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BOOK REVIEWS

Partial Differential Equations and Functional Analysis, In: *Memory of Pierre Grisvard,* edited by J. CEA, D. CHENAIS, G. GEYMONAT and J.-L. LIONS, *Progress in Nonlinear Differential Equations and Their Applications,* Vol. 22, Birkhäuser, Boston-Basel-Berlin, 1996, 264 pp., ISBN 0-8176-3839-3.

Pierre Grisvard, one of the most distinguished contemporary French mathematicians, died on April 22, 1994. A conference was held in November 1994 out of which grew the invited articles contained in this volume. All papers are related to functional analysis applied to partial differential equations, which was Grisvard's speciality. Indeed, his knowledge of this area was extremely broad. Functional analysis applied to partial differential equations is an area enjoying rebirth and reexamination in many parts of the world, particularly with respect to its recent thrusts into mathematical physics. Grisvard also became a specialist in the study of optimal regularity for partial differential equations with boundary conditions. He examined singularities coming from coefficients, boundary conditions, and mainly non-smooth domains. Grisvard left a legacy of precise results very useful in applied mathematics, which were published in journals and books. This volume contains a bibliography of his works as well as his last paper, which is published here posthumously.

The contents are as follows: P. Grisvard, *Problèmes aux limites dans des domaines avec points de rebroussement*; T. Apel and S. Nicaise, *Elliptic problems in domains with edges: anisotropic regularity and anisotropic finite element meshes*; M. S. Baouendi and L. Preiss Rothschild, *Unique continuation of harmonic functions at boundary points and applications to problems in complex analysis*; C. Bardos and M. Belyshev, *The wave shaping problem*; P. G. Ciarlet, *Modélisation mathématique des coques linéairement élastiques*; G. Da Prato, *Fully nonlinear equations by linearization and maximal regularity and applications*; M. Dauge, *Strongly elliptic problems near cuspidal points and edges*; L. Boutet de Monvel, *Star produit associé à un crochet de Poisson de rang constant*; P. Destuynder and F. Santi, *La méthode des lâchées de tourbillons pour le calcul des efforts aérodynamiques*; A. Favini, *Sum of operators' method in abstract equations*; G. Geymonat and O. Tcha-Kondor, *Constructive methods for abstract differential equations and applications;* V. A. Kondratiev and O. A. Oleinik, *On asymptotics of solutions of second order elliptic equations in cylindrical domains*; V. A. Kozlov and V. G. Maz'ya, *Singularities to solutions in mathematical physics problems in non-smooth domains*; J.-L. Lions and E. Sanchez-Palencia, *Problèmes sensitifs et coques élastiques minces*; M. T. Niane, *Contrôlabilité exacte frontière de l'équation des ondes en présence de singularités*; E. Sinestrari, *Interpolation and extrapolation spaces in evolution equations;* M. Zerner, *Localisation des singularités sur la frontière et partitions de l'unité.*

The research or expository articles included in this volume, many of them with direct references to Grisvard's contributions, will be of great interest for all specialists in nonlinear partial differential equations and their applications.

R. Precup

1991 AMS Subject Classification: 65B99.

YURI EGOROV, VLADIMIR KONDRATIEV, *On Spectral Theory of Elliptic Operators, Operator Theory,* Vol. 89, Birkhäuser, Basel, 1996, 338 pp., ISBN 3-7643-5390-2.

The book is devoted to the study of some classical problems of the spectral theory of elliptic differential equations. The results relate to estimates of the first eigenvalue and the negative spectrum of the Schrödinger operator, as well as to estimates of eigenvalues and eigenfunctions of elliptic operators. The authors' approach makes use of the variational principle and of some *a priori* estimates in Sobolev spaces.

The book has two distinct parts. The first part has an educational character and contains basic theory of Hilbert spaces, Sobolev spaces and classical theory of elliptic equations. It is essentially self-contained and addresses to senior university students. The second part of the book contains the authors' results obtained during the last two decades and partially published in mathematical journals, some of them not being translated in English before.

