# INTEGRO-DIFFERENTIAL EQUATION WITH TWO TIME MODIFICATIONS

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Abstract. We consider an integro-differential equation with two time modifications. Existence, uniqueness and monotony results of solution for the Cauchy problem are obtained using weakly Picard operator theory. In the last section we present a step method for this type of equation.

## 1. Introduction

This paper is concerned with the following integro-differential equation

$$x'(t) = g(t, x(t), x(t-\tau)) + \int_{t-h}^{t} K(s, x(s))ds,\ t \in I. \tag{1.1}$$

The aim of this paper is to obtain existence and uniqueness theorems using contraction principle, step method and monotony results for the Cauchy problem, see [7] and [11]. Such kind of results have been proved for an integro delay equation in [17]. The approach proposed in the present paper is different to the ones in [4], [17] and [18] and it is based on the different time modifications.

In our paper we consider $I = [0, \infty)$.

Regarding the two delays we have the following cases: $h > 0,\ \tau > 0,\ \tau > h$, discussed in [8], and here we take the case: $\tau < 0,\ h > 0,\ h = |\tau|$.

The equation becomes

$$x'(t) = g(t, x(t), x(t+h)) + \int_{t-h}^{t} K(s, x(s))ds,\ t \in [0, \infty[, \tag{1.2}$$

with the condition

$$x(t) = \varphi(t),\ t \in [-h, h]. \tag{1.3}$$

Relative to (1.2)–(1.3) we consider the following conditions:

- $(C_1)$ $(\mathbb{B}, |\cdot|)$ is a Banach space, $g \in C([0, \infty[\times \mathbb{B}^2, \mathbb{B}), K \in C([0, \infty[\times \mathbb{B}, \mathbb{B})$, $\varphi \in C([-h, h], \mathbb{B})$;
- $(C_1^{'})$ $(\mathbb{B}, |\cdot|)$ is a Banach space, $g \in C^{\infty}([0, \infty[\times \mathbb{B}^2, \mathbb{B}),\ K \in C^{\infty}([0, \infty[\times \mathbb{B}, \mathbb{B})$, $\varphi \in C^{\infty}([-h, h], \mathbb{B})$;
- $(C_2)$ there exists $L_1, L_2 > 0$ such that
$$|g(t, u_1, v_1) - g(t, u_2, v_2)| \leq L_1 |u_1 - u_2| + L_2 |v_1 - v_2|,\ u_i, v_i \in \mathbb{B}, t \in [0, \infty[;$$
- $(C_3)$ there exists $L_3 > 0$ such that
$$|K(s, u) - K(s, v)| \leq L_3 |u - v|,\ u, v \in \mathbb{B}, t \in [0, \infty[;$$
- $(C_4)$ $(L_1 + L_2 + 2L_3 h)h < 1$;
- $(C_5)$ $\varphi'(0) = g(h, \varphi(0), \varphi(h)) + \int_{-h}^{0} K(s, \varphi(s))ds$.

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In what follow we shall present some notions that will help us obtaining the results bellow.

Let $X$ be a nonempty set,

$$s(X) := \{(x_n)_{n\in\mathbb{N}^*} | x_n \in X, n \in \mathbb{N}^*\}$$

and

$$M(X) := \{(x_{ij})_1^\infty | x_{ij} \in X, i, j \in \mathbb{N}^*\}$$

where

$$(x_{ij})_1^\infty := \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots \\ x_{21} & x_{22} & x_{23} & \cdots \\ x_{31} & x_{32} & x_{33} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

is a infinite matrix.

For $A \in M(\mathbb{B})$ we denote

$$|A| := \sup_{1\le i\le\infty} \sum_{j\in\mathbb{N}^*} |a_{ij}|.$$

Let $d : X \times X \to s(\mathbb{B})$ be the generalized metric.

