REVUE D'ANALYSE NUMÉRIQUE ET DE THÉORIE DE L'APPROXIMATION

Tome 28, N° 2, 1999, pp. 163-172 such that any pair of two adjacent cells are situated either in the sume row or in

the same column and any system founded by three cells from the chain is not situated in the same row or in the same column. If the last cell of the chain is in

the same row or column with the first cell, the chain is called a cycle AN ALGORITHM FOR MULTICRITERIA TRANSPORTATION PROBLEMS It is known that, it a transportation problem admits a transport plan, there it

admins a teast an evelic transport plan. LIANA LUPSA, EUGENIA DUCA and DOREL I, DUCA $m+\kappa-1$, then the plan is called nondegenerated. If this number is smaller than

m+n-1, then the transport plan is called degenerated. If the transport plan I. PRELIMINARIES HIW I LIGHT SEE SHEET IN THE SEE STATES IN THE SEE STATES IN THE SECOND SEE (17) = Z.

It is known that many applications of linear programming in managing economic processes come to the solving some transportation problems. A such model was presented for instance in [8]. hoogamoo tast allowed thus alternate 1

A transportation problem (of the cost type) is a linear programming problem of the following type and state of the scheduling and sels are not sels. The family of the scheduling areas areas.

(C)
$$\min_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij}$$
 (X) log vid before the according to the accord

subject to
$$\sum_{j=1}^{n} x_{ij} = a_i, \ i \in \{1, ..., m\}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \ j \in \{1, ..., m\} \times \{1, ..., n\},$$

be denoted by Sol(X).

as usually represented in a table of the following type

ships (n + m) ods with	c_{11}	14897	c_{1n}	a_1	I the real numbers on the
	N. A. Stan	1000	(TAXA)		mosti (av. 11 av.) v
gulero o alconigar die Ta	c_{m1}	201020	C_{mn}	a_m	Annual Charles Har Warman - dist
pointed at a X wall pay	b_1	m \$126	b_n	BXS K	THEOREM I.I. (sec. J
is a pareaual plan with	A THIN IN THE	(56)	35.9.	6,172,1	pion il cad only il there is

Any feasible solution of the problem (C) is called a transport plan. A chain is any system of cells of the type

$$(i_1, j_1), (i_1, j_2), (i_2, j_2), (i_2, j_3), \dots$$

In the following,
$$y_1(j_1,j_2)$$
, (j_1,j_2) , a multicritaria linear programming problem in which

Forme 28, 17" Z. 1990, np. 163-172 such that any pair of two adjacent cells are situated either in the same row or in the same column and any system formed by three cells from the chain is not situated in the same row or in the same column. If the last cell of the chain is in the same row or column with the first cell, the chain is called a cycle.

A transport plan $X = (x_{ij})$ is acyclic if the cells that correspond to $x_{ii} > 0$ do not contain any cycle AFLIGORY MOTTATROSCHART

It is known that, if a transportation problem admits a transport plan, then it admits a least an cyclic transport plan.

If, in an acyclic transport plan $X = (x_{ij})$, the number of elements $x_{ij} > 0$ is m+n-1, then the plan is called nondegenerated. If this number is smaller than m + n - 1, then the transport plan is called degenerated. If the transport plan $X = (x_{ii})$ is degenerated, it will be convenient to add to the set

$$\{(i,j) \in \{1,...,m\} \times \{1,...,n\} \colon x_{ij} > 0\},$$
 geographic of the sum of the

some elements $(h, k) \in \{1, ..., m\} \times \{1, ..., n\}$ such that the new set has m + n -1 elements and the cells that correspond to it do not form a cycle; such a set is called a selection set generated by X. Obviously, in general, we can generate more selection sets. The family of the selection sets generated by the plan X will be denoted by Sel(X).

The acyclic transport plan $X = (x_{ij})$ is called potential with respect to a selection set $A \in Sel(X)$, if there exist the real numbers

$$u_1, ..., u_m, v_1, ..., v_n$$

which satisfy the conditions

(1)
$$v_j - u_i \le c_{ij} \text{ for all } (i, j) \in \{1, ..., m\} \times \{1, ..., n\}$$

and

and
(2)
$$v_j - u_i = c_{ij} \text{ for all } (i, j) \in A.$$

If the real numbers $u_1, \ldots, u_m, v_1, \ldots, v_n$ satisfy (1)–(2), then the (m + n)-tuple $(u_1, ..., u_m, v_1, ..., v_n)$ is called a potential system for A.

