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SOME REMARKS ON HILBERT'S INTEGRAL INEQUALITY

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Abstract. A generalization of the well-known Hilbert's inequality is given. Several other results of this type obtained in recent years follow as a special case of our result.

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1. INTRODUCTION

First, let us recall the well known Hilbert's integral inequality:

THEOREM A. *If $f,\ g \in L^2(0,\infty)$, then the following inequality holds:*

$$\int_0^\infty\int_0^\infty \tfrac{f(x)g(y)}{x+y}\,\mathrm{d}x\mathrm{d}y \leq \pi\left(\int_0^\infty f^2(x)\,\mathrm{d}x\int_0^\infty g^2(x)\,\mathrm{d}x\right)^{\frac{1}{2}}, \tag{1.1}$$

where π is the best constant.

In the recent years a lot of results with generalization of this type of inequality were obtained. Let us mention two of them which take our attention.

THEOREM B. (Gavrea, [1]) i) *Let a be a real number such that $a > 1$. If $f,\ g \in L^2\left(\frac{1}{a}, a\right)$, then:*

$$\int_{\frac{1}{a}}^{a}\int_{\frac{1}{a}}^{a} \tfrac{f(x)g(y)}{(x+y)^\lambda}\,\mathrm{d}x\mathrm{d}y \leq K_a(\lambda)\left(\int_{\frac{1}{a}}^{a} x^{1-\lambda}f^2(x)\,\mathrm{d}x\int_{\frac{1}{a}}^{a} x^{1-\lambda}g^2(x)\,\mathrm{d}x\right)^{\frac{1}{2}}, \tag{1.2}$$

where:

$$K_a(\lambda) = \int_{\frac{1}{a}}^{a} \tfrac{x^{\frac{\lambda-2}{2}}}{(1+x)^\lambda}\,\mathrm{d}x, \qquad \lambda > 0.$$

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ii) *Let* $0 < a < b$. *If* $f,\ g \in L^2(a,b)$, *then:*

$$\int_a^b\int_a^b \frac{f(x)g(y)}{(x+y)^\lambda}\,\mathrm{d}x\mathrm{d}y \le K_{\sqrt{\frac{b}{a}}}(\lambda)\left(\int_a^b x^{1-\lambda}f^2(x)\,\mathrm{d}x\int_a^b x^{1-\lambda}g^2(x)\,\mathrm{d}x\right)^{\frac{1}{2}}. \tag{1.3}$$

THEOREM C (Brnetić and Pečarić, [2]). *If* $f,\ g$ *are real functions and* $\lambda,\ p,\ q > 0$, $\frac{1}{p}+\frac{1}{q}=1$, *then the following inequalities hold and are equivalent:*

$$\int_0^\infty \frac{f(x)g(y)}{(x+y)^\lambda}\,\mathrm{d}x\mathrm{d}y < B\left(\frac{\lambda}{2},\frac{\lambda}{2}\right)\left(\int_0^\infty x^{p-1-\frac{p\lambda}{2}}f^p(x)\,\mathrm{d}x\right)^{\frac{1}{p}}\left(\int_0^\infty x^{q-1-\frac{q\lambda}{2}}g^q(x)\,\mathrm{d}x\right)^{\frac{1}{q}} \tag{1.4}$$

and

$$\int_0^\infty y^{\frac{\lambda p}{2}-1}\left(\int_0^\infty \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda}\right)^p \mathrm{d}y < B^p\left(\frac{\lambda}{2},\frac{\lambda}{2}\right)\int_0^\infty x^{p-1-\frac{p\lambda}{2}}f^p(x)\,\mathrm{d}x, \tag{1.5}$$

where B *is a beta-function.*

In this paper we generalize inequalities (1.2) and (1.3). As a special case of our results we obtain Theorem C.

In the rest of our paper we suppose that all integrals converge.