**Remark 1.1.** [13] *A functional $d : X \times X \to s(\mathbb{B})$, $(x, y) \mapsto (d_k(x, y))_{k\in\mathbb{N}^*}$ is a generalized metric of $X$ iff*

(a) *$d_k$ is a pseudometric, $\forall k \in \mathbb{N}^*$;*
(b) *$\forall x, y \in X$, $x \neq y$, there exist $k \in \mathbb{N}^*$ such as $d_k(x, y) \neq 0$.*

**Definition 1.2.** [13] *Let $(X, d)$ be a complete generalized metric space, $A : X \to X$ and $S \in M(\mathbb{B})$. The operator $A$ is a $S$-contraction iff:*

(i) *$S$ is row and column finite (meaning that there are only a finite number of nonzero elements in each row and each column);*
(ii) *$S$ is a Neumann matrix (meaning that if $S^n$ is definite for all $n \in \mathbb{N}$ and $\sum_{n\in\mathbb{N}} S^n$ converges for all $x, y \in X$);*
(iii) *$\sum_{n\in\mathbb{N}} S^n d(x, y)$ converges $\forall x, y \in X$;*
(iv) *$d(A(x), A(y)) \le Sd(x, y)$ $\forall x, y \in X$.*

We consider the space $X = C([-h, \infty[, \mathbb{B})$ endowed with the norm

$$\|\cdot\| : X \to s(\mathbb{R}_+) \ , \ \|x\| := \begin{pmatrix} \|x\|_0 \\ \vdots \\ \|x\|_m \\ \vdots \end{pmatrix},$$

where $\|x\|_0 = \max\limits_{-h\le t\le h} |x(t)|$ and $\|x\|_m = \max\limits_{mh\le t\le(m+1)h} |x(t)|, m \ge 1$.

This generalized norm induces a generalized metric, $d(x, y) := \|x - y\|$.

## 2. Preliminaries

Let $(X, d)$ be a generalized metric space and $A : X \to X$ an operator. In this paper we shall use the terminologies and notations from [13]–[15]. For the convenience of the reader we shall recall some of them.

We denote by $A_0 := 1_X$, $A^1 := A$, $A^{n+1} := A \circ A^n$, $n \in \mathbb{N}$, the iterate operators of the operator $A$. Also we shall use the following notations:

$F_A := \{x \in X \mid A(x) = x\}$ - the fixed point set of $A$;

$I(A) := \{Y \subset X \mid A(Y) \subset Y, Y \neq \emptyset\}$ - the family of the nonempty invariant subset of $A$;

**Definition 2.1.** *$A : X \to X$ is called a Picard operator (briefly PO) if:*

(i) $F_A = \{x^*\}$;
(ii) $A^n(x) \to x^*$ *as* $n \to \infty$, $\forall x \in X$.

**Definition 2.2.** *$A : X \to X$ is said to be a weakly Picard operator (briefly WPO) if the sequence $(A^n(x))_{n\in\mathbb{N}}$ converges for all $x \in X$ and the limit (which may depend on $x$) is a fixed point of $A$.*

If $A : X \to X$ is a WPO, then we may define the operator
$A^\infty : X \to X$ by

$$A^\infty(x) := \lim_{n\to\infty} A^n(x).$$

Obviously $A^\infty(X) = F_A$. Moreover, if $A$ is a PO and we denote by $x^*$ its unique fixed point, then $A^\infty(x) = x^*$, for each $x \in X$.

**Lemma 2.3.** *Let $(X, d, \le)$ be an ordered metric space and $A : X \to X$ an operator. We suppose that:*
(i) *$A$ is WPO;*
(ii) *$A$ is increasing.*

*Then, the operator $A^\infty$ is increasing.*

**Lemma 2.4.** *Let $(X, d, \le)$ an ordered metric space and $A, B, C : X \to X$ be such that:*
(i) *the operator $A, B, C$ are WPOs;*
(ii) *$A \le B \le C$;*
(iii) *the operator $B$ is increasing.*

*Then $x \le y \le z$ implies that $A^\infty(x) \le B^\infty(y) \le C^\infty(z)$.*

**Theorem 2.5.** [13] *Let $(X, d)$ be a complete metric space and $A : X \to X$ a $S$-contraction. Then we have*
(i) *$F_A = \{x^*\}$;*
(ii) *$A^n(x) \overset{d}{\to} x^*$, as $n \to \infty, \forall x \in X$;*
(iii) *$d(A^n(x), x^*) \le (E - S)^{-1} S^n d(x, A(x))$;*
(iv) *$d(x, x^*) \le (E - S)^{-1} d(x, A(x))$.*

In what follow we shall apply the above results to the problem (1.2)–(1.3). For other applications of these abstract results, see [2], [3], [8], [9], [12], [16].

