REVUE D'ANALYSE NUMÉRIQUE ET DE THÉORIE DE L'APPROXIMATION
Rev. Anal. Numér. Théor. Approx., vol. 35 (2006) no. 1, pp. 127–128
ictp.acad.ro/jnaat

BOOK REVIEWS

IVAN SINGER, *Duality for Nonconvex Approximation and Optimization*, Canadian Mathematical Society (CMS) Books in Mathematics, Vol. **24**, Springer 2006, ISBN 3-387-28394-3, XVIII+355 pp.

The book is concerned with duality in nonconvex approximation in normed spaces and with duality in nonconvex optimization in locally convex spaces. Nonconvex approximation problems in normed spaces comprise two classes of problems, called anticonvex by Jean-Paul Penot: the problem of maximization of the distance of an element to an arbitrary set (the problem of farthest points) and the problem of the minimization of the distance of an element to the complement of a convex set (called also the problem of best approximation from caverns).

After a period of time when the duality in approximation theory and duality in optimization developed in parallel, it was realized in the 1960s that the approximation theory is a particular case of more general optimization problems – minimization or maximization of a function. Starting with two seminal papers published by the author in 1974 and 1979 a new idea emerged, namely that many results in the approximation theory are sufficiently strong to yield new methods and results in optimization theory. This interplay between approximation and optimization – the duality for anticonvex approximation and for convex-anticonvex optimization – is the principal theme of the book. The approach varies from problems in normed linear spaces to more general problems in locally convex spaces, or within the framework of abstract convex analysis, as developed by the author in a previous monograph *Abstract Convex Analysis* (Wiley-Interscience 1997). Some preliminaries from convex analysis and elements of abstract convex analysis are included in the first chapter of the book *Preliminaries.*

We shall describe briefly the content of the book by chapters. Chapter 2, *Worst approximation,* is concerned with the problem of farthest points in normed spaces – existence, characterizations and duality formulas for the deviation function $\delta(x_0, G) = \sup\{\|x-g\| : g \in G\}$. Chapters 3, *Duality for quasi-convex supremization* and 4, *Optimal solutions for quasi-convex maximization,* are concerned with the problem of supremization, respectively maximization, of a quasi-convex function $f : X \to \overline{\mathbf{R}}$ over a subset G of the locally convex space X – unconstrained and constrained surrogate dual problems, unperturbational and perturbational Lagrangian, characterizations of optimal solutions. Chapter 5, *Reverse convex best approximation,* is concerned with the problem of best approximation by elements in the complement of a convex set: existence, characterization, duality formulas for the distance function. In Chapters 6, *Unperturbational duality for reverse convex infimization* and 7, *Optimal solutions for reverse convex optimization*, one considers the more general problem of reverse convex infimization, respectively minimization, of a function f over the complement of a convex subset G of a locally convex space X.

Chapter 8, *Duality for d.c. optimization problems,* is devoted to an important topic in modern optimization - optimization problems involving differences of convex functions (called d.c. functions). The methods of abstract convexity theory, which covers various generalizations of convex functions and convex sets, permit in Chapter 9, *Duality for optimization in the framework of abstract convexity,* the treatment of some optimization problems involving more general functions and sets.

The last chapter of the book, Chapter 10, *Notes and remarks,* contains some comments, bibliographical references and additional results for each of the preceding chapters.

Being the first in the literature devoted to duality for nonconvex approximation and optimization, the book is based entirely on journal articles. Some new results and new proofs to old ones are also included. The author is a well known specialist in both approximation theory and optimization theory with many original contributions (some of which are included in the present book) and several books which became standard references in the field: *Best Approximation in Normed Linear Spaces by Elements of Linear Subspaces* (Editura Academiei and Springer, 1970), *The Theory of Best Approximation and Functional Analysis* (SIAM, 1974), *Bases in Banach Spaces I, II* (Springer 1970, 1981), and the book on abstract convexity mentioned above.

The book is addressed to a large audience and can be used for research or as a reference book. Due to the detailed proofs and illustrations, it is also suitable for graduate courses or for self-study.

S. Cobzaş