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THERMAL STABILITY PROBLEMS IN A THIN POROUS PLATE

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Abstract. Some numerical and analytical aspects of the stability of the formal solution for the dynamical problem associated with the governing equations in a thin porous plate under a constant thermal source are discussed.

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1. THE PHYSICAL PROBLEM

Porous materials have become more and more important in the latest engineering technologies. Thus, silicon porous plates are used in nanotechnology researches [4], porous aluminia and zirconia plates [5] find use in the manufacture of fuel cell electrolyte substrates. Most of the times, the governing field equations and the constitutive relations are represented by nonlinear partial differential equations which are not amenable for a rigorous mathematical analysis. That is why, numerical methods that can lead us to an approximative solution or to some evaluations or some results on the stability of the solution of the associated dynamical problem play an important role.

Let us consider a porous medium, a rectangular plate that fulfills the domain $\Omega \subset \mathbb{R}^3$. Following Lord and Shulman [8], Green and Lindsay [7] and Iesan [9], the governing equations in a generalized thermoelastic solid with voids, without body forces, heat sources and extrinsic equilibrated body forces are [2]

$$\begin{cases} (\lambda+\mu)\dfrac{\partial}{\partial x_i}(\mathrm{div}\mathbf{u}) + \mu\Delta u_i + b\dfrac{\partial\varphi}{\partial x_i} - \beta\dfrac{\partial\theta}{\partial x_i} = \rho_0\ddot{u}_i, \quad i=\overline{1,3} \\ \alpha\Delta\phi - b(\mathrm{div}\mathbf{u}) - \xi\varphi + m\theta + \rho_0 l^* = \rho_0\chi\ddot{\phi} \\ T_0[\beta(\mathrm{div}\dot{\mathbf{u}}) + m\dot{\varphi} + a\dot{\theta}] = k\Delta\theta, \end{cases} \tag{1}$$

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on $\Omega \times (0, t_0)$, where $\mathbf{u} = (u_1, u_2, u_3)$ is the displacement field, θ stands for the variation of the absolute temperature, φ is the change in volume fraction field, ρ_0 is the density of the medium, λ, μ are the Lame's constants, k is the thermal conduction coefficient and a, b, m, α, β, ξ are the constitutive coefficients. The equations of the system (1) are the movement, the equilibrated forces equation and the energy equation, respectively.

According to the micropolar theory of thermoelasticity for elastic media with voids introduced by Eringen [6], we assume that

$$\begin{aligned} &\mathbf{u}^{(1)} = (x_3 v_1, x_3 v_2, w),\\ &\mathbf{u}^{(2)} = \text{ grad } (x_3 \Phi),\\ &\varphi = x_3 \psi,\ \theta = x_3 T \end{aligned} \tag{2}$$

where the functions v_1, v_2, w, Φ, ψ, T depends on x_1, x_2, t. In this manner, the system is reduced to the 2-dimensional case (in the median plane Σ $(x_1, x_2) \in \Sigma, t \in (0, t_0))$.

As a consequence of (2), the equilibrium equations can be reduced to [2]

$$\begin{cases} \dfrac{\partial^2 v_1}{\partial t^2} - \dfrac{\mu}{\rho_0} \Delta v_1 = 0 \\ \dfrac{\partial^2 v_2}{\partial t^2} - \dfrac{\mu}{\rho_0} \Delta v_2 = 0 \\ \dfrac{\partial^2 w}{\partial t^2} - \dfrac{\mu}{\rho_0} \Delta w = 0 \end{cases} \tag{3}$$

and

$$\begin{cases} \ddot{\Phi} = \dfrac{\lambda + 2\mu}{\rho_0} \Delta \Phi + \dfrac{b}{\rho_0} \psi - \dfrac{\beta}{\rho_0} T \\ \ddot{\psi} = \dfrac{\alpha}{\rho_0 \chi} \Delta \psi - \dfrac{b}{\rho_0} \Delta \Phi - \dfrac{\xi}{\rho_0} \psi + \dfrac{m}{\rho_0} T \\ \dot{T} = \dfrac{k}{\rho_0 c_l} \Delta T - \dfrac{\beta T_0}{\rho_0 c_l} \Delta \dot{\Phi} - \dfrac{m T_0}{\rho_0 c_l} \dot{\psi} \end{cases} \tag{4}$$