## 3. Existence and uniqueness

From Theorem 2.5 we have

**Theorem 3.1.** *In the condition $(C_1), (C_2)$ $(C_3)$, and $(C_4)$ the problem (1.2)–(1.3) has in $C([-h, \infty[, \mathbb{B})$ a unique solution $\overset{*}{x}$ which is the limit of the sequence of successive approximation.*

*Proof.* We consider the operator $A : X \to X$ defined by

$$A(x)(t) = \begin{cases} \varphi(t),\ t \in [-h, h] \\ \varphi(h) + \int_h^t g(\xi, x(\xi), x(\xi + h))d\xi + \\ \qquad + \int_h^t \int_{\xi-h}^{\xi} K(s, x(s))dsd\xi,\ t \in [h, \infty[. \end{cases} \tag{3.1}$$

$(X, d)$ is a complete metric space with $d = (\|\cdot\|_m)_{m\in\{-1,0,1,\dots\}}$ where

$$d(x, y) = \begin{pmatrix} d_0(x, y) \\ \vdots \\ d_m(x, y) \\ \vdots \end{pmatrix}.$$

For $t \in [-h, h]$ we have

$$\|A(x)(t) - A(y)(t)\|_0 = 0, \ \forall x, y \in X.$$

For $t \in [h, 2h]$ we have

$$\begin{aligned}
&|A(x)(t) - A(y)(t)|_1 \leq \\
&\leq L_1 \int_h^t |x(\xi) - y(\xi)|\, d\xi + L_2 \int_h^t |x(\xi + h) - y(\xi + h)|\, d\xi + \\
&\quad + L_3 \int_h^t \int_{\xi - h}^{\xi} |x(s) - y(s)|\, ds d\xi \\
&\leq L_1 h \|x - y\|_1 + L_2 h \|x - y\|_2 + L_3 \int_h^t (h \|x - y\|_0 + h \|x - y\|_1)\, d\xi \\
&\leq L_3 h^2 \|x - y\|_0 + (L_1 h + L_3 h^2) \|x - y\|_1 + L_2 h \|x - y\|_2 .
\end{aligned}$$

So, $\|A(x)(t) - A(y)(t)\|_1 \leq L_3 h^2 \|x - y\|_0 + (L_1 h + L_3 h^2) \|x - y\|_1 + L_2 h \|x - y\|_2 .$
For $t \in [2h, 3h]$ we have
$\|A(x)(t) - A(y)(t)\|_2 \leq L_3 h^2 \|x - y\|_1 + (L_1 h + L_3 h^2) \|x - y\|_2 + L_2 h \|x - y\|_3 .$
By induction, for $t \in [mh, (m + 1)h]$ we have that

$$\begin{aligned}
&\|A(x)(t) - A(y)(t)\|_m \leq \\
&\leq L_3 h^2 \|x - y\|_{m-1} + (L_1 h + L_3 h^2) \|x - y\|_m + L_2 h \|x - y\|_{m+1} .
\end{aligned}$$

Then

$$\begin{pmatrix} |A(x)(t) - A(y)(t)|_0 \\ |A(x)(t) - A(y)(t)|_1 \\ |A(x)(t) - A(y)(t)|_2 \\ \vdots \\ |A(x)(t) - A(y)(t)|_m \\ \vdots \end{pmatrix} \leq$$

$$\begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ L_3 h^2 & L_1 h + L_3 h^2 & L_2 h & \cdots & 0 & 0 & \cdots \\ 0 & L_3 h^2 & L_1 h + L_3 h^2 & \cdots & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & L_1 h + L_3 h^2 & L_2 h & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} \|x - y\|_0 \\ \|x - y\|_1 \\ \|x - y\|_2 \\ \vdots \\ \|x - y\|_m \\ \vdots \end{pmatrix}.$$

