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BOOK REVIEWS

Karl-Georg Steffens, *The History of Approximation Theory – From Euler to Bernstein*, Birkhäuser Verlag, Basel-Boston-Berlin, 2006, XIX + 219 pp., ISBN 0-8176-4532-2.

The purpose of this book is to describe the early development of Approximation Theory, from Leonhard Euler's cartographic investigations at the end of the 18th century, to the early 20th century contributions of Sergei Bernstein in defining a new branch of function theory: The Constructive Function Theory.

One sets as endpoint the year 1919, when de la Vallée-Poussin published his lectures *Leçons sur l'approximation des fonctions d'une variable réelle*, Gauthier-Villars, Paris, 1919.

The priority of the present investigations are the contributions of P. L. Chebyshev and of the St. Petersburg Mathematical School founded by him. Although some historical contributions on this subject have been published by V. L. Goncharov, N. I. Akhiezer, A. D. Gusak and O. N. Buts, the author considers that it will be useful to go into this topic for several reasons. He did not only want to describe the results of the St. Petersburg School but also to discuss its historico-philosophical background and how it interacted with other European schools. He also presents some interesting facts about the role played by the mathematicians from Götingen in spreading Russian contributions in approximation theory. He did not analyze the work of S. N. Bernstein because the German translation of Akhiezer's scientific biography of Bernstein was published only in 2000 by R. Kovacheva and H. Gonska.

The book consists of a Preface, an Introduction, and five chapters. The first chapter begins by an exposition of the history of the forerunners of modern approximation theory, meaning Euler, Laplace, and Fourier. The second chapter shifts to P. L. Chebyshev, his overall philosophy of mathematics and the St. Petersburg mathematical school, stressing in the third chapter on the role played by Zolotarev and the Markov brothers.

Chapter 4 contains a philosophical discussion on the contrasts between the development of approximation theory in Eastern (mainly Russia) and Western Europe, detailing the work of Weierstrass, as well as that of Göttingen School led by Hilbert and Klein.

Chapter 5 emphasizes the important work of the great Jewish mathematician Sergei Bernstein, whose constructive proof of the Weierstrass theorem and the extension of Chebyshev's work serve to unify East and West in their approaches to approximation theory.

The book contains also two appendices with biographical data of some eminent mathematicians from the St. Petersburg School, explanation of Russian nomenclature and academic degrees. The book ends by a Bibliography and an excellent Index.

It is recommended to mathematicians, historians interested in the history of science, and to a general public.

D. D. Stancu

Antonio Romano, Renato Lancellotta, Addolorata Marasco, *Continuum Mechanics using Mathematica®, Fundamentals, Applications and Scientific Computing*, Birkhäuser, Boston-Basel-Berlin, 2006; 388 pp. ISBN-10 0-8176-3240-9; ISBN-13 978-0-8176-3240-3; eISBN 0-8176-4458-X.

This book is devoted to the readers interested in understanding the basis of continuum mechanics and its fundamental applications from elasticity, fluid mechanics, plasticity, materials with memory, magneto-fluid mechanics, and state changes.

The book is divided into eleven chapters as follows. In the first chapter there are presented elements of linear algebra such as vectors, tensors, eigenvalues, eigenvectors, etc., that are further used in the book. The second chapter deals with the foundations of vector analysis related to curvilinear coordinates, differentiation of vector fields (covariant derivative, directional derivative), the Stokes and Gauss theorems, generalized polar coordinates. Chapter three is devoted to the finite deformation theory, while the next chapter is concerned with the kinetic principles, singular surfaces and the general differential formulas for surfaces and volumes. Chapter five deals with the general formulation of balance equations, mass conservation, momentum balance equation, as well as the Lagrangian formulation of balance equations. Balance equations are general relations whose validity does not depend on body properties. But, it is known from experience that two bodies with the same dimensions and shape may react differently when they are subjected to the same load and thermal conditions. Chapter six presents the constitutive axioms as well as the constitutive equations of continuous mechanics. Chapter eight provides the classification of a quasi-linear partial differential system in order to show under which circumstances there exist characteristic surfaces for which the Cauchy problem is ill posed. It is given a classification of the equations and systems, taking into account the existence and the number of these surfaces. This chapter also deals with shock waves and highlights the role of the second law of thermodynamics in the description of the phenomenon. The following two chapters cover the applications of the general principles presented in the previous chapters to ideal or viscous fluids, as well as to linearly elastic systems. Chapter ten also deals with the Fourier method as a tool for analyzing problems characterized by simple geometry. The last chapter offers a comparison of some proposed thermodynamic theories.