The contents are: Chapter 1 – *Hilbert Spaces* (24 pp.); Chapter 2 – *Functional Spaces* (84 pp.) – deals with L^p, W_p^m, $W_{p,0}^m$, H^s spaces, Sobolev's embedding theorem, Poincaré's inequality, interpolation inequalities, compactness of the embedding, Hardy's inequalities and Morrey embedding theorem; Chapter 3 – *Elliptic Operators* (23 pp.) – refers to elliptic and strongly elliptic equations, regularity, boundary value problems and smoothness of solutions up to the boundary; Chapter 4 – *Spectral Properties of Elliptic Operators* (20 pp.); Chapter 5 – *The Sturm-Liouville Problem* (54 pp.) – deals with the dependence of eigenvalues, in particular the first one, of the Sturm-Liouville problem for the equation $(P(x)y')' + \lambda Q(x)y = 0$ on the coefficients P and Q; the estimates of λ are given in terms of the integrals $\int P(x)^\alpha dx$ and $\int Q(x)^\beta dx$, where α and β are arbitrary reals; Chapter 6 – *Differential Operators of Any Order* (26 pp.) – is about the similar Sturm-Liouville problem for a differential equation of order m; Chapter 7 – *Eigenfunctions of Elliptic Operators in Bounded Domains* (42 pp.) – deals with solvability results for boundary value problems for strongly elliptic equations depending on the volume of the domain or its inner radius; there are also considered here estimates of the first eigenvalue and of eigenfunctions for general operator pencils; Chapter 8 – *Negative Spectra of Elliptic Operators* (44 pp.) – is devoted to the study of the negative spectrum of an elliptic operator of the Schrödinger type $L = L_0 - V(x)$ depending on V.

The book also contains a short introduction, a bibliography of 91 titles and a subject index.

It is very well written, with clear precise statements and complete proofs. We recommend it to all those interested in the spectral theory of differential operators as well as to the specialists in applied mathematics. It is equally a very nice introduction to partial differential equations and we warmly recommend it for the students' work.

R. Precup

JAN van NEERVEN, *The Asymptotic Behaviour of Semigroups of Linear Operators,* Birkhäuser Verlag, Basel-Boston-Berlin, 1996, 238 pp.

Most of what was known up to 1985 about the asymptotics of semigroups is presented in *One-parametric semigroups of positive operators*, edited by R. Nagel, *Lecture Notes in Mathematics,* Vol. 1184, Springer Verlag, 1986.

These notes can be seen as a follow-up to that book, reporting on the progress done over these ten years in the study of the asymptotic behaviour of semigroups. The book of Jan van Neerven is based on a course taught at the University of Tubingen in the academic year 1994-1995, and represents the first attempt to organize the available material in a book form.

The basic results are presented in Chapter 1 – *Spectral Bound and Growth Bound.* In Chapter 2 – *Spectral Mapping Theorem* – various spectral inclusion theorems and spectral mapping theorems for semigroups with an unbounded generator are proved, including C_0-semigroups with non-quasi-analytic growth. Chapter 3 – *Uniform Exponential Stability* – is concerned with the stability results of Datko-Pazy and Rolewicz. In Chapter 4 – *Boundedness of the Resolvent* – it is proved the convexity theorem of Weis and Wrobel and some individual stability results in some special Banach spaces (B-convex spaces with analytic Radon-Nikodym property) as well as in general Banach spaces are established. The last chapter of the book, Chapter 5 – *Countability of the Spectrum* –, is dealing with stability results of Arendt, Batty and Lubich, a Katzenelson-Tzafriri-type theorem for C_0-semigroups and Tauberian theorems for the Laplace transform.

For the convenience of the reader, it is included an Appendix containing some results on fractional powers, interpolation theory, Banach lattices and Banach function spaces.

Self-contained and providing complete proofs, this book will be of great interest to researchers and graduate students working in the fields of operator semigroups and evolution equation.

I. Raşa

LEON SIMON, *Theorems on Regularity and Singularity of Energy Minimizing Maps,* Lecture Notes in Mathematics, ETH Zürich, Birkhäuser Verlag, Basel-Boston-Berlin, 1996, 152 pp.

The aim of this monograph is to give an introduction to the basic regularity theory for energy minimizing maps, including recent developments concerning the structure of the singular set and asymptotics on approach to the singular set.