2. MAIN RESULTS

A generalization of inequalities (1.2) and (1.3) is given in the following theorem:

THEOREM 1. *Let* $0 < a < b$. *If* $f,\ g$ *are real functions and* $p,\ q > 0$, $\frac{1}{p}+\frac{1}{q}=1$, *then:*

$$\int_a^b\int_a^b \frac{f(x)g(y)}{(x+y)^\lambda}\,\mathrm{d}x\mathrm{d}y \le K_{\sqrt{\frac{b}{a}}}(\lambda)\left(\int_a^b x^{p-1-\frac{p\lambda}{2}}f^p(x)\,\mathrm{d}x\right)^{\frac{1}{p}}\left(\int_a^b x^{q-1-\frac{q\lambda}{2}}g^q(x)\,\mathrm{d}x\right)^{\frac{1}{q}}, \tag{2.1}$$

where

$$K_{\sqrt{\frac{b}{a}}}(\lambda) = \int_{\sqrt{\frac{a}{b}}}^{\sqrt{\frac{b}{a}}} \frac{x^{\frac{\lambda-2}{2}}}{(1+x)^\lambda}\,\mathrm{d}x, \qquad \lambda \in \mathbb{R}^+.$$

Proof. Our proof consists of two steps. In the first step we prove the following lemma:

Lemma 1. *Let a be a real number such that $a>1$. If $f,\ g$ are real functions and $p,\ q>0$, $\frac{1}{p}+\frac{1}{q}=1$, then:*

(2.2)
$$\int_{\frac{1}{a}}^{a}\int_{\frac{1}{a}}^{a} \frac{f(x)g(y)}{(x+y)^{\lambda}}\,\mathrm{d}x\mathrm{d}y \le K_a(\lambda)\left(\int_{\frac{1}{a}}^{a} x^{p-1-\frac{p\lambda}{2}} f^p(x)\,\mathrm{d}x\right)^{\frac{1}{p}}\left(\int_{\frac{1}{a}}^{a} x^{q-1-\frac{q\lambda}{2}} g^q(x)\,\mathrm{d}x\right)^{\frac{1}{q}},$$

where

$$K_a(\lambda)=\int_{\frac{1}{a}}^{a} \frac{x^{\frac{\lambda-2}{2}}}{(1+x)^{\lambda}}\,\mathrm{d}x, \qquad \lambda\in\mathbb{R}^{+}.$$

Proof. We start with the following equality:

$$\int_{\frac{1}{a}}^{a}\int_{\frac{1}{a}}^{a} \frac{f(x)g(y)}{(x+y)^{\lambda}}\,\mathrm{d}x\mathrm{d}y = \int_{\frac{1}{a}}^{a}\int_{\frac{1}{a}}^{a} \frac{f(x)\frac{x^{\frac{2-\lambda}{2q}}}{y^{\frac{2-\lambda}{2p}}}}{(x+y)^{\frac{\lambda}{p}}}\cdot\frac{g(y)\frac{y^{\frac{2-\lambda}{2p}}}{x^{\frac{2-\lambda}{2q}}}}{(x+y)^{\frac{\lambda}{q}}}\,\mathrm{d}x\mathrm{d}y. \tag{2.3}$$

By Hölder's inequality and (2.3), we have:

$$\int_{\frac{1}{a}}^{a}\int_{\frac{1}{a}}^{a} \frac{f(x)g(y)}{(x+y)^{\lambda}}\,\mathrm{d}x\mathrm{d}y \le$$

$$\le \left(\int_{\frac{1}{a}}^{a}\int_{\frac{1}{a}}^{a} \frac{f^p(x)}{(x+y)^{\lambda}}\cdot\frac{x^{\frac{p(2-\lambda)}{2q}}}{y^{\frac{2-\lambda}{2}}}\,\mathrm{d}x\mathrm{d}y\right)^{\frac{1}{p}}\left(\int_{\frac{1}{a}}^{a}\int_{\frac{1}{a}}^{a} \frac{g^q(y)}{(x+y)^{\lambda}}\cdot\frac{y^{\frac{q(2-\lambda)}{2p}}}{x^{\frac{2-\lambda}{2}}}\,\mathrm{d}x\mathrm{d}y\right)^{\frac{1}{q}}$$