Let us consider the boundary of Ω, $\partial\Omega = \Sigma^+ \bigcup \Sigma^- \bigcup\limits_{i=1}^{4} \Sigma_i$. Then the boundary conditions have the form:

$$\begin{cases} \sum\limits_{j=1}^{3} n_j \cdot t_{ij} = 0, \quad \text{on} \quad \partial\Omega \\ \overline{u} = 0, \quad \text{on} \quad \partial\Omega \\ \varphi = 0, \quad \text{on} \quad \partial\Omega \\ k \cdot \dfrac{\partial \theta}{\partial \overline{n}} + \varepsilon\sigma(\theta^4 - S^{*4}) = 0, \quad \text{on} \quad \Sigma^+ \\ k \cdot \dfrac{\partial \theta}{\partial \overline{n}} + c_t(\theta - S^*) = 0, \quad \text{on} \quad \bigcup\limits_{i=1}^{4} \Sigma_i \\ \theta = S^*, \quad \text{on} \quad \Sigma^- \end{cases} \tag{5}$$

with c_t the transfer coefficient, $\varepsilon \in (0,1]$ material constant ($\varepsilon = 1$ for a black body), S^* the source temperature (assumed constant), $\sigma = 5,6704 \cdot 10^{-8}$ $kg \cdot s^{-3} \cdot K^{-4}$ Stefan-Boltzmann constant and t_{ij} given by

$$t_{ij} = \lambda \cdot (\nabla \cdot \overline{u}) \cdot \delta_{ij} + 2\mu \cdot e_{ij} + b\varphi \cdot \delta_{ij} - \beta T \cdot \delta_{ij}, \quad i,j \in \{1,2,3\}.$$

The initial conditions have the form [2]:

$$\mathbf{u}(\mathbf{x},0) = 0,\ \varphi(\mathbf{x},0) = 0,\ \theta(\mathbf{x},0) = 0. \tag{6}$$

In [1] numerical evaluations on the thermal stresses in a thin porous plate due to the radiations of a thermal source are given. They are concerned with the evaluation of the absolute temperature, porosity and the vertical displacement [1]. All the deformations, the stresses and the change in volume fraction field depend on the temperature. It is showed that as the absolute temperature inside the plate is growing, the deformation of the plate take place until the reach of the equilibrium state [1]. The solution of the dynamical problem proved to be stationary.

In [2], using the theory of semigroups from Pazy [11], an existence and uniqueness theorem based on the representation theory is given. The idea of the theorem was to write the system (4) as an evolution system of order 1 given by a strongly elliptic differential operator on a Hilbert space. Bârsan [10] also gave an existence and uniqueness result for the problem with initial data and boundary conditions modelling the mechanical behavior of a thin porous plate, using the logarithmic convexity method.

A numerical modelling concerning the convexity of the median surface of the elastic thin porous plate is obtained in [3] using FreeFem++, i.e. the slopes and the curvatures in the directions of the coordinate axes are graphically represented. It is numerically showed that when the plate reaches its thermal equilibrium, the curvature touches its maximum value.

2. THE STABILITY ANALYSIS

Our main interest in this paper is in an analytical stability study of the dynamical problem. For this purpose the same method as in [11] was used. The following dimensionless variables, functions

$$\begin{cases} x' = \frac{\omega_1^*}{c_2} \cdot x_1, \quad y' = \frac{\omega_1^*}{c_2} \cdot x_2, \quad z' = \frac{\omega_1^*}{c_2} \cdot x_3, \quad \tau = \omega_1^* \cdot t \\ v_1 = \frac{\omega_1^*}{c_2} \cdot u_1, \quad v_2 = \frac{\omega_1^*}{c_2} \cdot u_2, \quad v_3 = \frac{\omega_1^*}{c_2} \cdot u_3, \quad \theta' = \frac{T}{T_0} \\ \psi = \frac{\omega_1^{*2} \cdot \chi}{c_2^2} \cdot \varphi, \quad c_2 = \sqrt{\frac{\mu}{\rho_0}}, \quad \omega_1^* = \frac{\rho_0 \cdot c_e \cdot c_2^2}{k}, \end{cases} \tag{7}$$