So $d(A(x), A(y)) \leq S d(x, y)$, where $S : s(\mathbb{R}) \to s(\mathbb{R})$, $\|S\| := \sup\limits_{i \in \mathbb{N}} \sum_{j=0}^{\infty} |L_{ij}| = (L_1 + L_2 + 2L_3 h)h$, which proves that $A$ is Lipschitz with

$$S = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ L_3 h^2 & L_1 h + L_3 h^2 & L_2 h & \cdots & 0 & 0 & \cdots \\ 0 & L_3 h^2 & L_1 h + L_3 h^2 & \cdots & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & L_1 h + L_3 h^2 & L_2 h & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}.$$

From condition $(C_4)$ we have that $A$ is $S$-contraction. Applying Theorem 2.5 we have the conclusion. □

**Remark 3.2.** *From the proof of Theorem 3.1, it follows that the operator $A$ is PO in $((C[-h, \infty], \mathbb{B}), d)$.*

## 4. Inequalities of Čaplygin type

In this section we shall study the relation between the solution of the problem (1.2)–(1.3) and the subsolution of the same problem.

Let $\overset{*}{x}$ be the unique solution of the problem (1.2)–(1.3) and $y$ a subsolution of the same problem, i.e.

$$y'(t) \leq g(t, y(t), y(t+h)) + \int_{t-h}^{t} K(s, y(s))ds,\ t \in [0, \infty[, \tag{4.1}$$

where $g$ and $K$ satisfy the conditions $(C_1)$–$(C_3)$ and

$$y(t) = \varphi(t),\ t \in [-h, h]. \tag{4.2}$$

In this section we consider an ordered Banach space $(\mathbb{B}, |\cdot|, \leq)$ and the operator $A$ defined by (3.1) on the ordered Banach space $X = ((C[a, b], \mathbb{B}), \|\cdot\|, \leq)$. We have the following theorem

**Theorem 4.1.** *We suppose that:*
*(a) the conditions $(C_1)$–$(C_4)$ are satisfied;*
*(b) $g(t, \cdot, \cdot) : \mathbb{B}^2 \to \mathbb{B}$ and $K(t, \cdot) : \mathbb{B} \to \mathbb{B}$ are increasing, $\forall t \in [0, \infty[$.*
*Then $y \leq \overset{*}{x}$ for all $t \in [0, \infty[$.*

*Proof.* In terms of the operator $A$ defined by the relation (3.1), we have $\overset{*}{x} = A(\overset{*}{x})$ and $y \leq A(y)$. On the other hand from condition $(b)$ and Lemma 2.3, we have that the operator $A^\infty$ is increasing. Hence $y \leq A(y) \leq A^2(y) \leq \cdots \leq A^\infty(y) \leq A^\infty(\overset{*}{x}) = \overset{*}{x}$. So, $y \leq \overset{*}{x}$. □

## 5. Data dependence: monotony

In this section we study the monotony of the system (1.2)–(1.3) with respect to $g$ and $K$. For this we use the abstract comparison Lemma from Section 2.

Consider the following equations

$$x_i'(t) = g_i(t, x_i(t), x_i(t+h)) + \int_{t-h}^{t} K_i(s, x_i(s))ds,\ t \in [0, \infty[,\ i = \overline{1,3} \tag{5.1}$$

with the conditions (1.3) for each problem and let $\overset{*}{x}_i,\ i = \overline{1,3}$ the unique solutions of these problems. Then we need the operators $A_i : X \to X$ defined by

$$A_i(x)(t) = \begin{cases} \varphi(t),\ t \in [-h, h] \\ \varphi(h) + \int_h^t g(\xi, x_i(\xi), x_i(\xi+h))d\xi + \\ \quad + \int_h^t \int_{\xi-h}^{\xi} K(s, x_i(s))dsd\xi,\ t \in [h, \infty[. \end{cases}$$

**Theorem 5.1.** *Let $g_i, K_i, i = \overline{1,3}$, that satisfy the conditions $(C_1)$–$(C_4)$.*
*We suppose that we have*
*(i) $g_1 \leq g_2 \leq g_3$;*
*(ii) $g(t, \cdot, \cdot) : \mathbb{B}^2 \to \mathbb{B}$ and $K(t, \cdot) : \mathbb{B} \to \mathbb{B}$ are increasing.*
*Let $\overset{*}{x}_i$ the solutions of the equations (5.1), $i = \overline{1,3}$.*
*Then $\overset{*}{x}_1(t) \leq \overset{*}{x}_2(t) \leq \overset{*}{x}_3(t), \forall t \in [0, \infty[$.*

