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BOOK REVIEWS

General Inequalities 7, Edited by C. BANDLE, W. N. EVERITT, L. LOSONCZI and W. WALTER, ISNM, Vol. 123, Birkhäuser Verlag, Basel–Boston–Berlin 1997, 604 pp., ISBN 3-7643-5722-3

Perhaps the last remaining field comprehended and used by mathematicians in all areas of the discipline, inequalities continue to play an essential role in mathematics. New inequalities and exciting applications are discovered every year, therefore surveys on the present-day state of the art are very useful.

This volume contains the proceedings of the *General Inequalities* 7 meeting held at Oberwolfach in November 1995. Among the participants in the conference we mention: J. Aczel, R. Ger, H. König, A. Kufner, R. J. Nessel, J. E. Pečarić. The contributions reflect the ramifications of general inequalities into several areas of mathematics and are classified in seven sections: 1. inequalities in analysis (4 articles); 2. inequalities for matrices and discrete problems (4 articles); 3. inequalities for eigenvalue problems (4 articles); 4. inequalities for differential operators (5 articles); 5. convexity (4 articles); 6. inequalities in functional analysis and functional equations (4 articles); 7. applications (5 articles).

There is also a lively final section entitled *Problems and remarks*, containing 5 items.

The volume includes the latest results presented by the participants and is a useful reference book for both lecturers and research workers.

I. Raşa

MIKHAIL I. KADETS, VLADIMIR M. KADETS, *Series in Banach Spaces. Conditional and unconditional convergence*, Operator Theory, Advances and applications, Vol. 94, Birkhauser Verlag, Basel–Boston–Berlin, 1997.

This well-written book is devoted to one of the most fruitful areas in functional analysis: conditionally and unconditionally convergent series in Banach spaces. The exposition is focussed on two main problems: 1. characterization of series in a Banach space that remain convergent after an arbitrary rearrangement of their terms (in this case all its rearrangements have the same sum); 2. description of the sum range for the rearrangement convergent series of a conditionally convergent series. The author's monograph *Rearrangements of Series in Banach Spaces*, Transl. of Math. Monographs 86, Amer. Math. Soc., Providence R. I., 1991, was the first attempt to cover, partially, the aforementioned subjects. In the meantime, new spectacular results have appeared in tis field, reflected in the book under review. Recent results and combinatorial methods of high level are also included. One of the fundamental results in this field is the Steinitz's theorem characterizing the sum range of a conditionally convergent series in finite-dimensional spaces. Pecherskii's recent theorem gives a characterization of the sum range for conditionally convergent series in infinite-dimensional spaces, when such a series is not perfectly divergent after rearrangements. An example of conditionally convergent series with a sum range consisting of two points is followed by Chobanian's theorem, which is another variant of Steinitz's theorem for infinite-dimensional spaces. A particular case in L_p-spaces is reflected in a theorem of M. I. Kadets. The well-known Dvoretzky-Rogers theorem is also presented and the image of unconditionally convergent series by certain operators is described. In this context it is proved the theorem of Grothendieck stating that a bounded linear operator $T: l_1 \to l_2$ is absolutely summing.

Some notions and results from the geometry and local theory of Banach spaces are used for obtaining unconditionally convergent series in certain infinite-dimensional spaces. The theorem of Dvoretzky about the finite representability of l_2, in any infinite-dimensional Banach spaces is also proved. The theorem of Bessaga-Pelczynski concerning weakly absolutely convergent series in Banach spaces not containing c_0 is also proved. Steinitz-type theorems for B-convex Banach spaces and a theorem of Banaszczyk for series in metrizable nuclear spaces are also presented.

The book includes an interesting appendix concerning the limit set of the Riemann integral sums for vector, valued functions defined on [0, 1], closely related to the main subject. The convexity of this limit set is proved for B-convex spaces. The case when this limit is only star-shaped is also treated. The authors are well-known specialists in the field and the book contains some of their contributions. It also includes a subject index and a bibliography with more than 100 items.

This volume may be recommended to graduates and post graduates specializing in functional analysis and to researchers in this field.

Ioan Şerb

NOVOZHILOV, I. V., *Fractional Analysis, Methods of Motion Decomposition*, Birkhäuser Verlag, 1997, hardcover, 232 pp., ISBN 0-8176-3889-X.

The title of this book may ask for an explanation. What the author means by "fractional analysis"? Well, it is not hard to explain. One of the most widely used methods for the approximate solution of nonlinear equations is the method of small parameter. The point is that, in general, the systems do not contain a small parameter at all, so we have to introduce by hand, and then, in the next step, we have to decompose the right-hand side of our equation in a principal part (the big part) and a perturbation (the small part) such that if we neglect the small part we can solve the equations only with the big part in the righ- hand side. This decomposition, in the case of dynamical equations, is called "motion decomposition", while the entire analysis is called "fractional analysis".