and coefficients

$$\begin{cases} a_1 = \frac{\lambda+\mu}{\mu}, \quad a_2 = \frac{b \cdot c_2^2}{\chi \cdot \mu \cdot \omega_1^{*2}}, \quad a_3 = \frac{\beta \cdot T_0}{\mu}, \quad a_4 = \frac{b \cdot \chi}{\alpha} \\ a_5 = \frac{\xi \cdot c_2^2}{\alpha \cdot \omega_1^{*2}}, \quad a_6 = \frac{m \cdot \chi \cdot T_0}{\alpha}, \quad a_7 = \frac{\rho_0 \cdot \chi \cdot c_2^2}{\alpha} \\ a_8 = \frac{m \cdot c_2^4}{k \cdot \chi \cdot \omega_1^{*3}}, \quad \varepsilon_1 = \frac{\beta \cdot c_2^2}{k \cdot \omega_1^*}, \end{cases} \tag{8}$$

are introduced. Then the dimensionless form of the dynamical problem (1) is

$$(9)\qquad \begin{cases} a_1 \cdot \frac{\partial}{\partial x}(\nabla \cdot \mathbf{v}) + \triangle v_1 + a_2 \cdot \frac{\partial \psi}{\partial x} - a_3 \cdot \frac{\partial \theta}{\partial x} = \frac{\partial^2 v_1}{\partial \tau^2} \\ a_1 \cdot \frac{\partial}{\partial y}(\nabla \cdot \mathbf{v}) + \triangle v_2 + a_2 \cdot \frac{\partial \psi}{\partial y} - a_3 \cdot \frac{\partial \theta}{\partial y} = \frac{\partial^2 v_2}{\partial \tau^2} \\ a_1 \cdot \frac{\partial}{\partial z}(\nabla \cdot \mathbf{v}) + \triangle v_3 + a_2 \cdot \frac{\partial \psi}{\partial z} - a_3 \cdot \frac{\partial \theta}{\partial z} = \frac{\partial^2 v_3}{\partial \tau^2} \\ \triangle \psi - a_4 \cdot (\nabla \cdot \mathbf{v}) - a_5 \cdot \psi + a_6 \cdot \theta = a_7 \cdot \frac{\partial^2 \psi}{\partial \tau^2} \\ \varepsilon_1 \cdot \frac{\partial}{\partial \tau}(\nabla \cdot \mathbf{v}) + a_8 \cdot \frac{\partial \psi}{\partial \tau} + \frac{\partial \theta}{\partial \tau} = \triangle \theta \end{cases}$$

for $(x, y, z) \in \Omega$. Here $\mathbf{v} = (v_1, v_2, v_3)$, $v_i, \psi \in C^2(\Omega \times \mathcal{T}) \bigcap C^1(\overline{\Omega} \times \mathcal{T})$ and $\theta \in C^{2,1}(\Omega \times \mathcal{T}) \bigcap C^1(\overline{\Omega} \times \mathcal{T})$. In (9) in order not to complicate the symbolic writing, we kept the same notations for the cartesian coordinates (x, y, z) (instead of (x', y', z')), for θ and for the entire Ω domain.