The first chapter contains analytic preliminaries: Hölder continuity, smoothing, functions with L^2 gradients, harmonic and weakly harmonic functions, the fundamental lemma on harmonic approximation, elliptic regularity and a technical regularity lemma on which the proof of a theorem for energy minimizing maps will be based.

In Chapter 2 the energy minimizing maps are defined, together with the variational equations. The ε-regularity theorem is proved. A monotonicity formula and the reverse Poincaré inequality, which are key tools in the study of energy minimizing maps, are also given. The chapter contains a nice compactness theorem for energy minimizing maps and some corollaries of the ε-regularity theorem.

Chapter 3 is devoted to approximation properties of the singular set. After the definition and properties of tangent maps, further properties of the singular set of a map u, including a geometric picture, are also given. We remark that the geometric picture gives good information about the structure of the top dimensional part of the singular set of u if the tangent map at each point of this set is unique. Some results on functionals on vector bundles and on the Euler-Lagrange operator for such a functional, including the Liapunov-Schmidt reduction and the Lojasiewicz inequality, are also given.

Chapter 4 contains rectifiability results for the singular set of energy minimizing maps. After a general rectifiability lemma, energy estimates are given and used to obtain L^2 estimates for energy minimizing maps.

We recommend this beautiful monograph as a good mathematical book for the specialists in partial differential equations and calculus of variations.

D. Trif

Learning and Geometry: Computational Approaches, edited by DAVID KUEKER and CARL SMITH, *Progress in Computer Science and Applied Logic*, Vol. 14, Birkhäuser, Boston-Basel-Berlin, 1996.

The papers in this volume are a partial record of the "Workshop on Learning and Geometry" hosted by the University of Maryland in January 1991. In addition to the research papers submitted for these proceedings, the program for the workshop, a list of participants and the introductory remarks concerning the purpose of the workshop prepared by Vincent Mirelli are included.

The volume contains the following papers:

LEARNING: J. Rissanen and Bin Yu, *MDL Learning;* Robert H. Sloan, *PAC Learning Noise and Geometry;* Sanjeev R. Kulkarni, *A Review of Some Extensions to the PAC Learning Model.*

GEOMETRY: Jürgen Bokowski, *Finite Point Sets and Oriented Matroids: Combinatorics in Geometry;* Shang-Ching Chou, *A Survey of Geometric Reasoning Using Algebraic Methods;* Walter Whiteley, *Synthetic versus Analytic Geometry for Computers;* Walter Whiteley, *Representing Geometric Configurations;* Wu Wen-Tsün, *Geometry Theorem Proving in Euclidean, Decartesian, Hilbertian and Computerwise Fashion.*

The aim of the workshop was to emphasize how results in artificial intelligence, namely learning techniques of geometric objects, can be incorporated into the geometric manipulation of data produced by a variety of sensors.

The workshop was organized by The Center for Night Vision in conjunction with The Systems Research Center (now called the Institute for Systems Research) at the University of Maryland. It was attended by eminent specialists in various fields such as computational linguistics, geometry theorem proving, synthetic, foundational and algebraic geometry.

By its interdisciplinary character, the present volume will be of interest to a broad audience, working on problems of pattern recognition and their applications.

J. Robu

VLADIMIR F. DEMYANOV, ALEXANDER M. RUBINOV, *Constructive Nonsmooth Analysis, Approximation & Optimization*, Vol. 7, Verlag Peter Lang, Frankfurt am Main, 416 pp.

Smooth analysis is dealing with the approximation of functions by linear functions and it can be applied to the study of extrema of smooth functions. But in many optimization problems there occur functions more general than the smooth ones, and their study requires more general notions of differentiability. The present book is concerned with various notions of differentiability and their relevance in finding necessary conditions as well as numerical algorithms for extremal problems.

The book contains six chapters, five appendices, a bibliography of 300 titles, a subject index, a glossary of notions and abbreviations, and a section of historical and bibliographical notes.

Chapter I – *Homogeneous Approximations of Sets and Mappings* – is concerned with approximation of sets by cones (the tangent cone, the Bouligand cone or the cone of feasible directions, the cone of admissible directions) and their analogues for multivalued mappings. Dini and Hadamard derivatives are applied to the study of the differentiability of maximum function with dependent constraints.

