Mixed convection in a vertical channel subject to Robin boundary condition

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Abstract

The steady mixed convection flow in a vertical channel is investigated for laminar and fully developed flow regime. In the modelling of the heat transfer the viscous dissipation term was also considered. Temperature on the right wall is assumed constant while a mixed boundary condition (Robin boundary condition) is considered on the left wall. The governing equations are expressed in non-dimensional form and then solved both analytically and numerically. It was found that there is a decrease in reversal flow with an increase in the mixed convection parameter.

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1 Introduction

Heat transfer in channels occurs in many industrial processes and natural phenomena. It has been, therefore, the subject of many detailed, mostly numerical studies for different flow configurations. Most of the interest in this subject is due to its practical applications, for example, in the design of cooling systems for electronic devices and in the field of solar energy collection. Some of the published papers, such as by Aung [1], Aung et al. [2], Aung and Worku [3, 4], Barletta [5, 6], and Boulama and Galanis [7], are concerned with the evaluation of the temperature and velocity profiles for the vertical parallel-flow fully developed regime. As is well known, heat exchangers technology involves convective flows in vertical channels. In most cases, these flows imply conditions of uniform heating of a channel, which can be modelled either by uniform wall temperature (UWT) or uniform wall heat flux (UHF) thermal boundary conditions. In the present paper, new types of boundary conditions are considered. The right wall is kept at constant temperature while a convective heat flux is considered on the left wall (see, Bejan[8]):

$$\left(k\frac{\partial T}{\partial y}\right)_{y=0} + h_a\left(T_a - T\right)_{y=0} = 0 \tag{1}$$

where k is the thermal conductivity, h_a is the external heat transfer coefficient and T_a is the external temperature (see Figure 1). This kind of boundary condition is appropriate to express matematically heat loosing in insullation problems.In addition we have taken in account in this paper the effect of viscous dissipation, see Barletta[9].

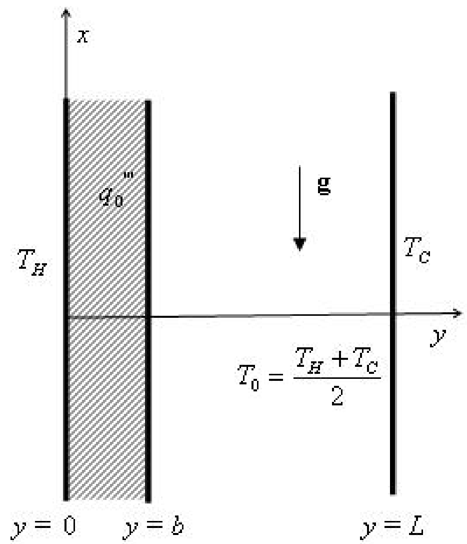

Figure 1: Geometry of the problem and the co-ordinate system

2 Basic Equations

Consider a viscous and incompressible fluid, which steadily flows between two infinite vertical and parallel plane walls. At the entrance of the channel the fluid has an entrance velocity U_0 parallel to the vertical axis of the channel. The geometry of the problem, the boundary conditions, and the coordinate system are shown in Fig. 1. The variation of density with temperature is given by the Boussinesq approximation and the fluid rises in the duct driven by buoyancy forces and initial velocity. Hence, the flow is due to difference in temperature and in the pressure gradient. The flow being fully developed the following relations apply here $v = 0$, $\partial v/\partial y = 0$, $\partial p/\partial y = 0$, where v is the velocity in the transversal direction and p is the pressure. Thus, from the continuity equation, we get $\partial u/\partial x = 0$ so that the velocity component along x -axis depends only by y, $u = u(y)$. Based on the fact that the flow is fully developed we can assume that the temperature $T = T(y)$. Under these assumptions the momentum and energy equations for the flow and heat transfer have the following form:

$$\nu \frac{d^2u}{dy^2} - \frac{1}{\rho}\frac{dp}{dx} + g\beta\,(T - T_0) = 0 \tag{2}$$

$$\alpha \frac{d^2T}{dy^2} + \frac{\nu}{c_p}\left(\frac{du}{dy}\right)^2 = 0 \tag{3}$$

subject to the boundary condition given by Eq. (1), noslip condition for for velocity at the walls and constant temperature at the left wall:

$$u(0) = 0, u(L) = 0, T(L) = T_w \tag{4}$$

where α is the thermal diffusivity of the viscous fluid, ρ is the fluid density and c_p is the specific heat at constant pressure. In the sistem (2) and (3) there is an additional unknown, the gradient of pressure, dp/dx. In order to close the above system subject to the boundary conditions (1) and (4) it is necessary to consider the equation of the mass flux conservation:

$$U_0 = \frac{1}{L}\int_0^L u(y)dy \tag{5}$$

where L is the channel width. Further, we introduce the following dimensionless variables (see Pop and Ingham[10] or Kohr and Pop[11]):

$$U=\frac{u}{U_0}, X=\frac{xRe}{L}, Y=\frac{y}{L}, \theta=\frac{T-T_0}{T_w-T_0}, P=\frac{L^2}{\rho\nu^2}p \tag{6}$$

where $Re = U_0 L/\nu$ is the Reynolds number and $T_0 = (T_a + T_w)/2$ is a characteristic temperature. Using (6) in the equations (2)-(3), in the boundary conditions (1) and (4) and in the mass flux conservation (5)we obtain:

$$\frac{d^2U}{dY^2} + \lambda\theta - \gamma = 0 \tag{7}$$

$$\frac{d^2\theta}{dY^2} + Br(\frac{dU}{dY})^2 = 0 \tag{8}$$

$$U(0)=0, U(1)=0, \left(\frac{d\theta}{dY}\right)_{Y=0} = \kappa(1+\theta)_{Y=0}, \theta(1)=1 \tag{9}$$

$$\int_0^1 U(Y)dY = 1; \tag{10}$$

In Eqs(7)-(10) γ is the pressure gradient in X direction, Br is the Brinkman number, λ is the mixed convection parameter and κ is the convection heat transfer parameter given by

$$\gamma=\frac{dP}{dX}, Br=PrEc=\frac{\mu U_0^2}{k(T_w-T_0)}, \lambda=\frac{Gr}{Re}=\frac{g\beta(T_w-T_0)L^2}{U_0\nu}, \kappa=\frac{h_aL}{k} \tag{11}$$

and Pr, Ec, Gr and Re are the Prandtl number, Eckert number, Grashoff number and Reynolds number,respectively, defined as:

$$Pr=\frac{\nu}{\alpha}, Ec=\frac{U_0^2}{c_p(T_w-T_0)}, Gr=\frac{g\beta(T_w-T_0)L^3}{\nu^2}, Re=\frac{U_0L}{\nu} \tag{12}$$

The physical quantity of interest in this problem are the skin friction coefficient C_f and the Nusselt number Nu, which are defined as:

$$C_f=\frac{\mu}{\rho U_0^2}\left(\frac{du}{dy}\right)_{y=0,L}, Nu=\left(\frac{h_fL}{k}\right)_{y=0,L} \tag{13}$$

In Eq. (13) h_f is the internal heat transfer coefficient which can be calculated from the heat transfer balance at the wall:

$$\left(k\frac{\partial T}{\partial \mathbf{n}}\right)_{wall} = h_f\left(T_{wall}-T_{fluid}\right))$$

where $\mathbf{n}$ is the normal to the wall. Using dimensionless varibles (6) we obtain:

$$C_fRe=\left(\frac{dU}{dY}\right)_{Y=0,1}, Nu|_{Y=0}=\kappa\left(\frac{\theta(0)+1}{\theta(0)-1}\right), Nu|_{Y=1}=-\frac{\left(\frac{d\theta}{dY}\right)_{Y=1}}{\theta(0)-1} \tag{14}$$

3 Results and Discussions

Equations (7) to (10) admit an analytical solution in two particular cases:

i) Case $Br = 0$
In this case the system (7) and (8) becomes:

$$\frac{d^2U}{dY^2} - \frac{dP}{dX} + \lambda\theta = 0 \tag{15}$$

$$\frac{d^2\theta}{dY^2} = 0 \tag{16}$$

subject to the boundary conditions (9). Further, from Eq.(15), (16) and condition (10) we obtain

$$\theta(Y) = \frac{2\kappa}{1+\kappa}Y + \frac{1-\kappa}{1+\kappa}$$
$$U(Y) = -\frac{\kappa\lambda}{1+\kappa}\frac{Y^3}{3} + (\gamma + \frac{1-\kappa}{1+\kappa}\lambda)\frac{Y^2}{2} + (\frac{\kappa\lambda}{3(1+\kappa)} - \frac{1}{2}(\gamma + \frac{1-\kappa}{1+\kappa}))Y \tag{17}$$
$$\gamma = -12 + \frac{\lambda}{1+\kappa}$$

