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ABOUT ONE DISCRETE ANALOG OF HAUSDORFF SEMI-CONTINUITY OF SUITABLE MAPPING IN A VECTOR COMBINATORIAL PROBLEM WITH A PARAMETRIC PRINCIPLE OF OPTIMALITY ("FROM SLATER TO LEXICOGRAPHIC")*

VLADIMIR A. EMELICHEV † and ANDREY A. PLATONOV†

Abstract. A multicriteria linear combinatorial problem is considered, principle of optimality of which is defined by a partitioning of partial criteria onto groups with Slater preference relation within each group and the lexicographic preference relation between them. Quasistability of the problem is investigated. This type of stability is a discrete analog of Hausdorff lower semicontinuity of the many-valued mapping that defines the choice function. A formula of quasistability radius is derived for the case of metric l_∞. Some conditions of quasistability are stated as corollaries.

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1. INTRODUCTION

Usually stability of an optimization problem is understood as continuous dependence of solution set on parameters of the problem. The most general approaches to stability analysis of optimization problems are based on properties of many-valued mappings that define optimality principles [1].

Mathematical analysis does not present methods sufficient to investigate stability of a discrete optimization problem. It is greatly due to complexity of discrete models, which can behave unpredictably under small variations of initial data. At the same time, if terminology of general topology is not used, then the formulation of a question of stability can be significantly simplified in the case of a space of acnodes. There are different types of stability of discrete optimization problems (see e.g. [2–6]). Stability of a discrete problem in the broad sense means that there exists a neighborhood of the initial data in the space of parameters such that any problem with parameters from this neighborhood possesses some invariance with respect to the initial problem. In

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†Belarussian State University, Minsk, Belarus, e-mail: emelichev@bsu.by, plabian@tut.by.

particular, upper (lower) semicontinuity of an optimal mapping is equivalent to nonappearance of new (preserving of initial) optimal solutions under ”small” perturbations of the mapping parameters. So concepts of stability [2–7] and quasistability [5,8–12] of discrete optimization problems arise.

In this article under parametrization of optimality principle (from Slater to lexicographic) we obtain a generalized formula of quasistability radius of a multicriteria linear combinatorial problem with partitioning of criteria into groups according to their importance. Slater preference relation is defined within each group and the lexicographic preference relation – between them. We consider the case, where metric l_∞ is defined in the space of problem parameters.

Note that similar formulas have been derived earlier in [13–21] for stability and quasistability radii of vector trajectorial and game-theoretic problems with other parametric principles of optimality (“from Condorset to Pareto”, “from Pareto to Slater”, “from Pareto to Nash”, “from lexicographic to Nash” and others).

2. DEFINITIONS AND PROPERTIES

We consider typical vector (n-criteria) combinatorial problem. Let a vector criterion

$$f(t,A) = (f_1(t,A_1), f_2(t,A_2), \ldots, f_n(t,A_n)) \to \min_{t\in T}$$

with partial criteria

$$f_i(t,A_i) = \sum_{j\in N(t)} a_{ij}, \quad i \in N_n = \{1,2,...,n\},\ n \geq 1,$$

be defined on a system of subsets (trajectories) $T \subseteq 2^E$, $|T| \geq 2$, of a finite set $E = \{e_1, e_2, \ldots, e_m\}$, $m \geq 2$. Here $N(t) = \{j \in N_m : e_j \in t\}$, A_i is the i-th row of a matrix $A = [a_{ij}] \in \mathbf{R}^{n\times m}$. Put $f_i(\emptyset, A_i) = 0$.

A specific peculiarity of vector optimization is the existence of many principles of optimality (or functions of choice) which could be defined with binary relations of preference.

Let us define three binary relations in the space $\mathbf{R}^p$ of arbitrary dimension $p \in \mathbf{N}$ according to formulas

$$y \underset{Sl}{\succ} y' \Leftrightarrow y_i > y'_i,\ i \in N_p,$$

$$y \underset{Sm}{\succ} y' \Leftrightarrow y_i \geq y'_i,\ i \in N_p,$$

$$y \underset{lex}{\succ} y' \Leftrightarrow y_k > y'_k,$$

where k=min$\{i \in N_p : y_i \neq y'_i\}$, y=$(y_1, y_2, \ldots, y_p)$, y'=$(y'_1, y'_2, \ldots, y'_p)$.