In the boundary conditions $\sum\limits_{j=1}^{3} n_j \cdot t_{ij} = 0$, $i \in \{1, 2, 3\}$ from (5), t_{ij} is replaced by its nondimensional expression t'_{ij}, where

$$(10)\qquad t'_{ij} = \tfrac{a_1 - 1}{a_3} \cdot (\nabla \cdot \mathbf{v}) \cdot \delta_{ij} + \tfrac{2}{a_3} \cdot e_{ij} + \tfrac{a_2}{a_3} \psi \cdot \delta_{ij} - \theta \cdot \delta_{ij}. \quad i, j \in \{1, 2, 3\}.$$

It is convenient to transform the boundary conditions (5), written in the nondimensional form, in periodically conditions with respect to $\bigcup\limits_{i=1}^{4} \Sigma_i$, by imposing that $\Sigma_1 : y = 0$, $\Sigma_2 : x = L$, $\Sigma_3 : y = l$, $\Sigma_4 : x = 0$. This allows us to split the problem (9) in two problems: one is the stationary problem associated to the dynamical one with the boundary conditions

$$(11)\qquad \begin{cases} \sum_{j=1}^{3} n_j \cdot t'_{ij} = 0, \quad (i \in \overline{1,3}) \quad \text{on} \quad \partial \Omega' \\ \psi = 0, \quad \text{on} \quad \partial \Omega' \\ k \cdot \frac{\partial \theta}{\partial n} + \epsilon \cdot \sigma \cdot (\theta^4 - S^{*4}) = 0, \quad \text{on} \quad \Sigma^+ : z = \frac{h_0}{2} \\ k \cdot \frac{\partial \theta}{\partial n} + c_t \cdot (\theta - S^*) = 0, \quad \text{on} \quad \bigcup\limits_{i=1}^{4} \Sigma_i \\ \theta = S^*, \quad \text{on} \quad \Sigma^- : z = -\frac{h_0}{2} \end{cases}$$

and the other problem is the initial one but with homogeneous initial and boundary conditions. When applied to Σ^+ and Σ^-, the micropolar theory, led us the decomposition of the stationary problem in two problems, for each of them the solution is easy to obtain. On the median plane Σ the equations have the same form on $\Sigma^{\pm}$, i.e. relatively to the Σ^+ and Σ^-, we decompose the

$$\left\{\begin{array}{l} \triangle V_1 = 0, \\ \triangle V_2 = 0, \\ \triangle W = 0, \\ \triangle \Phi = 0, \end{array}\right. \tag{12}$$

with the boundary conditions

$$\left\{\begin{array}{l} \frac{\partial V_1}{\partial x}(L,y) = \frac{\partial V_1}{\partial x}(0,y), \quad V_1(L,y) = V_1(0,y), \quad y \in [0,l], \\ \\ \frac{\partial V_2}{\partial y}(x,l) = \frac{\partial V_2}{\partial y}(x,0), \quad V_2(x,l) = V_2(x,0), \quad x \in [0,L], \\ \\ \frac{\partial W}{\partial x}(L,y) = \frac{\partial W}{\partial x}(0,y), \quad \frac{\partial \Phi}{\partial x}(L,y) = \frac{\partial \Phi}{\partial x}(0,y), \quad y \in [0,l], \\ \\ \frac{\partial W}{\partial y}(x,l) = \frac{\partial W}{\partial y}(x,0), \quad \frac{\partial \Phi}{\partial x}(x,0) = \frac{\partial \Phi}{\partial y}(x,l), \quad x \in [0,L]. \end{array}\right. \tag{13}$$

On Ω the decomposition leads to

$$\left\{\begin{array}{l} \triangle \theta = 0, \\ \left(\frac{h_0}{2} \pm z\right) \cdot \triangle \psi_1 - a_5 \cdot \left(\frac{h_0}{2} \pm z\right) \cdot \psi_1 + a_6 \cdot \theta = 0, \end{array}\right. \tag{14}$$

for $\Sigma^{\pm}$ with boundary conditions

$$\left\{\begin{array}{l} \psi_1 = 0, \quad \text{on} \quad \partial\Sigma \\ k \cdot \frac{\partial \theta}{\partial \overline{n}} + \epsilon \cdot \sigma \cdot (\theta^4 - S^{*4}) = 0, \quad \text{on} \quad \Sigma^+ : z = \frac{h_0}{2} \\ k \cdot \frac{\partial \theta}{\partial \overline{n}} + c_t \cdot (\theta - S^*) = 0, \quad \text{on} \quad \bigcup\limits_{i=1}^{4} \Sigma_i \\ \theta = 0, \quad \text{on} \quad \Sigma^- : z = -\frac{h_0}{2} \end{array}\right. \tag{15}$$