$$= \left(\int_{\frac{1}{a}}^{a}\int_{\frac{1}{a}}^{a} \frac{f^p(x)}{(x+y)^{\lambda}}\cdot\frac{x^{\frac{p(2-\lambda)}{2q}}\cdot x^{\frac{2-\lambda}{2}}}{x^{\frac{2-\lambda}{2}}\cdot y^{\frac{2-\lambda}{2}}}\,\mathrm{d}x\mathrm{d}y\right)^{\frac{1}{p}}\times$$

$$\times\left(\int_{\frac{1}{a}}^{a}\int_{\frac{1}{a}}^{a} \frac{g^q(y)}{(x+y)^{\lambda}}\cdot\frac{y^{\frac{q(2-\lambda)}{2p}}\cdot y^{\frac{2-\lambda}{2}}}{y^{\frac{2-\lambda}{2}}\cdot x^{\frac{2-\lambda}{2}}}\,\mathrm{d}x\mathrm{d}y\right)^{\frac{1}{q}}$$

$$= \left(\int_{\frac{1}{a}}^{a} f^p(x)x^{\frac{(2-\lambda)(p-q)}{2q}}\left(\int_{\frac{1}{a}}^{a}\frac{\left(\frac{y}{x}\right)^{\frac{\lambda-2}{2}}}{(x+y)^{\lambda}}\,\mathrm{d}y\right)\mathrm{d}x\right)^{\frac{1}{p}}\times$$

$$\times\left(\int_{\frac{1}{a}}^{a} g^q(y)y^{\frac{(2-\lambda)(q-p)}{2p}}\left(\int_{\frac{1}{a}}^{a}\frac{\left(\frac{x}{y}\right)^{\frac{\lambda-2}{2}}}{(x+y)^\lambda}\,\mathrm{d}x\right)\mathrm{d}y\right)^{\frac{1}{q}}$$

$$(2.4)\qquad =\left(\int_{\frac{1}{a}}^{a} f^p(x)x^{\frac{(2-\lambda)(p-q)}{2q}} I_x\,\mathrm{d}x\right)^{\frac{1}{p}}\left(\int_{\frac{1}{a}}^{a} g^q(y)y^{\frac{(2-\lambda)(q-p)}{2p}} I_y\,\mathrm{d}y\right)^{\frac{1}{q}},$$

where we denote $I_x=\int_{\frac{1}{a}}^{a}\left(\frac{y}{x}\right)^{\frac{\lambda-2}{2}}(x+y)^{-\lambda}\,\mathrm{d}y$, $I_y=\int_{\frac{1}{a}}^{a}\left(\frac{x}{y}\right)^{\frac{\lambda-2}{2}}(x+y)^{-\lambda}\,\mathrm{d}x$.

Using the change of variables $y=xt$, $\mathrm{d}y=x\,\mathrm{d}t$ we have for I_x:

$$I_x=\int_{\frac{1}{ax}}^{\frac{a}{x}}\frac{t^{\frac{\lambda-2}{2}}}{(x+xt)^\lambda}x\,\mathrm{d}t=x^{1-\lambda}\int_{\frac{1}{ax}}^{\frac{a}{x}}\frac{t^{\frac{\lambda-2}{2}}}{(1+t)^\lambda}\,\mathrm{d}t=x^{1-\lambda}h(x)$$

and similarly: $I_y=y^{1-\lambda}h(y)$.