These relations create the following well-known objects of vector optimization:

Slater set (weakly efficient trajectories)

$$Sl^n(A) = \{t \in T : \ \forall t' \in T \ (f(t,A) \underset{Sl}{\overline{\succ}} f(t',A))\},$$

Smale set (strictly efficient trajectories)

$$Sm^n(A) = \{t \in T : \ \forall t' \in T \setminus \{t\} \ (f(t,A) \underset{Sm}{\overline{\succ}} f(t',A))\}$$

and set of lexicographically optimal trajectories

$$L^n(A) = \{t \in T : \ \forall t' \in T \ (f(t,A) \underset{lex}{\overline{\succ}} f(t',A))\}.$$

Here $\overline{\succ}$ is the negation of the relation $\succ$. Obviously that $Sm^n(A) \subseteq Sl^n(A)$. Due to transitivity of relation $\underset{Sl}{\succ}$ and finiteness of set T, set $Sl_n(A) \neq \emptyset$ for any matrix $A \in \mathbf{R}^{n\times m}$. But Smale set can be empty, because binary relation $\underset{Sm}{\succ}$ is not transitivity.

Let $s \in N_n$, $\mathcal{I} = \{I_1, I_2, \ldots, I_s\}$ be a partition of the set N_n into s nonintersecting nonempty subsets, i.e.

$$N_n = \bigcup_{r \in N_s} I_r,$$

where $I_r \neq \emptyset$, $r \in N_s$; $p \neq q \Rightarrow I_p \cap I_q = \emptyset$. For any such partitioning we define the binary relation $\Omega^n_{\mathcal{I}}$ of strict preference in the space $\mathbf{R}^n$ between different vectors $y = (y_1, y_2, \ldots, y_n)$ and $y' = (y'_1, y'_2, \ldots, y'_n)$ as follows:

$$y \ \Omega^n_{\mathcal{I}} \ y' \quad \Leftrightarrow \quad y_{I_k} \succ y'_{I_k},$$

where $k = \min\{i \in N_s : \ \ y_{I_i} \neq y'_{I_i}\}$; y_{I_k} and y'_{I_k} are the projections of the vectors y and y' correspondingly onto the coordinate axes of the space $\mathbf{R}^n$ with numbers from the subset I_k.

The introduced binary relation $\Omega^n_{\mathcal{I}}$ determines ordering of the shaped subsets of criteria such that any previous subset is significantly more important that any consequent subset. This relation generates the set of $\mathcal{I}$-optimal trajectories

$$T^n(A, \mathcal{I}) = \{t \in T : \forall t' \in T \quad (f(t,A) \ \overline{\Omega^n_{\mathcal{I}}} \ f(t',A))\}.$$

It is evident that $T^n(A, \mathcal{I}_{Sl})$, where $\mathcal{I}_{Sl} = \{N_n\}$ $(s = 1)$, is Slater set $Sl^n(A)$, and $T^n(A, \mathcal{I}_L)$, where $\mathcal{I}_L = \{\{1\}, \{2\}, \ldots, \{n\}\}$ $(s = n)$, is the set of lexicographically optimal trajectories.

So under the parametrization of optimality principle we understand assigning a characteristic to binary relation which in special cases induces well-known Slater and lexicographic optimality principles.

It is easy to show that the binary relation $\Omega^n_{\mathcal{I}}$ is antireflexive, asymmetric, transitive, and hence it is a cyclic. And since the set T is finite, the set $T^n(A, \mathcal{I})$ is non-empty for any matrix A and any partitioning $\mathcal{I}$ of the set N_n.

Hereinafter by $Z^n(A, \mathcal{I})$ we denote the problem of finding the set $T^n(A, \mathcal{I})$.

Clearly, $T^1(A,\{1\})$ is the set of optimal trajectories of the scalar linear trajectories problem $Z^1(A,\{1\})$, where $A \in \mathbf{R}^m$. Many extreme combinatorial problems on graphs, boolean programming and scheduling problems and others are reduced to $Z^1(A,\{1\})$ (see e.g. [4, 6, 7]).

Denote

$$Sl_1(A) = \{t \in T : \forall t' \in (f_{I_1}(t,A) \underset{Sl}{\succeq} f_{I_1}(t',A))\}$$

The following properties follow directly from the above definitions.

Property 2.1. $T^n(A,\mathcal{I}) \subseteq Sl_1(A) \subseteq T$.

Property 2.2. If $f_{I_1}(t,A) \underset{Sl}{\succ} f_{I_1}(t',A)$, then $f(t,A)\ \Omega^n_{\mathcal{I}}\ f(t',A)$.

Property 2.3. If $f(t,A)\ \Omega^n_{\mathcal{I}}\ f(t',A)$, then $f_{I_1}(t,A) \underset{Sl}{\succ} f_{I_1}(t',A) \vee f_{I_1}(t,A) = f_{I_1}(t',A)$.