for Σ^+ and

$$\left\{\begin{array}{l} \psi_1 = 0, \quad \text{on} \quad \partial\Sigma \\ \theta = 0, \quad \text{on} \quad \Sigma^+ : z = \frac{h_0}{2} \\ k \cdot \frac{\partial \theta}{\partial \overline{n}} + c_t \cdot (\theta - S^*) = 0, \quad \text{on} \quad \bigcup\limits_{i=1}^{4} \Sigma_i \\ \theta = S^*, \quad \text{on} \quad \Sigma^- : z = -\frac{h_0}{2} \end{array}\right. \tag{16}$$

for Σ^-. Here

$$\left\{\begin{array}{l} \mathbf{v} = \mathbf{v}^{(1)} + \mathbf{v}^{(2)}, \quad \nabla' \cdot \mathbf{v}^{(1)} = 0, \quad \nabla' \times \mathbf{v}^{(2)} = 0, \\ \\ \mathbf{v}^{(1)} = ((\frac{h_0}{2} + z) \cdot V_1, (\frac{h_0}{2} + z) \cdot V_2, W), \quad \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} = 0 \\ \\ \mathbf{v}^{(2)} = ((\frac{h_0}{2} + z) \cdot \frac{\partial \Phi}{\partial x}, (\frac{h_0}{2} + z) \cdot \frac{\partial \Phi}{\partial y}, \Phi), \\ \\ \psi = (\frac{h_0}{2} + z) \cdot \psi_1, \\ \\ \theta = A \cdot (\frac{h_0}{2} + z) \cdot \theta_1 \end{array}\right. \tag{17}$$

with A, B two real constants and the functions V_1, V_2, W, Φ, ψ_1, θ_1 depending on (x, y, τ).

For each case, i.e. (12-13-14-15), (12-13-14-16), the solution is easy to obtain and the general form of the solution of the stationary problem is the sum of the two solution groups (relatively to Σ^+ and Σ^-). In our case, this solution has the form

$$(18)\quad \begin{cases} v_1(x,y,z) = a_3 f(L,S^*,h_0,c_t)\cdot\frac{\cosh\frac{2\pi y}{L}-\cosh\frac{2\pi(l-y)}{L}}{\sinh\frac{2\pi l}{L}}\sin\frac{2\pi x}{L} \\ v_2(x,y,z) = -a_3 f(L,S^*,h_0,c_t)\cdot\frac{\sinh\frac{2\pi y}{L}+\sinh\frac{2\pi(l-y)}{L}}{\sinh\frac{2\pi l}{L}}\cos\frac{2\pi x}{L} \\ v_3(x,y,z) = \frac{a_3L^2S^*}{4\pi^2h_0}\cdot(\frac{A}{c_t}+\frac{3}{2})\frac{\cosh\frac{2\pi\cdot y}{L}-\cosh\frac{2\pi(l-y)}{L}}{\sinh\frac{2\pi l}{L}}\cos\frac{2\pi x}{L} \\ \psi(x,y,z) = \frac{8a_6\pi f(L,S^*,h_0,c_t)}{\sqrt{L^3\cdot l}}\cdot\sum\limits_{k,j=1}^{\infty}\frac{\sin\frac{k\pi x}{L}\sin\frac{j\pi y}{l}}{a_5+(\frac{k\pi}{L})^2+(\frac{j\pi}{l})^2} \\ \theta(x,y,z) = \frac{4\pi f(L,S^*,h_0,c_t)}{L}, \end{cases}$$

with $f(L,S^*,h_0,c_t) = \frac{L\cdot S^*}{2\pi\cdot h_0}\cdot[\frac{1}{2}\cdot(\frac{h_0}{2}-z)+\frac{A}{c_t}\cdot(\frac{h_0}{2}+z)]$ for all $x\in[0,L]$, $y\in[0,l]$, $z\in[-\frac{h_0}{2},\frac{h_0}{2}]$. Then we impose

$$v_1(x,y,z,\tau)=0, v_2(x,y,z,\tau)=0,\ v_3(x,y,z,\tau)=0,$$
$$\psi(x,y,z,\tau)=0,\ \theta(x,y,z,\tau)=0,\quad \text{on}\quad \Omega\times(0,t_0).$$

The analytical study led us to the following general stability result.