Chapter II – *The Clarke Derivatives and Subdifferentials* – is dealing with Clarke upper and lower derivatives and subdifferentials for locally Lipschitz functions and with Clarke's tangent cone. (F. Clarke is responsible for the term "nonsmooth analysis".) Generalized Jacobians in the sense of F. Clarke and fans, a notion due to A. Ioffe, are also considered.

In Chapter III – *Quasidifferential Calculus* – a new concept of differential (called quasidifferential) for directionally differentiable functions is defined and its relations to Clarke and Penot subdifferentials are studied. In fact, another book by the same authors – *Quasidifferentiable Calculus*, New York, 1986 – is devoted to this notion.

In Chapter IV – *Codifferentiable Functions* – a related notion, called codifferentiability, is considered. Although the classes of quasidifferentiable and codifferentiable functions agree, the quasidifferential and the codifferential of a function may differ.

The codifferential has the advantage over the quasidifferential in allowing the definition of continuously codifferentiable functions with respect to Hausdorff metric. (Quasidifferential and codifferential are defined as a pair of convex compact sets satisfying some maximality conditions.) Twice codifferentiable functions are also considered.

In Chapter V – *Extremal Problems* – the developed machinery is applied to the study of extremal values of nonsmooth functions, including second-order conditions.

In the last chapter – *Implicit Functions Theorems* – an extension of Kakutani fixed point theorem for multivalued mappings is applied to prove inverse and implicit function theorems for Lipschitz, quasidifferentiable or multivalued mappings.

Although all the results are presented in finite dimensional setting, the possible extensions to ordered normed spaces (Kantorovich spaces) are briefly discussed in Appendix III. Other appendices are dealing with convex analysis (Appendix I), multivalued mappings (Appendix II), quasilinear algebra (Appendix IV), and some open problems for differences of sets in $\mathbf{R}^n$ (Appendix V).

The book is clearly written, contains a rich amount of material, and all the notions are illustrated by concrete examples and are applied to appropriate extremal problems.

The authors are well-known specialists in the field, having published many research papers and several books (almost all translated in English). The present book is based on the book by V. F. Demyanov and L. V. Vasiliev – *Nonsmooth Optimization*, Nauka, Moscow, 1981 (in Russian) – but it is essentially a new book. Some parts are omitted, others are added, and to some results there are given new presentations, reflecting the progress made in the area since the publication of the Russian edition.

The book is fairly self-contained so that it can be used both as a textbook by students (and teachers) and as a reference book by the specialists.

S. Cobzas

FRANÇOISE CHAITIN-CHATELIN, VALÉRIE FRAYSSÉ, *Lectures on Finite Precision Computations*, SIAM, Philadelphia, 1996.

The book approaches the border domain between numerical analysis and computer science. The authors make a deep study of different problems that appear when using on computers some numerical algorithms which theoretically have properties assuring their consistency and stability, though, in some practical cases, these algorithms may become numerically unstable. Such problems are posed frequently both by the numerical analysis and by the domains where it is used.

From this point of view, the present book is an excellent presentation of the rigorous theoretical aspects concerning the numerical stability of the numerical algorithms.

The exposition is based on the solving of the equations having the form $F(x) = y$, where $F: X \to Y, X, Y$ being normed spaces. If there exists a solution of such an equation, $x = F^{-1}(y)$, there is posed the problem of approximating it by using a numerical algorithm for computing the value $G(y) = F^{-1}(y)$. In this respect, there is considered a family of approximating equations $F_\theta(x) = y_\theta$, where $0 \le \theta \le 1$. For this family there follow the corresponding solutions $(*) x_\theta = F_\theta^{-1}(y_\theta) = G_\theta(y_\theta)$. There are analyzed the conditions for which $x_\theta \to x$ as $\theta \to 0$. For this purpose, the authors consider two types of errors, namely, the direct error, given by the equality $x - x_\theta = G(y) - G_\theta(y_\theta)$, and the residual error, given by $y - F(x_\theta) = y - F(G_\theta(y_\theta))$. Using these errors, there are studied the conditions of consistency, stability and convergence of the algorithm given by (*). There are analyzed the conditions in which the algorithm is finite (for example, the Gauss method for linear systems) or iterative. Then, for each case, it is studied the influence of rounding errors.