Property 2.4. A trajectory $t \notin T^n(A,\mathcal{I})$ if and only if there exists a trajectory $t' \neq t$, such that $f(t,A)\ \Omega^n_{\mathcal{I}}\ f(t',A)$.

Property 2.5. A trajectory $t \in T^n(A,\mathcal{I})$ if and only if for any trajectory t' the relation $f(t,A)\ \overline{\Omega^n_{\mathcal{I}}}\ f(t',A)$ holds.

Denote

$$Sm_1(A) = \{t \in Sl_1(A) : \ t' \in T \setminus \{t\} \quad (f_{I_1}(t,A) \neq f_{I_1}(t',A))\}.$$

Property 2.6. $Sm_1(A) \subseteq T^n(A,\mathcal{I})$.

Assume the converse, i.e. $t \in Sm(A)$ and $t \notin T^n(A,\mathcal{I})$. Then according to property 2.4 there exists a trajectory $t' \neq t$, such that

$$f(t,A)\,\Omega^n_{\mathcal{I}}\,f(t',A).$$

Hence due to property 2.3 we have

$$f_{I_1}(t,A) \underset{Sl}{\succ} f_{I_1}(t',A) \vee f_{I_1}(t,A) = f_{I_1}(t',A).$$

Taking into account the inclusion $t \in Sl_1(A)$, we obtain

$$f_{I_1}(t,A) = f_{I_1}(t',A),$$

i. e. $t \notin Sm_1(A)$, which contradicts to the assumption.

Directly from definition of set Sm_1 we obtain

Property 2.7. $\forall\ t \in Sm_1(A) \quad \forall\ t' \in T \setminus \{t\} \quad \exists\ i \in I_1 \quad (f_i(t',A_i) > f_i(t,A_i))$.

Let us consider such variant of the problem $Z^n(A,\mathcal{I})$ stability, which is the discrete analog of Hausdorff lower semicontinuity in the point A of the many-valued mapping

$$\mathbf{R}^{n\times m} \to 2^T,$$

that puts set $T^n(A,\mathcal{I})$ to any collection of parameters of the problem from the metric space $\mathbf{R}^{n\times m}$ with metric l_∞. For a vector problem $Z^n(A,\mathcal{I})$, lower semicontinuity means that set $T^n(A,\mathcal{I})$ can only expand under "small" perturbations of elements of matrix A.

For any number $\varepsilon > 0$, define the set of perturbing matrixes

$$\Psi(\varepsilon) = \{A' \in \mathbf{R}^{n\times m} : \ ||A'|| < \varepsilon\},$$

where $||A'|| = \max\{|a'_{ij}| \ : \ (i,j) \in N_n \times N_m\}, \ A' = [a'_{ij}]$.

According to definitions from [5, 8–12], we will give following definitions.

Definition 2.8. *Vector problem $Z^n(A,\mathcal{I})$, $n \geq 1$, is called quasistabile (under perturbations of elements of matrix A), if there exists $\varepsilon > 0$, such that for any perturbing matrix $A' \in \Psi(\varepsilon)$ the inclusion*

$$T^n(A,\mathcal{I}) \subseteq T^n(A+A',\mathcal{I})$$

holds.

Definition 2.9. *Under the quasistability radius of the vector problem $Z^n(A,\mathcal{I})$, $n \geq 1$, we understand the number*

$$\rho^n(A,\mathcal{I}) = \begin{cases} \sup \Xi & \text{, if } \Xi \neq \emptyset, \\ 0 & \text{, if } \Xi = \emptyset, \end{cases}$$

where

$$\Xi = \{\varepsilon > 0 : \forall\ A' \in \Psi(\varepsilon)\ (T^n(A,\mathcal{I}) \subseteq T^n(A+A',\mathcal{I}))\}.$$

3. LEMMAS

For any different trajectories t any t' we define the numbers:

$$\Delta(t,t') = |(t \cup t') \setminus (t \cap t')|,$$

$$d^n(t,t',A) = \max_{i\in I_1} \frac{f_i(t',A_i) - f_i(t,A_i)}{\Delta(t,t')}.$$

Obviously, that $\Delta(t,t') > 0$.

