Conjugate heat transfer in a vertical channel filled with a nanofluid adjacent to a heat generating solid domain

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Abstract

The effect of thermal dispersion in the conjugate steady free convection flow of a nanofluid in a vertical channel is investigated numerically using a single phase model. Considering the laminar and fully developed flow regime a simplified mathematical model is obtained. In the particular cases when solid phase and thermal dispersion effects are neglected the problem was solved analytically. The numerical solution is shown to be in excellent agreement with the close form analytical solution. Nusselt number enhancement with the Grashof number, volume fraction, aspect ratio parameter and thermal diffusivity constant increasing has been found.

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Keywords: nanofluid, vertical channel, free convection, conjugate heat transfer, heat generation

1 Introduction

Heat transfer in channels occurs in many industrial processes and natural phenomena. It has been the subject of many studies for different flow configurations. We mention some practical applications of convective heat transfer in channels: design of cooling systems for electronic devices, insulation, ventilation, grain storage, geothermal energy recover, solar energy collection, etc. Some classical papers, such as by Aung [1], Aung et al. [2], Barletta [3], Kumar et al. [8], Vajravelu and Sastri [12], are concerned with the evaluation of the temperature and velocity profiles for the vertical parallel-flow fully developed regime. Enhancement of heat transfer is essential in improving performances and compactness of electronic devices. Usual cooling agents (water, oil, etc.) have relatively small thermal conductivities and therefore heat transfer is not very efficient. Suspensions of nanoparticles in fluids improve physical properties and increase the heat transfer. Small fraction of nanoparticles added in a base fluid leads to a large increase of the fluid thermal conductivity. Good description and classification of the nanofluids characteristics can be found in papers such as: Daungthongsuk and Wongwises [5], Wang and Mujumdar [14] and Kumar et al. [9]. The chaotic movement of the nanoparticles and sleeping between the fine particles and fluid generate the thermal dispersion effect and this leads to an increase in the energy exchange rates in fluid. Xuan and Roetzel [13] proposed a thermal dispersion model for a single phase nanofluid. Thermal dispersion effects in nanofluids flow in enclosure using a single phase model were analyzed by Khanafer et al. [7] and Kumar et al. [9] for a differentially heated rectangular cavity, Khaled and Vafai [6] studied the heat transfer enhancement through control of thermal dispersion effects in a horizontal channel, while Mokmeli and Saffar-Avval [10] numerically studied nanofluid heat transfer in a straight tube. In all these paper the enhancement of heat transfer due to nanofluids special properties was reported. In the present paper, the effect of the thermal dispersion on the steady free convection flow in a long vertical channel, using the fully developed flow assumptions, is investigated using a single phase thermal dispersion model similar with the model considered by Khanafer et al. [7].

2 Basic equations

Consider the fully developed steady flow of an incompressible nanofluid in vertical channel. The left wall of the channel have a thickness b and thus we have to consider a conjugate heat transfer problem. The geometry of the problem, the boundary conditions, and the coordinate system is shown in Fig. 1.

The fluid flows up in the channel driven by buoyancy forces, so that the flow is due only to the difference in temperature gradient. The flow being fully developed the following relations apply here $v = 0$ and $\partial v/\partial y = 0$, where v is the velocity in the transversal direction. Thus, from the continuity equation, we get $\partial u/\partial x = 0$ so that the velocity along the channel is $u = u(y)$. Based on the fact that the flow is fully developed we can assume that the temperature depends only by y, i.e., $T = T(y)$. The physical properties of the nanofluid are considered constant except for density, which is given by the Boussinesq approximation.

We use in this study the heat capacity and the thermal expansion coefficient of

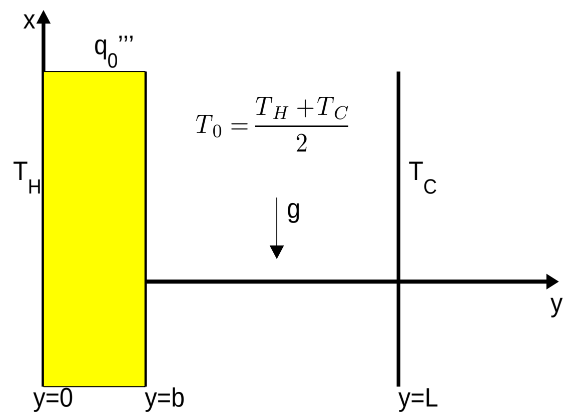

Figure 1: Geometry of the problem and the coordinate system

the nanofluid given in Kanafer et al. [7] as:

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \tag{2.1}$$

$$(\rho\beta)_{nf} = (1-\phi)\rho_f\beta_f + \phi\rho_s\beta_s, \tag{2.2}$$

where ρ is the density, c_p is the specific heat at constant pressure, ϕ is the volume fraction of suspension particles, β is the expansion coefficient, while subscripts nf, f and s stand for nanofluid, fluid and solid, respectively.