THEOREM 1.1. (see, for example [11]). The transport plan X is an optimal plan if and only if there is a set $A \in Sel(X)$ such that X is a potential plan with

2. MULTICRITERIA TRANSPORTATION PROBLEMS

In the following we call a multicriteria transportation problem of the cost type, denoted by (MTP), a multicriteria linear programming problem in which the objective function is a vector function $f = (f_1, ..., f_p) : \mathbb{R}^{m \times n} \to \mathbb{R}^p$, given by

$$f_k(X) = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^k x_{ij} \quad k \in \{1, \dots, p\},$$
 for all $X = (x_{ij}) \in \mathbb{R}^{m \times n}$, and the constraints are

$$\sum_{j=1}^{n} x_{ij} = a_i, i \in \{1, ..., m\}$$

$$\sum_{j=1}^{n} x_{ij} = b_j, j \in \{1, ..., n\}$$

$$x_{ij} \ge 0, (i, j) \in \{1, ..., m\} \times \{1, ..., n\}$$
We assume that since $\{x_{ij} \ge 0, (i, j) \in \{1, ..., m\} \times \{1, ..., n\}$

By analogy with the scalar case, a multicriteria transportation problem (MTP) will be represented by

The following result	c^1_{11},\dots,c^p_{13}	ben Ha	c^1_{ln},\ldots,c^p_{ln}	a_{1}	for each $(i,j) \in \{1,\}$ gives a sufficient coa
plan of the problem	$c_{m1}^1, \dots, c_{m1}^p$	∵ ₩	$c_{mn}^{1},\ldots c_{mn}^{p}$	a_n	THEOREM 2.1. (IL), with respect on
	b_1		b_n		(f)
	10 15 11 11 11	11 11	A B MAY WELL	W	ir un '> 0 , um illia

The set of the feasible solution of the problem (MTP) will be denoted by S. A transport plan $X \in S$ is called Pareto (or min-efficient) if there is no $Y \in S$ such that

$$f_k(Y) \le f_k(X), k \in \{1, ..., p\}.$$

at least one of the inequalities being strict. Because any Pareto transport plan is a Pareto solution of a multicriteria linear programming problem, some interesting properties of the Pareto transport plan set can be found for instance in [7]. On the other hands, for the determination of a Pareto transport plan, we can use any algorithms given in [2], [9], [10] etc. If, in addition, $x_{ii}((i, j) \in \{1, ..., m\} \times \{1, ..., n\})$ must be integer, the algorithm given in [7] allows us to determine all the equivalence classes of the Pareto transport plans. Note that we can also solve a multicriteria transportation problem using the r-balance points [6].

The particular form of the multicriteria linear programming problem which corresponds to the multicriteria transportation problem (MTP) allows us to elaborate specific algorithms, as we can see below.

In the following we will denote by (T_k) $(k \in \{1, ..., p\})$ the scalar trans- $\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{k} x_{ij}$ vd stoned sW portation problem

$$\sin \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{k} x_{ij} \qquad \text{i.e.}$$

subject to

 $\sum_{j=1}^{n} x_{ij} = a_i, \ i \in \{1, ..., m\}$ $\sum_{i=1}^m x_{ij} = b_j, \ j \in \{1, ..., n\}$ where $\sum_{i=1}^m x_{ij} = b_i$ is a substantial of the second $\sum_{i=1}^m x_{ij} = k$. He so $x_{ii} \ge 0, (i, j) \in \{1, ..., m\} \times \{1, ..., n\}.$ ha Il Ni .ax = ar