The book begins with a chapter devoted to dimensional analysis and small parameters. The author shows, by examples, how we can obtain differential equations in adimensional quantities and how the small parameter enters our analysis of these adimensional equations.

In the second chapter it is shown how to get an approximate solution for a regularly perturbed system, on a finite interval of normalized dimensionless time.

In order to solve the problem for a large time interval, the motion is decomposed into the main motion and small components of it. This decomposition, for different systems (with fast phase,

boundary layer and discontinuous characteristics), is the subject of Chapters 4 – 6. The last Chapter is devoted to problems related to passage to a limit with respect to a small parameter.

The volume is written having in mind especially graduates or researchers wanting to see at work the methods of fractional analysis. The emphasis is on specific examples and not on the general theory. The examples are taken from the approximate mathematical models for the mechanics of gyroscopes, transport or robots. It is not examined phenomena resonance, nor other problems which are more or less specific to celestial mechanics.

The book reflects the research interest of the author. He has published a number of papers and books on this subject.

Being well written, with examples which are worked out in all the necessary details, the book will certainly be of a great help especially to those working in applied mechanics, but, the range of persons who could benefit from the reading of this book is much larger.

Cristina Blaga

LECH MALIGRANDA, *Orlicz Spaces and Interpolation*, Seminarios de Matematicà, Vol. 5, Univ. de Campinas (UNICAMP), Brasil, 1989, 206 pp.

Orlicz spaces, introduced and studied by the Polish mathematician W. Orlicz between 1932–1936, and named Orlicz spaces by C. Zaanen in 1949, are natural and far-reaching extensions of Lebesgue classical spaces. They form one of the most important classes of function spaces, having deep applications in various branches of mathematics such as, e.g., integral and differential equations. The first monograph exposing systematically the theory is that of M. A. Krasnoselski and Ja. B. Rutícki, *Convex Functions and Orlicz Spaces*, published first in Russian, at Moscow, in 1958, and translated in English by Nordhooff Groningen, Amsterdam, in 1961. Orlicz spaces are also treated in C. Zaanen's book, *Integration*, North-Holland, Amsterdam, 1967. The related and more general theory of modular spaces, developed by the Polish and Japanese schools, is exposed in the books of J. Musielak, *Orlicz Spaces and Modular Spaces*, Springer Verlag, Berlin, 1983, and of H. Nakano, *Modulared Semi-ordered Linear Spaces*, Maruzen, Tokyo, 1950.

The book is based on some lectures given by the author between 1987–1989 at the Universities Central de Venezuela, Campinas (Brasil), and Complutense de Madrid. As the author kindly informed us, in 1989 only a small number of copies were printed (due to the economic crisis facing Brazil) and only now is available to a broader audience. In the meantime two other books have been published: M. M. Rao and Z. D. Ren, *Theory of Orlicz spaces*, M. Dekker, New York 1991, and Shutao Shen, *Geometry of Orlicz Spaces*, Dissertationes Mathematicae, Vol. 356, Warszawa 1996.

The volume under review is focused on the presentation of basic results on Orlicz spaces and on interpolation of operators between them. It is divided into 15 sections. The first three ones are devoted to the presentation of basic definitions and results concerning Orlicz and modular spaces. Section 4 is concerned with separability of Orlicz spaces, while Section 5 is devoted to the study of convexity and boundedness properties of Orlicz spaces (local convexity and local boundedness). Existence and non-existence of non-trivial continuous linear functionals or compact operators are discussed in Sections 6 and 7. The triviality of the dual of the space $L^p[0, 1]$, $0 < p < 1$, is obtained as a particular case of these results. The representation of continuous linear functionals on Orlicz spaces is presented in Section 9. Sections 10–12 are concerned with topics such as product of convex Orlicz functions, indices of Orlicz spaces and Orlicz spaces generated by Young functions. Interpolation of non-linear operators between Orlicz spaces and Calderon-Lozanovski spaces are presented in Sections 13–15.

There are included, in a relatively small number of pages, a lot of results, some of them belonging to the author. Numerous examples, illustrating the abstract notions or delimiting them, are presented. The book also contains a number of open (at that time) problems.

The book is considered to be a good introductory course, preparing the reader for the approach to more advanced texts and research papers in the field.

S. Cobzaş

Multivariate Approximation and Splines, edited by: GÜNTHER NÜRNBERGER, JOCHEN W. SCHMIDT and GUIDO WALZ, ISNM, Vol. 125, Birkhäuser Verlag, Basel–Boston–Berlin, 1997, 336 pages.