For the effective viscosity we consider the model proposed by Brinkman [4], which is valid for high volume fraction ($\phi > 0.05$):

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{2.3}$$

where μ is the dinamyc viscosity.

The effective stagnant thermal conductivity is approximated by the Maxwell-Garnetts model, see Wang and Mujumdar [14], which applies for spherical type particles:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}, \tag{2.4}$$

where k is the thermal conductivity.

The effective thermal conductivity includes also the thermal dispersion enhancement

$$k_{eff} = k_{nf} + k_d, \tag{2.5}$$

where the term due to thermal dispersion, k_d, is given by, see Khaled and Vafai [6]:

$$k_d = C(\rho C_p)_{nf}|u|\phi L, \tag{2.6}$$

where L is the thickness of the channel and C is a constant depending on the diameter of the nanoparticle and its surface geometry.

We limit the study in this paper to water based nanofluids containing Cu nanoparticles. Nanofluids thermo-physical properties are shown in the Table 1, see Oztop and Abu-Nada [11].

Property	H_2O	Cu
$C_p(J/kgK)$	4179	385
$\rho(kg/m^3)$	997.1	8933
$k(W/mK)$	0.613	400
$\alpha \times 10^7(m^2/s)$	1.47	1163.1
$\beta \times 10^{-5}(1/K)$	21	1.67

Table 1: Physical properties of fluid and Cu nanoparticles

In the assumption of the fully developed flow the governing equations for the flow and heat transfer have the following form:

$$\alpha_s \frac{d^2 T_s}{dy^2} + \frac{q_0'''}{(\rho C_p)_s} = 0; \tag{2.7}$$

$$\mu_{nf} \frac{d^2 u}{dy^2} + (\rho\beta)_{nf} g(T_f - T_0) = 0; \tag{2.8}$$

$$\frac{d}{dy}(k_{eff} \frac{dT_f}{dy}) = 0; \tag{2.9}$$

subject to the boundary conditions:

$$T_s|_{y=0} = T_H; \quad T_f|_{y=L} = T_C; \tag{2.10}$$

$$T_f|_{y=b} = T_s|_{y=b}; \tag{2.11}$$

$$k_s \frac{\partial T_s}{\partial y}|_{y=b} = k_{nf} \frac{\partial T_f}{\partial y}|_{y=b}; \tag{2.12}$$

$$u(b) = u(L) = 0; \tag{2.13}$$

where g is the gravitational acceleration, T is the temperature, u is the velocity, q_0''' is the heat generation and α is the thermal diffusivity.

In order to solve equations (2.7)-(2.9), subject to the boundary conditions (2.10)-(2.13), we introduce the following non-dimensional variables used also by Kumar et al. [8]:

$$\Theta_s = \frac{k_s(T_s - T_0)}{q_0''' L^2}, \quad \Theta_f = \frac{k_s(T_f - T_0)}{q_0''' L^2}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{U_c}, \tag{2.14}$$

where U_c and T_0 are the characteristic velocity and temperature given by:

$$T_0 = \frac{T_H + T_C}{2}, \quad U_c = \frac{g\beta_f(\frac{q_0'''L^2}{k_s})L^2}{\nu_f}. \tag{2.15}$$

Using (2.14) in equations (2.7)-(2.9) we obtain the following dimensionless ordinary differential equations:

$$\frac{d^2\Theta_s}{dY^2} + 1 = 0; \tag{2.16}$$

$$\frac{d^2U}{dY^2} + \lambda_\phi\Theta_f = 0; \tag{2.17}$$

$$\frac{d}{dY}[(k_\phi + C_\phi|U|)\frac{d\Theta_f}{dY}] = 0; \tag{2.18}$$

subject to

$$\Theta_s|_{Y=0} = q; \tag{2.19}$$

$$\Theta_s|_{Y=r} = \Theta_f|_{Y=r}; \tag{2.20}$$

$$\frac{d\Theta_s}{dY}|_{Y=r} = K\frac{d\Theta_f}{dY}|_{Y=r}; \tag{2.21}$$

$$\Theta_f|_{Y=1} = -q; \tag{2.22}$$

$$U(r) = U(1) = 0; \tag{2.23}$$

where:

$$r = \frac{b}{L}, \quad \lambda_\phi = (1-\phi)^{2.5}[(1-\phi) + \phi\frac{\rho_s\beta_s}{\rho_f\beta_f}], \quad q = \frac{k_s(T_H - T_C)}{2q_0'''L^2}, \tag{2.24}$$

$$K = \frac{k_{nf}}{k_s}, \quad k_\phi = \frac{k_{nf}/k_f}{1 - \phi + \phi\frac{(\rho C_p)_s}{(\rho C_p)_f}}, \quad C_\phi = C\phi PrGr \tag{2.25}$$

are constants depending on the properties of the nanofluid and $Pr = \nu_f/\alpha_f$, $Gr = g\beta_f q_0'''L^3/\nu_f^2$ are *Prandtl number* and *Grashof number*, respectively.

The physical quantity of interest in this problem is the Nusselt number, which for the conjugate wall is defined as:

$$Nu = \frac{hL}{k_f}|_{y=b} \tag{2.26}$$

where the convective heat transfer coefficient, h, is obtained from the relation:

$$-k_{eff}\frac{dT}{dy}|_{y=b} = h(T|_{y=b} - T_0). \tag{2.27}$$

Substituting (2.27) in (2.26) the dimensionless form of the Nusselt number becomes:

$$Nu = -\frac{k_{nf}}{k_f}\frac{1}{\Theta_f|_{Y=r}}\frac{d\Theta_f}{dY}|_{Y=r} \tag{2.28}$$

3 Results and discussions

In the case when thermal dispersion effect is neglected, i.e. $C = 0$, the problem has an analytical solution, which is given by

$$\Theta_s(Y) = -\frac{Y^2}{2} + (r + Ka_1)Y + q; \tag{3.1}$$

$$\Theta_f(Y) = a_1 Y - a_1 - q; \tag{3.2}$$

$$U(Y) = -\lambda_\phi[a_1\frac{Y^3}{6} - (a_1 + q)\frac{Y^2}{2}] + a_2 Y + a_3; \tag{3.3}$$

where:

$$a_1 = \frac{\frac{1}{2}r^2 + 2q}{r(1-K) - 1}; a_2 = \frac{\lambda_\phi}{6}[a_1(r^2 - 2r - 2) - 3q(r+1)];$$

$$a_3 = \frac{\lambda_\phi}{6}r[-a_1(r-2) + 3q].$$

In this particular case, the Nusselt number, has the form $Nu = -ak_{nf}/k_f$, where $a = a_1/(a_1 r - a_1 - q)$, and depends only by thermal characteristics of the nanofluid.

Equations (2.16)-(2.23) were solved numerically using finite difference discretization for different volume fractions of Cu nanoparticles, $\phi = 0$, 0.05, 0.1 and 0.2, and thermal conductivity ratio parameter, $K = 0.001$, 0.01 and 0.1. In this study we consider fixed values for q and r (i.e. $q = 1$, $r = 0.1$) and, following Khaled and Vafai [6], the values for constant C_ϕ were taken 0, 100, 250, 500, 1000, 5000 and 10000.

We compared the numerical method with the analytical solutions (3.1)-(3.3) and a very good agreement was found. In Figs. 2 to 4 the analytical solutions are also presented using a dot marker. Thus, we are confident that the numerical method works fine.

Tables 2 to 4 show the Nusselt number for different values of the above parameters. We mention that the value of Nusselt number increases with the increase of constant C_ϕ and thermal conductivity parameter K. Due to the conjugate heat transfer and thermal dispersion Nusselt number does not present a monotone behavior in respect with volume fraction ϕ.

Table 5 presents the variation of the maximum temperature in solid with C_ϕ and K. The maximum of the temperature in solid increases with the decrease of C_ϕ and K.

Figs. 2 to 4 present the velocity and temperature profiles for $\phi = 0.2$ and different values of C_ϕ and K. The reversed character of the flow becomes less important with the increasing of parameter C_ϕ for $K = 0.001$ (see Fig. 2a) while for $K = 0.1$ the flow is down for large values of C_ϕ (see Fig. 2b). Figs. 3 and 4 present the temperature profiles for $K = 0.1$ and $K = 0.001$ in solid and nanofluid. We observe a decrease of the temperature in solid and an increase of the temperature near the cold wall with the increase of C_ϕ for both values of K.

