v_{3n+d}}{v_{3n}}.$$

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