PROPOSITION 1. *The solution of the dynamical problem* (9) *with the boundary conditions* (5) *and the initial conditions* (6) *is stable in time.*

Proof. Using the micropolar theory [6], we obtain the expressions of the functions v_1, v_2, v_3, ψ, θ. This allows us to perform an evaluation of the solution (18) of the dynamical problem with respect to the $L^2(\Omega)$ norm, i.e. $||u||_{L^2(\Omega)} = \Big(\int_\Omega u^2 \mathrm{d}\omega\Big)^{\frac{1}{2}}$. We get

$$||v_1||^2_{L^2(\Omega)} = \frac{a_3^2\cdot S^{*2}\cdot L^3\cdot h_0}{96\pi^2}\cdot(1+\frac{2A}{c_t}+\frac{4A^2}{c_t^2})\cdot\frac{\sinh^2\frac{\pi l}{L}}{\sinh^2\frac{2\pi l}{L}}\cdot(\frac{L}{\pi}\cdot\sinh\frac{2\pi l}{L}-2l);$$

$$||v_2||^2_{L^2(\Omega)} = \frac{a_3^2\cdot S^{*2}\cdot L^3\cdot h_0}{96\pi^2}\cdot(1+\frac{2A}{c_t}+\frac{4A^2}{c_t^2})\cdot\frac{\sinh^2\frac{\pi l}{L}}{\sinh^2\frac{2\pi l}{L}}\cdot(\frac{L}{\pi}\cdot\sinh\frac{2\pi l}{L}+2l);$$

$$||v_3||^2_{L^2(\Omega)} = \frac{a_3^2\cdot S^{*2}\cdot L^5}{32\pi^4\cdot h_0^2}\cdot(\frac{A}{c_t}+\frac{3}{2})^2\cdot\frac{\sinh^2\frac{\pi l}{L}}{\sinh^2\frac{2\pi l}{L}}\cdot(\frac{L}{\pi}\cdot\sinh\frac{2\pi l}{L}-2l);$$

$$||\psi||^2_{L^2(\Omega)} = \frac{a_6^2\cdot S^{*2}\cdot h_0}{3}\cdot(1+\frac{2A}{c_t}+\frac{4A^2}{c_t^2})\cdot\sum_{k,j=1}^{\infty}\frac{1}{[a_5+(\frac{k\pi}{L})^2+(\frac{j\pi}{l})^2]^2};$$

$$||\theta||^2_{L^2(\Omega)} = \frac{h_0^3}{3}\cdot(1+\frac{2A}{c_t}+\frac{4A^2}{c_t^2}).$$

All the expressions of the $L^2(\Omega)$ norms of v_1, v_2, v_3, ψ, θ, depend on the thermal source S^* by a constant A which is a solution of the algebraic equation

$$k \cdot \frac{2S^*}{h_0 \cdot c_t} \cdot A + \epsilon \cdot \sigma \cdot (\frac{2S^*}{h_0 \cdot c_t})^4 \cdot A^4 - \epsilon \cdot \sigma \cdot S^{*4} = 0.$$

It is easy to see that all the above norms are bounded, which imply that the solution is stable in time. □

3. CONCLUSIONS

In this paper the stability of the solution of the associated dynamical problem for a thin porous plate under a constant thermal source is investigated using methods from micropolar plates theory. The main idea of the method is to split the dynamical problem in two problems obtained by taking into account the decomposition of $\partial\Omega$, the boundary of the domain Ω. Then, for each of these problems, the solution is easy to reach.

This analytical stability result sustain some previous works of the first author, mostly numerical studies.

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