Let $X = (x_{ii})$ be an acyclic transport plan and let $A \in Sel(X)$. For each $k \in \{1, ..., n\}$ p}, let $(u_1^k, ..., u_m^k, v_1^k, ..., v_n^k)$ be a solution of the system

$$(w, u) \times (w, v_j^k - u_i^k) = c_{ij}^k, (i, j) \in A.$$

We denote by (917M) moldoig nonerrogeneri eineligitting a
$$\alpha_{ij}^k = v_j^k - u_i^k - c_{ij}^k$$
 takes our thin yeolaan yellow

for each $(i, j) \in \{1, ..., m\} \times \{1, ..., n\}$ and $k \in \{1, ..., p\}$. The following result gives a sufficient condition for the Pareto-efficiency:

THEOREM 2.1. Let $k \in \{1, ..., p\}$. If X is a potential plan of the problem (T_t) , with respect to $A \in Sel(X)$ and if

$$\alpha_{ij}^k = v_j^k - u_i^k - c_{ij}^k < 0$$

for each personal will be set of the reaching and the more demonstration and the set of the set of

Leave point $X \in A \setminus \{n, ..., l\} \times \{m, ..., l\} \ni (i, i)$ there is no $I \in S$ such that

$$(u_1^k, ..., u_m^k, v_1^k, ..., v_n^k)$$

is a solution of the system, an employed using the control of the system and the system of the syste

Pareto solution of a rank
$$(r, t)$$
 $\lim_{i \to 0} \frac{1}{i} y = \frac{1}{i}$

then X is a Pareto transport plan for the multicriteria transportation problem rithmas given in $\{21, \{9\}, \{10\} \text{ etc.} 11 \text{ in addition, } x_1 ((i, j) \in [1], \text{ etc.} \text{ in } X \in \mathbf{M})$,

Proof. If X is a potential plan for the problem (T_t) , then it is an optimal plan for the problem (T_k) . From (3), it follows that X is the unique optimal transport plan for (T_k) . Hence X is a Pareto transport plan for the multicriteria transportation problem (MTP).

Let $X = (x_{ij})$ be a potential plan for the problem (T_1) and let $A \in Sel(X)$. For each $k \in \{1, ..., p\}$ let $(u_1^k, ..., u_m^k, v_1^k, ..., v_n^k)$ be a solution of the system

portation problem

(4)
$$v_j^k - u_i^k = c_{ij}^k, \quad (i, j) \in A$$
We denote by
$$\alpha_{ij}^k = v_j^k - u_i^k - c_{ij}^k$$

We denote by

$$\alpha_{ij}^k = \nu_j^k - u_i^k - c_i$$

for each $(i, j) \in \{1, ..., m\} \times \{1, ..., n\}$ and $k \in \{1, ..., p\}$. Obviously $\alpha_{ij}^1 < 0$, for all $(i,j) \in A_X^0 := \{1,...,m\} \times \{1,...,n\}$. Let $A_X^1 = \{(i, j) \in A_X^0 : \alpha_{ij}^1 = 0\}$.

$$A_X^1 = \{(i, j) \in A_X^0 : \alpha_{ii}^1 = 0\}$$

We presume that $\alpha_{ij}^2 \le 0$, for all $(i, j) \in A_X^1$, and we denote by $A_X^2 = \{(i, j) \in A_X^1 : \alpha_{ij}^2 = 0\}.$

$$A_X^2 = \{(i, j) \in A_X^1 : \alpha_{ii}^2 = 0\}.$$

We continue in the same way: if there is $k \in \{1, ..., p-1\}$, for which $\alpha_{ij}^k \le 0$ for all $(i, j) \in A_X^{k-1}$, we denote by

$$A_X^k = \{(i, j) \in A_X^{k-1} : \alpha_{ii}^k = 0\}.$$

A necessary condition for the Pareto-efficiency is given by the following result.