This book is accompanied by a disc that contains several programs written with Mathematica®, which cover the topics in the book.

The book is very clearly written, in a pleasant and accessible style. It is warmly recommended to advanced undergraduate and graduate students, researchers in applied mathematics, mathematical physics, and engineering, that are interested in modern topics in this field.

Mirela Kohr

Farid M.L. Amirouche, *Fundamentals of Multibody Dynamics: Theory and Applications*, Birkhäuser, Boston-Basel-Berlin, 2006, XVIII+684 pp., ISBN 0-8176-4236-6.

Multibody dynamics has grown in the past two decades to be an important tool for designing, prototyping, and simulating complex articulated mechanical systems. This is mainly due to its versatility in analyzing a broad range of applications. This textbook – a result of the author's many years of research and teaching – brings together diverse concepts of dynamics, combining the efforts of many researchers in the field of mechanics. Bridging the gap between dynamics and engineering applications such as microrobotics, virtual reality simulation of interactive mechanical systems, nanomechanics, flexible biosystems, crash simulation, and biomechanics, the book puts into perspective the importance of modelling in the dynamic simulation and solution of problems in these fields.

To help engineering students and practicing engineers understand the rigid-body dynamics concepts needed for the book, the author presents a compiled overview of particle dynamics and Newton's second law of motion in the first chapter. A particular strength of the work is its use of matrices to generate kinematic coefficients associated with the formulation of the governing equations of motion, facilitating the computational investigation of the presented problems.

Junior/senior undergraduates and first-year graduate engineering students taking a course in dynamics, physics, control, robotics, or biomechanics will find this a useful book with a strong computer orientation towards the subject. The work may also be used as a self-study resource or research reference for practitioners in the above-mentioned fields.

Ferenc Szenkovits

Piermarco Cannarsa and Carlo Sinestrari, *Semiconcave Functions, Hamilton-Jacobi Equations, and Optimal Control,* Progress in Nonlinear Differential Equations and Their Applications, Birkhäuser, Boston-Basel-Berlin, 2004, XII+304 pp., ISBN 0-8176-4336-2.

A semiconcave function is a continuous function $u : A \to \mathbb{R}^n$, where $A \subset \mathbb{R}^n$ is open, such that (SC): $u(x+h)+u(x-h)-2u(x) \leq C|h|^2$, for $x, h \in \mathbb{R}^n$ with $[x-h, x+h] \subset A$. Semiconcave functions admit several characterizations: (1) the function $u(x) - 2^{-1}C|x|^2$ is concave; (2) u can be written in the form $u = u_1 + u_2$ with $u_1 : A \to \mathbb{R}$ concave and $u_2 \in C^2(A)$ satisfying $\|D^2u\|_\infty \leq C$; (3) u can be represented in the form $u(x) = \inf\{u_i(x) : i \in I\}$, for some set $\{u_i : i \in I\}$ of functions in $C^2(A)$. In the second chapter, 2. *Semiconcave functions,* one considers a more general class of semiconcave functions satisfying the inequality (SC) with $|h|^2$ replaced by $|h|\omega(|h|)$, where $\omega : \mathbb{R}_+ \to \mathbb{R}_+$ is a modulus, i.e. a nondecreasing upper semicontinuous function such that $\lim_{\rho\to 0+} \omega(\rho) = 0$. Chapter 3. *Generalized gradients and semiconcavity,* is devoted to the superdifferentiability properties of semiconcave functions with applications to marginal functions and to distance function. Singularities, meaning points where a semiconcave function is not differentiable, are studied in the fourth chapter. By Rademacher's theorem the singularity set of a semiconcave function is of Lebesgue measure zero, but the author study finer properties of singularities sets expressed in terms of Hausdorff measure and dimension, as well as the propagation of singularities.

Thee rest of the book, Chapters 5. *Hamilton-Jacobi equations,* 6. *Calculus of variations,* 7. *Optimal control problems,* and 8. *Control problems with exit time,* are devoted to applications, emphasizing the key role the semiconcave functions play in the study of the corresponding topics.

The prerequisites are minimal: a standard background in real analysis and PDEs. For the convenience of the reader, some results from convex analysis, Hausdorff measure and dimension, set-valued analysis, and BV functions, are collected in an appendix, with proofs for some of them, or with exact references for more advanced ones.

The book is clearly written and fairly self-contained. It can be used for graduate or post-graduate courses on the calculus of variations, dynamic optimization, optimal control or Hamilton-Jacobi equations.

S. Cobzaş