This book contains the refereed papers presented at the international conference on "Multivariate Approximation and Splines" held in Mannheim, Germany, on September 7–10, 1996. Fifty well-known experts from Bulgaria, Great Britain, France, Israel, The Netherlands, Norway, Poland, Switzerland, Ukraine, USA and Germany participated in the symposium.

The aim of the conference was to give an overview of recent developments in multivariate approximation with special emphasis on spline methods. The contributions cover a variety of topics from the rapidly developing branches of multivariate approximation, including interpolation, data fitting, splines, radial basis functions, neural networks, computed aided design methods, wavelets, subdivision, optimization, differential equations and numerical integration.

The research has applications in areas such as industrial production, visualization, pattern recognition, image and signal processing, cognitive systems and modelling in medicine, geology, physics, biology and in other natural sciences. An example the modelling of high resolution radar imaging and clinical magnetic resonance tomography with the aid of coherent wavelets, which is the subject of the paper of W. Schempp. It is shown that the construction of matched filter banks depends on Kepler's spatiotemporal strategy applied to quantum holography and can be described by Fourier analysis of the Heisemberg nilpotent Lie group.

Gheorghe Micula

HANS TRIEBEL, *Fractals and Spectra-Related to Fourier Analysis and Function Spaces*, Monographs in Math. Vol. 91, Birkhäuser Verlag, Basel–Boston–Berlin, 1997, hardcover, vii+272 pp, ISBN 3-7643-5776-2.

This monograph deals with the symbiotic relationship between Fourier analysis, theory of function spaces, fractal geometry, and spectral theory of (pseudo)differential operators. It may be considered a continuation of two previous books written by the same author –*Theory of Function Spaces*, Vols I and II, Birkhäuser, 1983 and 1992, and *Function Spaces, Entropy Numbers, Differential Operators*, Cambridge University Press 1996 (with D. E. Edwards). The main goals of the book are to give estimates for the entropy numbers of compact embeddings between function spaces and to study their relations with the distribution of eigenvalues in fractal setting.

The volume comprises five chapters. In the first chapter some basic facts about fractal geometry (without proofs) are presented. Isotropic, anisotropic and non-isotropic d-sets and related self-affine fractals are discussed in detail. Chapter II deals with entropy numbers in (weighted) l^p-spaces. Chapter III is devoted to some function spaces on $\mathbb{R}^n$. Some new results of the author are applied especially to so-called harmonic and local characterizations for both scalar- and vector-valued function spaces. It is the aim of Chapter IV to discuss the interrelation between fractals and function spaces. This paves the way to a spectral theory of fractal differential operators which is developed in Chapter V. The author first deals with elliptic operators generated by a quadratic form in $H^s(\Gamma)$, where Γ is a compact d-set in $\mathbb{R}^n$ with $0 < d < n$, followed by a detailed study of pseudodifferential operators with fractals coefficients. The third (and the most interesting) type of operators appears in a mixed situation – on the one hand the operators may live on a compact d-set, but on the other hand they communicate with the surrounding space $\mathbb{R}^n$. The last part of the book is concerned with Schrödinger operators having fractal potentials and with some related nonlinear problems.

The presentation of the material is clear and concise. The book is recommended mainly to professionals, but it is also useful to graduates having a basic knowledge of fractal geometry, functional analysis and partial differential equations. Further developments of this new subject are expected.

J. Kolumbán

C. E. D'ATTELLIS, E. M. FERNÁNDEZ-BERDAGUER (Eds), *Wavelet Theory and Harmonic Analysis in Applied Sciences*, Applied and Numerical Harmonic Analysis, Series Editor: JOHN J. BENEDETTO), Birkhäuser Verlag, Boston–Basel–Berlin, 1997, xviii + 346 pp., ISBN 0-8176-3953-5, 3-7643-3953-5.

This book contains a selection of fourteen papers presented at The First Latin-American Conference on Mathematics in Industry and Medicine, held in Buenos Aires, Argentina, from November 27 to December 1, 1995. The editors published in this volume the works focused on the theory and applications of wavelet and harmonic analysis.

The contributed papers are divided into three parts. Part I, *Theory and Implementations*, is addressed to specialists in the recent theoretical developments in wavelet theory and harmonic analysis. Singular operators, wavelet characterization of functions, multiwavelet bases, frame and Riesz bases, Fourier analysis and multiresolution analyses are considered.

Part II, *Applications in Biomedical Sciences*, deals with concrete applications of wavelet and spectral analysis to human heart electric activity, study of cardiorespiratory signals and analysis of electroencephalogram signals.

Papers in Part III, *Applications in Physical Sciences*, are devoted to applications such as modelling nonlinear processes, computing the electrostatic potential, wave propagation in solids and magnetotellurics.

Petru Blaga