THEOREM 2.2. If there exists $q \in \{1, ..., p-1\}$ such that

$$\alpha_{ij}^{k} \le 0$$
, for all $(i, j) \in A_X^{k-1}$, and $k \in \{1, ..., q\}$,

and if there exists $(r, s) \in A_X^q = \{(i, j) \in A_X^{q-1} : \alpha_{ii}^q = 0\}$, with $\alpha_{rs}^{q+1} > 0$, then there is a transport plan Y having the property that

and
$$f_k(Y) = f_k(X)$$
 for each $k \in \{1, ..., q\}$

$$f_{q+1}(Y) < f_{q+1}(X),$$

which means that X is not Pareto-efficient. $[w, L, L] = \frac{1}{4}K$ but $L = X \log 4W$

Proof. We introduce the cell (r, s) in the transport plan $X = (x_{ij})$. This will have a cycle \mathscr{C} . We travel through this cycle, starting from the cell (r, s) and we assign to its cells the sign + and -, alternatively, starting with the cell (r, s) which gets the + sign. The cells of the cycle denoted by + form a semichain L^+ , and the cells of the cycle denoted by – form a semichain L^- . We analyse the elements x_{ij} of the transport plan X situated in the semichain L^- and we define

$$\theta = \min\{x_{ij} : (i, j) \in L^-\},\$$

d. We consider. which is attained, for example, in the cell (u, t). From the elements x_{ij} which correspond to the semichain L- we substract the number θ , and to the elements x_{ij} corresponding to the semichain L^+ we add the number θ . The other elements, which are not in the cycle \mathscr{C} , remain the same. We obtain a new transport plan Yto which we attach the set with MASA All and when the set with the set

We have

(6)
$$B = A \cup \{(r, s)\} \setminus \{(u, t)\}.$$

Let us denote by M the set of the cells which are not in the cycle \mathscr{C} . Then, for each $k \in \{1, ..., q\}$, we have

$$\begin{split} f_k(Y) &= \sum_{(i,j) \in M} c_{ij}^k y_{ij} + \sum_{(i,j) \in L} c_{ij}^k y_{ij} + \sum_{(i,j) \in L^-} c_{ij}^k y_{ij} = \\ &= \sum_{(i,j) \in M} c_{ij}^k x_{ij} + \sum_{(i,j) \in L^+} c_{ij}^k (x_{ij} + \theta) + \sum_{(i,j) \in L^-} c_{ij}^k (x_{ij} - \theta) = \\ &= f_k(X) - \theta \alpha_{rs}^k = f_k(X). \end{split}$$

Calculating $f_{q+1}(Y)$, we obtain

$$f_{q+1}(Y) = \sum_{(i,j) \in M} c_{ij}^{q+1} y_{ij} + \sum_{(i,j) \in L} c_{ij}^{q+1} y_{ij} + \sum_{(i,j) \in L} c_{ij}^{q+1} y_{ij} =$$

$$= \sum_{(i,j) \in M} c_{ij}^{q+1} x_{ij} + \sum_{(i,j) \in L} c_{ij}^{q+1} (x_{ij} + \theta) + \sum_{(i,j) \in L} c_{ij}^{q+1} (x_{ij} - \theta) =$$

$$= f_{q+1}(X) - \theta \alpha_{rs}^{q+1} < f_{q+1}(X).$$

Hence, the transport plan X is not a Pareto one. \square

areally used. A second of the second of the

Using theorems 2.1 and 2.2, we can state the following algorithm for the determination of a Pareto transport plan for multicriteria transportation problems.

Algorithm

- 1. We put k = 1 and $A_K^0 = \{1, ..., m\} \times \{1, ..., n\}$.
- 2. One determines an cyclic optimal transport plan $X = (x_{ij})$ for the problem (T_1) and we attach to it a set $A \in Sel(X)$.
- 3. We determine a potential system

and bins
$$A$$
 directions a mode $\{(u_1^k,...,u_m^k), v_1^k,...,v_n^k\}$ (less off a gas $+$ off some given by

(7)
$$v_j - u_i = c_{ij}^k$$
, $(i,j) \in A$.