*Proof.* From Theorem 3.1 the operators $A_i$ are POs. From the condition $(ii)$ it follows that the operator $A_2$ is monotone increasing and from condition $(i)$ we have $A_1 \leq A_2 \leq A_3$. But $\overset{*}{x}_1 = A_1{}^\infty(\overset{*}{x}_1)$, $\overset{*}{x}_2 = A_2{}^\infty(\overset{*}{x}_2)$ and $\overset{*}{x}_3 = A_3{}^\infty(\overset{*}{x}_3)$.

By applying the abstract comparison Lemma 2.4 follows that the unique solution of the problem (1.2)–(1.3) is increasing with respect to $A$. □

**Remark 5.2.** *The conclusion of the Theorem 5.1. means that the unique solution of (1.2)–(1.3) is increasing with respect to the right hand.*

## 6. Step method

Next we apply the step method for (1.2)–(1.3). Let the conditions $(C_1'), (C_2), (C_3)$ and $(C_5)$ and we suppose also the condition

$(C_6)$ For all $t \in [-h, \infty)$, $u_1, u_2, u_3 \in \mathbb{B}$ there exists a unique $u_2 \in \mathbb{B}$, $u_2 = f(t, u_1, u_3)$ such as $u_3 = g(t, u_1, u_2) + \int_{t-h}^{t} K(s, u_1)ds$.

Note that if $x \in C^1(\mathbb{B})$ is a solution for (1.2)–(1.3) then, by mathematical induction, follows that $x \in C^\infty(\mathbb{B})$.

**Theorem 6.1.** *Suppose that we have $(C_1'), (C_2), (C_3), (C_5)$ and $(C_6)$. Then the problem (1.2)-(1.3) has a solution if and only if*

$$\varphi^{(n+1)}(0) = g^{(n)}(0, \varphi(0), \varphi(h)) + \left[\int_{t-h}^{t} K(s, \varphi(s))ds\right]^{(n)}\Bigg|_{t=0}, \ n \in \mathbb{N}.$$

*More, the solution is unique.*

*Proof.* By the step method we have

(p₀) $x_0(t) = \varphi(t), t \in [0, h]$.

Also we have

$$x_0'(t) = g(t, x_0(t), x(t+h)) + \int_{t-h}^{t} K(s, x_0(s))ds$$

or

$$\varphi'(t) = g(t, \varphi(t), x(t+h)) + \int_{t-h}^{t} K(s, \varphi(s))ds.$$

From condition $(C_6)$ we have that

$$x(t) := x_1(t) = f(t-h, \varphi(t-h), \varphi'(t-h)), \forall t \in [h, 2h].$$

From the regularity condition we have that $x(t) \in C^\infty[-h, 2h]$ where

$$x(t) = \begin{cases} \varphi(t), \ t \in [-h, h] \\ f(t-h, \varphi(t-h), \varphi'(t-h)), \ t \in [h, 2h]. \end{cases} \tag{6.1}$$

The next step is

($p_1$) $x_1'(t) = g(t, x_1(t), x(t+h)) + \int_{t-h}^{t} K(s, x_1(s))ds$.

From condition $(C5)$ we have that

$$x(t) := x_2(t) = f(t-h, x_1(t-h), x_1'(t-h)), \forall t \in [2h, 3h].$$

From the regularity condition we have that $x(t) \in C^\infty[-h, 3h]$ where

$$x(t) = \begin{cases} \varphi(t), \ t \in [-h, h] \\ f(t-h, \varphi(t-h), \varphi'(t-h)), \ t \in [h, 2h] \\ f(t-h, x_1(t-h), x_1'(t-h)), \ t \in [2h, 3h]. \end{cases} \tag{6.2}$$

By induction we can obtain the solution on $[-h, \infty[$ of the form

$$x(t) = \begin{cases} \varphi(t), \ t \in [-h, h] \\ x_1, \ t \in [h, 2h] \\ x_2, \ t \in [2h, 3h] \\ \cdots \\ x_n, \ t \in [nh, (n+1)h]. \end{cases} \tag{6.3}$$