Lemma 3.1. *If $d^n(t,t',A) \geq \varphi > 0$, then the following relation holds for any perturbing matrix $A' \in \Psi(\varphi)$:*

$$f(t,A+A')\,\overline{\Omega^n_{\mathcal{I}}}\,f(t',A+A').$$

Proof. Directly from the definition of the number $d^n(t,t',A)$, there exists index $k \in I_1$, such that

$$f_k(t',A_k) - f_k(t,A_k) \geq \varphi\Delta(t,t'). \tag{1}$$

Further suppose that the assertion of the lemma is false, i.e. that there exists a matrix $A^* = [a^*_{ij}] \in \Psi(\varphi)$ such that $f(t,A+A^*)\ \Omega^n_{\mathcal{I}}\ f(t',A+A^*)$. Then by virtue of property 2.3 and linearity of the functions $f_i(t,A)$, $i \in N_n$, we derive

$$0 \geq f_i(t',A_i+A^*_i) - f_i(t,A_i+A^*_i) = f_i(t',A_i) - f_i(t,A_i) + f_i(t',A^*_i) - f_i(t,A^*_i) \geq$$
$$\geq f_i(t',A_i) - f_i(t,A_i) - ||A^*_i||\Delta(t,t') > f_i(t',A_i) - f_i(t,A_i) - \varphi\Delta(t,t'), \quad i \in I_1,$$

i.e. for any index $i \in I_1$ the inequality

$$f_i(t', A_i) - f_i(t, A_i) < \varphi\Delta(t, t'),$$

holds, which contradicts to (1). □

LEMMA 3.2. *Let $t \in T^n(A, \mathcal{I})$, $t' \in T \setminus \{t\}$. For any number $\alpha > d^n(t, t', A)$ there exists a matrix $A^* \in \mathbf{R}^{n\times m}$ with norm $||A^*|| = \alpha$, such that*

$$f(t, A + A^*)\ \Omega^n_{\mathcal{I}}\ f(t', A + A^*). \tag{2}$$

Proof. We construct the perturbing matrix $A^* = [a^*_{ij}] \in \mathbf{R}^{n\times m}$ by the formula:

$$a^*_{ij} = \begin{cases} -\alpha & \text{, if } i \in I_1,\ e_j \in t' \setminus t, \\ \alpha & \text{, if } i \in I_1,\ e_j \in t \setminus t', \\ 0 & \text{, otherwise.} \end{cases}$$

Then $||A^*|| = \alpha$ and equitably equalities

$$f_i(t', A^*_i) - f_i(t, A^*_i) = -\alpha\Delta(t, t'), \quad i \in I_1.$$

From here we get

$$\frac{1}{\Delta(t, t')}(f_i(t', A_i + A^*_i) - f_i(t, A_i + A^*_i)) = \frac{f_i(t', A_i) - f_i(t, A_i)}{\Delta(t, t')} - \alpha \leq$$

$$\leq d^n(t, t', A) - \alpha < 0, \quad i \in I_1,$$

i.e. $f_{I_1}(t, A + A^*) \underset{Sl}{\succ} f_{I_1}(t', A + A^*)$, this implies (2) by virtue of property 2.2. □

4. THEOREM

THEOREM 4.1. *For any partitioning $\mathcal{I} = \{\{I_1\}, \{I_2\}, \ldots, \{I_s\}\}$ of the set N_n, $n \geq 1$, into s subsets, $s \in N_n$, the quasistability radius $\rho^n(A, \mathcal{I})$ of a problem $Z^n(A, \mathcal{I})$ is expressed by the formula*

$$\rho^n(A, \mathcal{I}) = \min_{t\in T^n(A,\mathcal{I})} \ \min_{t'\in T\setminus\{t\}} \ d^n(t, t', A). \tag{3}$$

Proof. Before proving of the theorem, let us note that from the non-emptiness of $T^n(A, \mathcal{I})$ and $T \setminus \{t\}$ and from the definition of $d^n(t, t', A)$, the right side of formula (3), further define it by φ, is positive and correctly defined.

At first we prove the inequality

$$\rho^n(A, \mathcal{I}) \geq \varphi. \tag{4}$$

Without loss of generality assume that $\varphi > 0$ (otherwise inequality (4) is obvious). From the definition of the number φ, it follows that for any trajectories $t \in T^n(A, \mathcal{I})$ and $t' \neq t$ the inequalities

$$d^n(t, t', A) \geq \varphi > 0$$

hold. Applying lemma 3.1 we get

$$\forall\ A' \in \Psi(\varphi) \quad \forall\ t \in T^n(A, \mathcal{I}) \quad \forall\ t' \in T \quad (f(t, A + A')\ \overline{\Omega^n_{\mathcal{I}}}\ f(t', A + A')).$$

Therefore by virtue of property 2.5 we have $t \in T^n(A + A', \mathcal{I})$. Thus we conclude

$$\forall\ A' \in \Psi(\varphi) \quad (T^n(A, \mathcal{I}) \subseteq T^n(A + A', \mathcal{I})),$$

this formula proves (4).