4. We consider the second of t

$$\alpha_{ij}^k = v_j^k - u_{ij}^k - c_{ij}^k$$

for each $(i,j) \in A_X^{k-1}$ in Wheelman and contradict we will investigate the formula of the second states of t

5. We compare to zero each of the numbers A attachment and or guilband person

Which are not in the cycle
$$\mathbb{Z}_{ij}^{k}$$
, $(i,j) \in A_X^{k-1} \setminus A_{ij}$, alone as an interport of X_{ij}^{k} .

is a transport plan: V having the preparty that

- a) If $\alpha_{ii}^k < 0$, for each $(i, j) \in A_X^{k+1} \setminus A$, then X is a Pareto transport plan for the multicriteria transportation problem (MTP) and the algorithm stops.
- b) If there is $(r, s) \in A_X^{k-1} \setminus A$ so that $\alpha_{rs}^k > 0$, then we go to step 10.
- c) If there is $(r, s) \in A_X^{k-1} \setminus A$ such that $\alpha_{rs}^k = 0$ and $\alpha_{ii}^k > 0$, for each $(i, j) \in A_X^{k-1} \setminus A$, then we go to step 6.
- 6. We compare k to p.
 - a) If k = p, then X is a Pareto transport plan for the multicriteria transportation problem (MTP) and the algorithm stops.
 - b) If k < p, then we go to step 7.9
- 7. We put $A_X^k = \{(i, j) \in A_X^{k-1} : \alpha_{ii}^k = 0\}$.
- 8. We compare $A_X^k \setminus A$ to the empty set. a) If $A_X^k \setminus A = \emptyset$, then X is a Pareto transport plan for the multicriteria transportation problem (MTP) and the algorithm stops.
 - b) If $A_X^k \setminus A \neq \emptyset$, then we go to step 9.
- 9. k increase with 1 and we go back to step 3.
- 10. We introduce the cell (r, s) in the transport plan X. This will have a cycle W. We travel through this cycle, starting from the cell (r, s) and we assign to its cells by the sign + and -, alternatively, starting with the cell (r, s)which gets the + sign. The cells of the cycle denoted by + form a semichain L^+ , and the cells of the cycle denoted by – form a semichain L^- . We analyse the elements x_{ij} of the transport plan X situated in the semichain L and we define $(u_1^2, u_2^2, u_3^2, v_4^2, v_3^2, v_3^2, v_4^2) = (0, -1, -3, 3, 1, 3, 8).$

$$\theta = \min\{x_{ij} : (i, j) \in L^-\},\$$

 $\theta = \min\{x_{ij} : (i, j) \in L^-\},$ which is attained, for example, in the cell (u, t). From the elements x_{ij} which correspond to the semichain L^- we substract the number θ , and to the elements x_{ii} corresponding to the semichain L^+ we add the number θ . The other elements, which are not in the cycle &, remain the same. We obtain a new transport plan X to which we attach the set

$$A := A \cup \{(r,s)\} \setminus \{(u,t)\}.$$

We go back to step 3.2 We have the state of the state of

In order to illustrate the above algorithm we conclude with the following numerical example. $A = \{(1, 1), (2, 1), (2, 2), (2, 4), (3, 1), (3, 4),$

Example. Let us consider the (MTP) problem given by Table 1. In this case we have p = 3, m = 3, n = 4. We put k = 1 and $A_X^0 = \{1, 2, 3\} \times$ $\{1, 2, 3, 4\}$. An acyclic optimal transport plan X for the problem (T_1) is

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Tor each	1 2		3 4	3 5	6	7	9	8	1	172	

a) If
$$k = p$$
, then X is $\begin{pmatrix} 48 \text{r.0.0} & 9 \text{ras8} \\ 0 \text{ i.o. problem (MTP):a} \\ 0 \text{ i.o. problem (MTP):a} \\ 0 \text{ i.o. set en we give then we give the problem of the$

о. We сопираге к ю-р.

We have

$$A = \{(1, 1), (1, 4), (2, 1), (2, 2), (3, 2), (3, 3)\}.$$

A solution of the system (7) is such to solution of the system (7) is

tem (7) is a constant of the (1744) matrix in the property
$$(u_1^1, u_2^1, u_3^1, v_1^1, v_2^1, v_3^1, v_4^1) = (0, 2, 0, 4, 3, 5, 7)$$

Since there is $(1, 2) \in A_X^0 \setminus A$ such that $\alpha_{12}^1 = 0$ and $\alpha_{ij}^1 \le 0$ for each $(i,j) \in A_X^0 \setminus A_i$ and k < p, we go to step 7. We have

$$(i, j) \in A_X^{\vee} \setminus A$$
 and $k \in \mathcal{P}$, we go to step \mathcal{P} . We have:
$$A_X^{\vee} = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 4), (3, 1), (3, 2)\}.$$