C_ϕ		ϕ	
	0.05	0.1	0.2
0	2.593686	2.984831	3.912962
100	4.383934	4.209832	4.597601
250	8.156642	7.365368	6.165330
500	12.796259	11.836095	9.605016
1000	19.102887	18.263184	15.442523
5000	31.875747	34.480343	37.022310
10000	34.522407	38.455023	45.131964

Table 2: Values of Nusselt number for $K = 0.1$

C_ϕ		ϕ	
	0.05	0.1	0.2
0	2.567849	2.955098	3.873984
100	3.878139	3.912190	4.532541
250	5.801081	5.747647	5.559927
500	7.698974	7.757198	7.510472
1000	9.637866	10.024039	10.138831
5000	12.575802	14.017470	16.581315
10000	13.106638	14.830093	18.361529

Table 3: Values of Nusselt number for $K = 0.01$

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C_ϕ		ϕ	
	0.05	0.1	0.2
0	2.565294	2.952158	3.870129
100	3.885839	3.920925	4.526447
250	5.813239	5.758804	5.571575
500	7.716117	7.774421	7.525006
1000	9.655428	10.044972	10.161326
5000	12.575822	14.022163	16.600783
10000	13.100287	14.825970	18.367821

Table 4: Values of Nusselt number for $K = 0.001$

C_ϕ		K	
	0.001	0.01	0.1
0	1.004777	1.003020	1.000000
100	1.004739	1.002726	1.000000
250	1.004678	1.002283	1.000000
500	1.004564	1.001548	1.000000
1000	1.004403	1.000760	1.000000
5000	1.003992	1.000000	1.000000
10000	1.003870	1.000000	1.000000

Table 5: Maximum value for the temperature in solid

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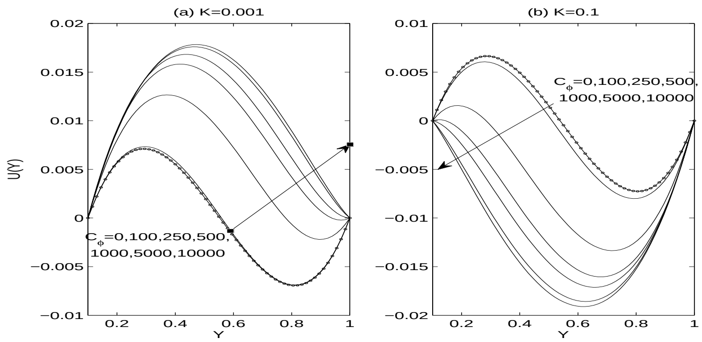

Figure 2: Velocity profile for different values of parameter C_ϕ

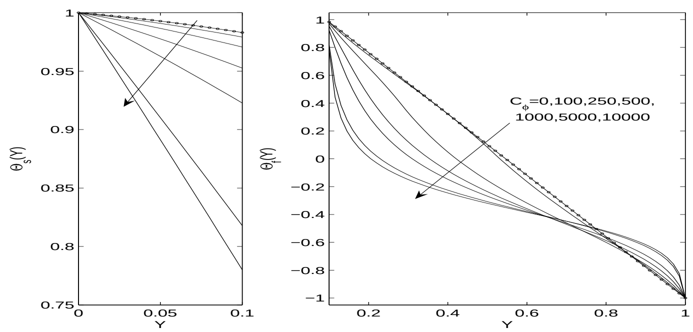

Figure 3: Temperature profile in solid (left) and fluid (right) for different values of parameter C_ϕ and $K = 0.1$

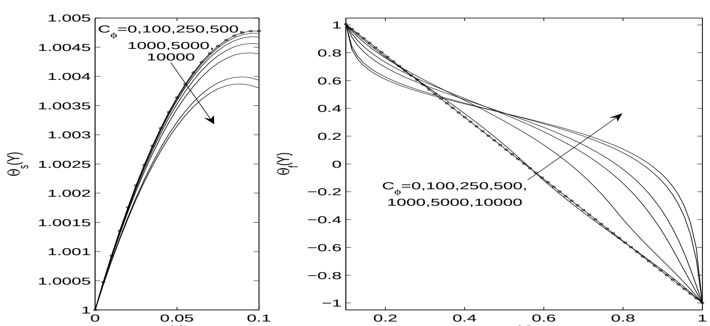

Figure 4: Temperature profile in solid (left) and fluid (right) for different values of parameter C_ϕ and $K = 0.001$