It remains to show that

$$\rho^n(A, \mathcal{I}) \leq \varphi. \tag{5}$$

Let $\varepsilon > \alpha > \varphi$ and trajectories $t \in T^n(A, \mathcal{I})$ and $t' \neq t$ be such that $d^n(t, t', A) = \varphi$. Then according to lemma 3.2 there exists a matrix A^* with norm $||A^*|| = \alpha$ such that (2) holds, i.e. $t \notin T^n(A + A^*, \mathcal{I})$. Hence we have

$$\forall \varepsilon > \varphi \quad \exists A^* \in \Psi(\varepsilon) \quad (T^n(A, \mathcal{I}) \not\subseteq T^n(A + A^*, \mathcal{I}))\,,$$

which proves inequality (5), said above and taking into (4) we obtain (3). $\square$

5. COROLLARIES

Series of corollaries follows directly from theorem.

COROLLARY 5.1. *The quasistability radius of the problem $Z^n(A, \mathcal{I}_{Sl})$, $n \geq 1$, of finding Slater set $Sl^n(A)$ is expressed by the formula*

$$\rho^n(A, \mathcal{I}_{Sl}) = \min_{t \in Sl^n(A)} \ \min_{t' \in T \setminus \{t\}} \ \max_{i \in N_n} \frac{f_i(t', A_i) - f_i(t, A_i)}{\Delta(t, t')}.$$

The formula of quasistability radius, led in corollary 5.1, easily pass into the formula of quasistability radius of the scalar trajectorial problem with linear criterion [4]

COROLLARY 5.2 ([22]). *The quasistability radius of the problem $Z^n(A, \mathcal{I}_L)$, $n \geq 1$, of finding the set of lexicographically optimal trajectories $L^n(A)$ is expressed by the formula*

$$\rho^n(A, \mathcal{I}_L) = \min_{t \in L^n(A)} \ \min_{t' \in T \setminus \{t\}} \frac{f_1(t', A_1) - f_1(t, A_1)}{\Delta(t, t')}.$$

COROLLARY 5.3. *For any partitioning $\mathcal{I}$ of the set N_n, $n \geq 1$, into s subsets, $s \in N_n$, the following statements are equivalent for a problem $Z^n(A, \mathcal{I})$, $n \geq 1$:*

(i) *the problem $Z^n(A, \mathcal{I})$ is quasistable,*
(ii) $\forall\ t \in T^n(A, \mathcal{I}) \quad \forall\ t' \in T \setminus \{t\} \quad \exists\ i \in I_1 \quad (f_i(t', A_i) > f_i(t, A_i))$,
(iii) $T^n(A, \mathcal{I}) = Sm_1(A)$.

Proof. Equivalence of statements (i) and (ii) follows directly from the theorem.

The implication (ii) $\Rightarrow$ (iii) is proved by contradiction. Suppose that (ii) holds but (iii) does not.

From properties 2.1 and 2.6 we get

$$Sm_1(A) \subseteq T^n(A, \mathcal{I}) \subseteq Sl_1(A).$$

Then (since $T^n(A,\mathcal{I}) \neq Sm_1(A)$ is assumed) there exists a trajectory $t \in T^n(A,\mathcal{I}) \subseteq Sl_1(A)$, such that $t \notin Sm_1(A)$. It follows that there exists a trajectory $t' \in Sl_1(A)$ such that

$$t' \neq t, \quad f_{I_1}(t,A) = f_{I_1}(t',A),$$

which contradicts to statement (ii).

The implication (iii) $\Rightarrow$ (i) is obvious by virtue of property 2.7. $\square$

From corollary 5.3, we easily get the following attendant results.

Corollary 5.4. *The problem $Z^n(A,\mathcal{I}_{Sl}), n \geq 1,$ of finding Slater set $Sl^n(A)$ is quasistable if and only if $Sl^n(A)$ and $Sm^n(A)$ are coinciding.*

It is easy to understand that for a scalar linear trajectorial problem the coincidence of Slater and Smale sets is equivalent to existence of a unique optimal solution. Therefore partial case of corollary 5.4 is

Corollary 5.5 ([4]). *Singlecriterion (scalar) linear trajectorial problem is quasistable if and only if it has a unique optimal solution.*

Corollary 5.6 ([22]). *The problem $Z^n(A,\mathcal{I}_L), n \geq 1,$ of finding the set $L^n(A)$ of lexicographically optimal trajectories is quasistable if and only if*

$$|L^n(A)| = \left|\mathrm{Arg}\ \min_{t\in T}\ f_1(t,A_1)\right| = 1.$$

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