Since $A_X^{1/2} \setminus A \neq \emptyset$ we put k = 2 and we go to step 3. A solution of the system (7) is

$$(u_1^2, u_2^2, u_3^2, v_1^2, v_2^2, v_3^2, v_4^2) = (0, -1, -3, 3, 1, 3, 8).$$

(7) is $(u_1^2, u_2^2, u_3^2, v_1^2, v_2^2, v_3^2, v_4^2) = (0, -1, -3, 3, 1, 3, 8).$ Since there is $(2, 4) \in A_X^1 \setminus A$ such that $\alpha_{24}^2 = 4 > 0$, we go to step 10. We have $L^{+} = \{(1, 1), (2, 4)\}$ and $L^{-} = \{(2, 1), (1, 4)\}, \theta = 54$ and

$$A = \{(1, 1), (2, 1), (2, 2), (2, 4), (3, 1), (3, 2), (3, 3)\}.$$

A solution of the system (7) is array of the cover of the system (7) is array of the cover of the system (7) is array of the cover of the system (7) is array of the cover of

$$(u_1^2, u_2^2, u_3^2, v_1^2, v_2^2, v_3^2, v_4^2) = (0, -1, -3, 3, 1, 3, 4).$$

Since there is $(3, 1) \in A_X^1 \setminus A$ so that $\alpha_{31}^2 = 4 > 0$, we go to step 10. We have L^+ $= \{(2, 2), (3, 1)\}, L^{-} = \{(2, 1), (3, 2)\} \text{ and } \theta = 49. \text{ The new } A \text{ is an observed}$

$$A = \{(1, 1), (2, 1), (2, 2), (2, 4), (3, 1), (3, 2), (3, 3)\}.$$

A solution of the system (7) is [GTM] and replaced to L. Solution of the system (7) is

A solution of the system (7) is
$$\{(a_1, b_1)_{1 = 1}, (b_1, b_2)_{1 = 1}, (a_1^2, a_2^2, a_3^2, v_1^2, v_2^2, v_3^2, v_4^2) = (0, 3, 1, 3, 5, 7, 8).$$

Since there is $(1, 2) \in A_X^1 \setminus A$ so that $\alpha_{12}^2 = 0$ and $\alpha_{ii}^2 \le 0$ for each $(i, j) \in$ $\in A_X^1 \setminus A$ and k < p, we go to step 7. We have Russiany Moscow Mauka, 1982.

$$A_X^2 = \{(1, 1), (1, 2), (1, 4), (2, 2), (2, 4), (3, 1), (3, 2), (3, 3)\}.$$

11(1) S. L. Johnsvielle, J.J. E.A. Vecesse, Lancar-qual Convex Jones Lundons. Since $A_X^2 \setminus A \neq \emptyset$ we put k = 3 and go to step 3. A solution of the system (7) is

$$(u_1^3, u_2^3, u_3^3, v_1^3, v_2^3, v_3^3, v_4^3) = (0, 5, 4, 15, 7, 11, 13).$$

There is $(1, 4) \in A_X^2 \setminus A$ such that $\alpha_{14}^3 = 9 > 0$. Then we go to step 10. We have

$$L^{+} = \{(1, 4), (2, 2), (3, 1)\}, L^{-} = \{(1, 1), (2, 4), (3, 2)\},$$

 $\theta = 40$.

$$A = \{(1, 1), (1, 4), (2, 2), (2, 4), (3, 1), (3, 3)\}.$$

(8)
$$X = \begin{pmatrix} 62 & 0 & 0 & 40 \\ 0 & 120 & 0 & 14 \\ 89 & 0 & 123 & 0 \end{pmatrix}$$

A solution of the system (7) is

$$(u_1^3, u_2^3, u_3^3, v_1^3, v_2^3, v_3^3, v_4^3) = (0, -4, 4, 5, -2, 11, 4).$$

Since $\alpha_{ij}^3 < 0$, for all $(i, j) \in A_X^2 \setminus A$, we have that X given by (8) is a Pareto transport plan for the multicriteria transportation problem given by Table 1.