A special importance is given to the cases when the application F possesses singularities in the neighbourhood of x.

The book contains many examples which complete and motivate the exposed theory.

The thorough analysis of these problems and also of others is done in 12 chapters: 1. *General Presentation;* 2. *Computability in Finite Precision;* 3. *Measures of Stability for Regular Problems;* 4. *Computation in the Neighbourhood of Singularity;* 5. *Arithmetic Quality of Reliable Algorithms;* 6. *Numerical Stability in Finite Precision;* 7. *Software Tools for Round-off Error Analysis in Algorithms;* 8. *The Toolbox PRECISE for Computer Experimentation;* 9. *Experiments with PRECISE;* 10. *Robustness to Nonnormality;* 11. *Qualitative Computing;* 12. *More Numerical Illustration with PRECISE.*

The exposition ends with a large bibliography.

The material presented in this book may serve both to the specialists in numerical analysis and to those in computer science.

I. Păvăloiu

NICHOLAS J. HIGHAM, *Accuracy and Stability of Numerical Algorithms*, SIAM, Philadelphia, 1996.

The book contains extensive, updated results concerning the behavior of the methods used in finite precision by the linear numerical algebra.

The perturbation theory and the rounding error analysis, thoroughly developed for each topic, permit the author to give a better explanation regarding the failure of some algorithms to produce the expected results and their improvement or replacement with some better ones.

The book is divided into 26 chapters, followed by 5 appendices, a bibliography, a name index, and a subject index.

The topics treated are as follows: The first two chapters deal with the principles of the floating point arithmetic. Several examples show the essential differences that may appear between the exact value of an expression or algorithm and its actual computing in finite precision. The IEEE Standard is also described.

The following two chapters deal with inner and outer products and matrix multiplications, different techniques and error bounds being presented.

Next, the evaluation of one variable polynomials and their derivatives is treated, from the perspective of the rounding errors.

The preparation for linear systems is completed by the presentation of the vector and matrix norms. The perturbation theory for the linear systems is given in the beginning, followed by the study of the general direct methods (LU and block LU factorizations) and of the direct methods for special systems (the Cholesky factorization, methods for triangular systems).

The iterative refinement, the matrix inversion problem and the condition number estimation are also dealt with, in Chapters 11, 13 and 14.

The Sylvester equation is studied next, and then the stationary iterative methods.

Bounds for $\|A^n\|$ are given both for the exact and the finite precision case in Chapter 17.

The following chapter consists of an error analysis for the QR factorizations and the related methods.

The perturbation theory for the least squares problem and an error analysis are given for different methods of solving. From the same point of view there are analyzed the undetermined systems and the algorithms for Vandermonde systems.

In Chapter 22, the error analysis is given for fast matrix multiplication by Winograd, Strassen and bilinear noncommutative algorithms.

Error bounds are given next, for the Cooley-Tuckey radix 2 Fast Fourier Transform and for circulant linear systems.

Chapter 24 – *Automatic Error Analysis* – contains an analysis of the direct search methods applied to several problems.

Finally, there are described some subtleties in floating point arithmetic and some test matrices, together with their properties.

The eigenproblem is not treated, since the book already contains 688 pages with 1,134 references.

The topics are exposed in a unitary manner, for each one being presented known and recent results, some of them even new. The significance of the results is clearly stated and the problems proposed at the end of each chapter challenge any reader interested in a deeper understanding of the subject. The historical notes throughout the book help anyone who wishes to complete his/her vision on the numerical analysis, and the references to LAPACK enhance even more the practical value of the book.

The large area covered and the deepness of the results recommend this volume as a fundamental textbook for every scientist in numerical analysis.

By their profoundness and/or subtlety, humour, oldness, etc., the quotations at the beginning of each chapter may alone hurry the reader to get sooner to the next chapter. For example, the Piet Hein's one may constitute, in fact, the motto of this book:

"The road to wisdom?
Well, it's plain and simple to express:
Err
and err
and err again
but less
and less
and less."

E. Cătinaş