In order to prove the necessity of the regularity condition we have $x \in C^\infty[-h, \infty[$ a solution of the problem (1.2)–(1.3). By successive derivations we have

$$x^{(n+1)}(t) = g^{(n)}(t, \varphi(t), \varphi(t+h)) + \left[\int_{t-h}^{t} K(s, \varphi(s))ds\right]^{(n)}, \ n \in \mathbb{N}.$$

For $t = 0$ follows that

$$\varphi^{(n+1)}(0) = g^{(n)}(0, \varphi(0), \varphi(h)) + \left[\int_{-h}^{0} K(s, \varphi(s))ds\right]^{(n)}.$$

□

**Remark 6.2.** *If $\mathbb{B} = \mathbb{R}^n$, then (1.2) is a finite system of equations, see* [6], [10].

**Remark 6.3.** *If $\mathbb{B} = l^p$, then (1.2) is a infinite system of equations, see* [1], [5], [19].

## References


[1] R.G. Cooke, *Infinite matrices and sequence spaces*, London, Macmillan, 1950

[2] A. Chiş-Novac, *Data dependence of fixed points in gauge spaces*, Fixed Point Theory, 12 (2011), No. 1, 49-56

[3] C. Crăciun, M.-A. Şerban, *A nonlinear integral equation via picard operators*, Fixed Point Theory, 12 (2011), No. 1, 57-70

[4] M. Dobriţoiu, I.A. Rus, M.A. Şerban, *An integral equation arising from infectious diseases, via Picard operator*, Studia Univ. "Babeş-Bolyai", Mathematica, Vol. LII, No. 3, 2007, 81-83

[5] I. Gohberg, S. Goldberg, M.A. Kaashoek, *Basic classes of linear operators*, Birkhuser, Basel, 2003

[6] D. Guo, V. Lakshmikantham, X. Liu, *Nonlinear Integral Equations in Abstract Spaces*, Kuwer Academic Publishers, Dordrecht, Boston, London, 1996

[7] V.A. Ilea, *Functional differential equations of first order with advanced and retarded arguments*, Presa Universitară Clujeană, 2006 (in Romanian)

[8] V.A. Ilea, D. Otrocol, M.A. Şerban, D. Trif, *Integro-differential equation with two time lags*, to appear

[9] V.A. Ilea, D. Otrocol, *On a D.V. Ionescu problem for functional-differential equations*, Fixed Point Theory, 10 (2009), No. 1, 125-140

[10] V. Kolmanovskii, A. Mishkis, *Applied Theory of Functional Differential Equations*, Kluwer Academic Publisers, 1992

[11] D. Otrocol, *Lotka-Volterra systems with retarded argument*, Presa Universitară Clujeană, 2007 (in Romanian)

[12] D. Otrocol, V.A. Ilea, C. Revnic, *An iterative method for a functional-differential equation with mixed type argument*, Fixed Point Theory, 11 (2010), No. 2, 327-336

[13] I.A. Rus, *The theory of a metrical fixed point theorem: theoretical and applicative relevances*, Fixed Point Theory, 9 (2008), No. 2, 541-559

[14] I.A. Rus, *Picard operators and applications*, Sciantiae Mathematicae Japonicae, 58 (2003), No. 1, 191-219

[15] I.A. Rus, *Picard operators and applications*, Seminar on Fixed Point Theory, Cluj-Napoca, 2 (2001), 41-58

[16] I.A. Rus, *Some nonlinear functional differential and integral equations, via weakly Picard operator theory: a survey*, Carpathian J. Math., 26 (2010), no. 2, 230-258

[17] I.A. Rus, M.A. Şerban, D. Trif, *Step method for some integral equations from biomathematics*, to appear

[18] S. Sakata, T. Hara, *Stability regions for linear differential equations with two kinds of time lags*, Funkcialaj Ekvacioj, 47 (2004), 129-144

[19] Sze-Bi Hsu, *Ordinary differential equations with applications*, Series on Applied Mathematics, vol. 16, Word Scientific, 2006



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