REFERENCES

- [1] R. Avram-Nitchi, About the Multiple Objective Fuzzy Operatorial Transportation Problems, Seminarul itinerant de ecuații funcționale, aproximare și convexitate. Cluj-Napoca, 21-26 mai,
- [2] A. Baciu, A. Pascu, E. Puşcaş, Applications of the Operations Research (in Roumanian), București, Ed. Militară, 1988.
- [3] A. Gupta, S. Khanna, M. C. Puri, Paradoxical Situations in Transportation Problems, Cahiers du C.E.R.O., 34, / (1992), 37–49.
- [4] D. S. Hochbaum, S. P. Hong, On the Complexity of the Production-Transportation Problem, SIAM J. Optimization, 6, 1 (1996), 250-264.
- [5] L. Lupșa, E. Duca, D. I. Duca, On the Structure of the Set of Points Dominated and Nondominated in an Optimization Problem. Revue d'anal. num. et la théorie de l'approximation, 22, 2 (1993), 193-199.
- [6] L. Lupsa, D. I. Duca, E. Duca, On the Balanced and Nonbalanced Vector Optimization Problems, Revue d'anal, num, et la théorie de l'approximation, 24, 1 (1995), 112-124.
- [7] L. Lupsa. D. I. Duca, E. Duca, Equivalence Classes in the Set of Efficient Solutions, Revue d'anal, num, et la théorie de l'approximation, 25, 1-2 (1996), 127-136.

Ob == 0.

[8] L. Lupsa, D. I. Duca, E. Duca, On Bicriterial Transportation Problems, Revue d'anal, num. et la théorie de l'approximation, 27, 1 (1998), 81-90.

[9] V. V. Podinovsky, V. D. Noghin, Pareto-Optimal Solutions of Multicriteria Problems (in Russian), Moscow, Nauka, 1982.

[10] M. Zeleny, Multiple Criteria Decision Making, New York, McGraw-Hill 1982.

[11] S. I. Zuhovicki, L. I. Avdeeva, Linear and Convex Programming (in Russian), Moscow, Since AF VA # 10 we put k = k and go to step 8.1% solution of the 1964 Auch Nauka 1964.

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A solution of the system (To is at a W T que to open the form $\{(3,1),\ldots(1,5),(m^2_1,m^2_2,m^2_3,m^2_4,m^2_4,m^2_4,m^2_4)\}=\{(0,-3,4),5,\cdots(2,1),4\},$

Since all Co. For all (i, j) E Ar A. we save that X given by (8) is a Parato transport plan for the mubilionicina transportation problem given by Table 1 - -

Some there is the army of \$32,000 and \$33.50 we contribute the wearest

- 11 R. Avrime-Vitebi, About the Mainight Objective Euger Operational Transportation Problems. Sentinaria (tinerga) de ceartit funccionale, aproximares i convextare. Claj Majora: 21-26 mai.
- [2] A. Bacin, A. Pascu, E. Pascas, Applications of the Operations Revealed (in Roumanian), By cured, Ed. Millaria, 1988.
- [3] A. Gapta, S. Schattat, M. C. Ruit, Paradysical Strictous in Transpondion Publicas, Caluers do C.E.R.O., 34, 7 (1992), 17-49.
- [4] D. S. Hochbraum, S. P. Hong, On the Complexity of the Production Languagements Problem. Shift L'Opinionion, 6, 2 (1996), 2 sur 26t
- 151 L. Lupga, E. Dara, E. L. Duca, On the Structure of the So. of Points Dominated and Nordonni nated in an Optimization Problem Feranci and more or lashen's de l'apprayoration, 22.
- [6] E. Lapsa, D. J. Duca, E. Duca, Cu the Balancet and Noolsdomed Vector Optimization Problems. Result Tanak nam er la thlarve de Faganarini dios. 24.14 (1995) (132-124.0 H. 194 - A.
- 1711 Lupsa 17 1 Duca, E. Puca, E. Quivalence Clarge is the Secol Efficient Solution, Parine d'unul. - ann 20 le drien is de L'approvanterent 25 : 1-2 (1966); 127-136