Since $x\in\left(\frac{1}{a},a\right)$ we conclude that $h(x)=\int_{\frac{1}{ax}}^{\frac{a}{x}}t^{\frac{\lambda-2}{2}}(1+t)^{-\lambda}\,\mathrm{d}t\geq 0$, and for $\lambda>0$ h strictly increasing on $\left(\frac{1}{a},1\right)$ and strictly decreasing on $(1,a)$ (see also [1]). Hence:

$$(2.5)\qquad h(x)\leq h(1)=\int_{\frac{1}{a}}^{a}\frac{t^{\frac{\lambda-2}{2}}}{(1+t)^\lambda}\,\mathrm{d}t=K_a(\lambda).$$

Using this inequality, (2.4) can be rewritten as:

$$\int_{\frac{1}{a}}^{a}\int_{\frac{1}{a}}^{a}\frac{f(x)g(y)}{(x+y)^\lambda}\,\mathrm{d}x\mathrm{d}y\leq$$

$$\leq\left(\int_{\frac{1}{a}}^{a} f^p(x)x^{\frac{(2-\lambda)(p-q)}{2q}+1-\lambda}h(1)\,\mathrm{d}x\right)^{\frac{1}{p}}\left(\int_{\frac{1}{a}}^{a} g^q(y)y^{\frac{(2-\lambda)(q-p)}{2p}+1-\lambda}h(1)\,\mathrm{d}y\right)^{\frac{1}{q}}$$

$$=K_a(\lambda)\left(\int_{\frac{1}{a}}^{a} f^p(x)x^{p-1-\frac{p\lambda}{2}}\,\mathrm{d}x\right)^{\frac{1}{p}}\left(\int_{\frac{1}{a}}^{a} g^q(y)y^{q-1-\frac{q\lambda}{2}}\,\mathrm{d}y\right)^{\frac{1}{q}}$$

and the Lemma is proved. □

Remark 1. Inequality (2.2) is a generalization of Theorem B i). We obtain (1.2) by putting $p=q=2$ in (2.2). □

We continue proving our Theorem 1 by rearranging (2.2). We put $\sqrt{\frac{b}{a}}$ instead a, $b > a$, and obtain:

$$\int\limits_{\sqrt{\frac{a}{b}}}^{\sqrt{\frac{b}{a}}}\int\limits_{\sqrt{\frac{a}{b}}}^{\sqrt{\frac{b}{a}}} \frac{f_1(x_1)g_1(y_1)}{(x_1+y_1)^\lambda}\,\mathrm{d}x_1\mathrm{d}y_1 \le \le K_{\sqrt{\frac{b}{a}}}(\lambda)\left(\int\limits_{\sqrt{\frac{a}{b}}}^{\sqrt{\frac{b}{a}}} f_1^p(x_1)x_1^{p-1-\frac{p\lambda}{2}}\,\mathrm{d}x_1\right)^{\frac{1}{p}}\left(\int\limits_{\sqrt{\frac{a}{b}}}^{\sqrt{\frac{b}{a}}} g_1^q(x_1)x_1^{q-1-\frac{q\lambda}{2}}\,\mathrm{d}x_1\right)^{\frac{1}{q}}. \tag{2.6}$$

Now we transform $X_1OY_1 \to XOY$ by putting $x = x_1\sqrt{ab}$, $y = y_1\sqrt{ab}$. Taking into account $\begin{vmatrix} \frac{\partial x_1}{\partial x} & \frac{\partial y_1}{\partial x} \\ \frac{\partial x_1}{\partial y} & \frac{\partial y_1}{\partial y}\end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{ab}} & 0 \\ 0 & \frac{1}{\sqrt{ab}}\end{vmatrix} = \frac{1}{ab}$, $\sqrt{\frac{a}{b}}\sqrt{ab} = a$, $\sqrt{\frac{b}{a}}\sqrt{ab} = b$, we obtain:

$$\int\limits_a^b\int\limits_a^b \frac{f_1\left(\frac{x}{\sqrt{ab}}\right)g_1\left(\frac{y}{\sqrt{ab}}\right)}{\left(\frac{x}{\sqrt{ab}}+\frac{y}{\sqrt{ab}}\right)^\lambda}\cdot\frac{\mathrm{d}x\mathrm{d}y}{ab} \le \le K_{\sqrt{\frac{b}{a}}}(\lambda)\left(\int\limits_a^b \frac{x^{p-1-\frac{p\lambda}{2}}}{(ab)^{\frac{1}{2}\left(p-1-\frac{p\lambda}{2}\right)}} f_1^p\left(\frac{x}{\sqrt{ab}}\right)\frac{\mathrm{d}x}{\sqrt{ab}}\right)^{\frac{1}{p}} \times \times\left(\int\limits_a^b \frac{x^{q-1-\frac{q\lambda}{2}}}{(ab)^{\frac{1}{2}\left(q-1-\frac{q\lambda}{2}\right)}} g_1^q\left(\frac{x}{\sqrt{ab}}\right)\frac{\mathrm{d}x}{\sqrt{ab}}\right)^{\frac{1}{q}}. \tag{2.7}$$

Replacing $f_1\left(\frac{x}{\sqrt{ab}}\right)$ with $f(x)$ and $g_1\left(\frac{y}{\sqrt{ab}}\right)$ with $g(y)$ we rewrite inequality (2.7):

$$\int\limits_a^b\int\limits_a^b \frac{f(x)g(y)}{(x+y)^\lambda}\cdot\frac{(ab)^{\frac{\lambda}{2}}}{ab}\,\mathrm{d}x\mathrm{d}y \le \le K_{\sqrt{\frac{b}{a}}}(\lambda)\left(\int\limits_a^b x^{p-1-\frac{p\lambda}{2}} f^p(x)\frac{(ab)^{-\frac{1}{2}\left(p-1-\frac{p\lambda}{2}\right)}}{\sqrt{ab}}\,\mathrm{d}x\right)^{\frac{1}{p}} \times$$

$$\times \left(\int_a^b x^{q-1-\frac{q\lambda}{2}} g^q(x) \frac{(ab)^{-\frac{1}{2}\left(q-1-\frac{q\lambda}{2}\right)}}{\sqrt{ab}} \,\mathrm{d}x \right)^{\frac{1}{q}}.$$

By dividing both sides of the last inequality with

$$(ab)^{\frac{\lambda}{2}-1} = (ab)^{-\frac{1}{2}+\frac{1}{2p}+\frac{p\lambda}{4p}-\frac{1}{2p}-\frac{1}{2}+\frac{1}{2q}+\frac{q\lambda}{4q}-\frac{1}{2q}}$$

we obtain (2.1), thus completing the proof of Theorem 1. □

Remark 2. Inequality (2.1) is a generalization of Theorem B ii). We obtain (1.3) by putting $p = q = 2$ in (2.1). □

A generalization of inequalities (1.4) and (1.5) is given in the following theorem:

Theorem 2. *Let* $0 < a < b$. *If* f *is a real function and* $p > 1$, *then:*

$$\int_a^b y^{\frac{\lambda p}{2}-1} \left(\int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda} \right)^p \mathrm{d}y \le K^p_{\sqrt{\frac{b}{a}}}(\lambda) \int_a^b x^{p-1-\frac{p\lambda}{2}} f^p(x)\,\mathrm{d}x. \tag{2.8}$$

Also, (2.1) *and* (2.8) *are equivalent.*

Proof. Let us show that (2.1) and (2.8) are equivalent.

First suppose that inequality (2.1) is valid. Denoting

$$g(y) = \left(\int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda} \right)^{p-1} y^{\frac{\lambda p}{2}-1}$$

we have:

$$\begin{aligned}
&\int_a^b y^{\frac{\lambda p}{2}-1} \left(\int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda} \right)^p \mathrm{d}y = \\
&\quad = \int_a^b y^{\frac{\lambda p}{2}-1} \left(\int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda} \right)^{p-1} \int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda}\,\mathrm{d}y \\
&\quad = \int_a^b g(y) \int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda}\,\mathrm{d}y = \int_a^b \int_a^b \frac{f(x)g(y)}{(x+y)^\lambda}\,\mathrm{d}x\mathrm{d}y \\
&\quad \le K_{\sqrt{\frac{b}{a}}}(\lambda) \left(\int_a^b x^{p-1-\frac{p\lambda}{2}} f^p(x)\,\mathrm{d}x \right)^{\frac{1}{p}} \left(\int_a^b y^{q-1-\frac{q\lambda}{2}} g^q(y)\,\mathrm{d}y \right)^{\frac{1}{q}} \\
&\quad = K_{\sqrt{\frac{b}{a}}}(\lambda) \left(\int_a^b x^{p-1-\frac{p\lambda}{2}} f^p(x)\,\mathrm{d}x \right)^{\frac{1}{p}} \times
\end{aligned}$$

$$\times\left(\int_a^b y^{q-1-\frac{q\lambda}{2}} y^{\frac{\lambda pq}{2}-q}\left(\int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda}\right)^{(p-1)q} \mathrm{d}y\right)^{\frac{1}{q}}$$

$$= K_{\sqrt{\frac{b}{a}}}(\lambda)\left(\int_a^b x^{p-1-\frac{p\lambda}{2}} f^p(x)\,\mathrm{d}x\right)^{\frac{1}{p}} \times$$

$$\times\left(\int_a^b y^{\frac{\lambda p}{2}-1}\left(\int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda}\right)^{p+q-q} \mathrm{d}y\right)^{\frac{1}{q}}.$$

By putting:

$$I = \int_a^b y^{\frac{\lambda p}{2}-1}\left(\int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda}\right)^p \mathrm{d}y,$$

we can write the last inequality in the following form:

$$I \le K_{\sqrt{\frac{b}{a}}}(\lambda)\left(\int_a^b x^{p-1-\frac{p\lambda}{2}} f^p(x)\,\mathrm{d}x\right)^{\frac{1}{p}} \times I^{\frac{1}{q}},$$

wherefrom we have (2.8).

Now let us suppose that inequality (2.8) is valid. By applying Hölder's inequality (in one variable) and (2.8) we have:

$$\int_a^b\int_a^b \frac{f(x)g(y)}{(x+y)^\lambda}\,\mathrm{d}x\mathrm{d}y =$$

$$= \int_a^b y^{-\frac{q-1-\frac{q\lambda}{2}}{q}}\left(\int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda}\right) y^{\frac{q-1-\frac{q\lambda}{2}}{q}} g(y)\,\mathrm{d}y$$

$$= \int_a^b\left(\int_a^b y^{-\frac{q-1-\frac{q\lambda}{2}}{q}} \frac{f(x)}{(x+y)^\lambda}\,\mathrm{d}x\right) y^{\frac{q-1-\frac{q\lambda}{2}}{q}} g(y)\,\mathrm{d}y$$

$$\le \left(\int_a^b y^{-\frac{p\left(q-1-\frac{q\lambda}{2}\right)}{q}}\left(\int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda}\right)^p \mathrm{d}y\right)^{\frac{1}{p}}\left(\int_a^b y^{q-1-\frac{q\lambda}{2}} g^q(y)\,\mathrm{d}y\right)^{\frac{1}{q}}$$

$$= \left(\int_a^b y^{\frac{\lambda p}{2}-1}\left(\int_a^b \frac{f(x)\,\mathrm{d}x}{(x+y)^\lambda}\right)^p \mathrm{d}y\right)^{\frac{1}{p}}\left(\int_a^b y^{q-1-\frac{q\lambda}{2}} g^q(y)\,\mathrm{d}y\right)^{\frac{1}{q}},$$

wherefrom we have (2.1). Since (2.1) has already been proved, the inequality (2.8) holds, too. □

Remark 3. For $a \to 0$, $b \to \infty$ we have $K_{\sqrt{\frac{b}{a}}}(\lambda) \to B\left(\frac{\lambda}{2}, \frac{\lambda}{2}\right)$ and we obtain Theorem C as a special case of our result. $\square$

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