ii) Case $\lambda = 0$
For $\lambda = 0$ the forced convection only is considered. The system (7) and (8) takes the following form:

$$\frac{d^2U}{dY^2} - \gamma = 0 \tag{18}$$

$$\frac{d^2\theta}{dY^2} + Br(\frac{dU}{dY})^2 = 0 \tag{19}$$

Taking in account that γ is constant, using the boundary conditions (9) and mass flux conservation (10) we have:

$$U(Y) = -6Y^2 + 6Y$$
$$\theta(Y) = -12BrY^4 + 24BrY^3 - 18BrY^2 + \frac{2\kappa}{1+\kappa}(1+3Br)Y + \frac{1+6Br-\kappa}{1+\kappa} \tag{20}$$
$$\gamma = -12$$

Equations (7) and (8) subject to (9) and (10) were solved numerically for different values of the parameters , λ, κ and Br (λ = 0, 100, 250, 500; κ = 0.01, 0.1, 1, 10; Br = 0, 0.001, 0.01, 0.025) using an implicit finite-difference method for velocity and a Gauss-Seidel iteration for temperature. Dimensionless velocity profiles, U(Y), and temperature profiles,$\theta(Y)$, are presented in Figs. 2 to 7 for different values of the above parameters. Analytical solutions (λ= 0, Br = 0) are also presented on figures with a circle marker.

The variation of the velocity $U(Y)$ and temperature $\theta(Y)$ with the mixed convection parameter $\lambda$is presented in Figs. 2 and 5. We notice

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λ	κ	$Br = 0$	$Br = 0.001$	$Br = 0.01$
0	0.1	5.940594	5.940594	5.940594
	1	5.940594	5.940594	5.940594
	10	5.940594	5.940594	5.940594
100	0.1	4.470594	4.501963	4.799567
	1	-2.144405	-2.134825	-2.047795
	10	-8.759405	-8.784130	-9.001301
500	0.1	-1.409405	-1.27279	-0.125966
	1	-34.484405	-32.905781	-25.265381
	10	-67.559405	-68.912398	-68.791864

Table 1: Friction coefficient $C_f Re|_{Y=0}$

λ	κ	$Br = 0$	$Br = 0.001$	$Br = 0.01$
0	0.1	-5.940594	-5.940594	-5.940594
	1	-5.940594	-5.940594	-5.940594
	10	-5.940594	-5.940594	-5.940594
100	0.1	-7.410594	-7.370828	-6.999151
	1	-14.025594	-13.992628	-13.700667
	10	-20.640594	-20.606004	-20.288425
500	0.1	-13.290594	-13.053063	-11.199059
	1	-46.365594	-43.025955	-27.986534
	10	-79.440594	-74.606586	-42.694129

Table 2: Friction coefficient $C_f Re|_{Y=1}$

λ	κ	$Br = 0$	$Br = 0.001$	$Br = 0.01$
0	0.1	-0.999999	-1.032983	-1.451739
	1	-1.000000	-1.005839	-1.059969
	10	-1.000000	-1.003203	-1.032116
100	0.1	-0.999999	-1.027827	-1.377468
	1	-1.000000	-1.006256	-1.063425
	10	-1.000000	-1.009053	-1.092552
500	0.1	-0.999999	-1.031473	-1.360956
	1	-1.000000	-1.133834	-2.040945
	10	-1.000000	-1.276971	-3.552673

Table 3: Nusselt number on the left wall $Nu|_{Y=0}$

λ	κ	$Br = 0$	$Br = 0.001$	$Br = 0.01$
0	0.1	1.000000	0.967016	0.548260
	1	1.000000	0.994160	0.940030
	10	1.000000	0.996796	0.967883
100	0.1	1.000000	0.959813	0.498805
	1	1.000000	0.981394	0.813917
	10	0.999999	0.978624	0.787469
500	0.1	1.000000	0.906917	0.033431
	1	1.000000	0.810584	-0.328300
	10	0.999999	0.683490	-0.854004

Table 4: Nusselt number on the right wall $Nu|_{Y=1}$

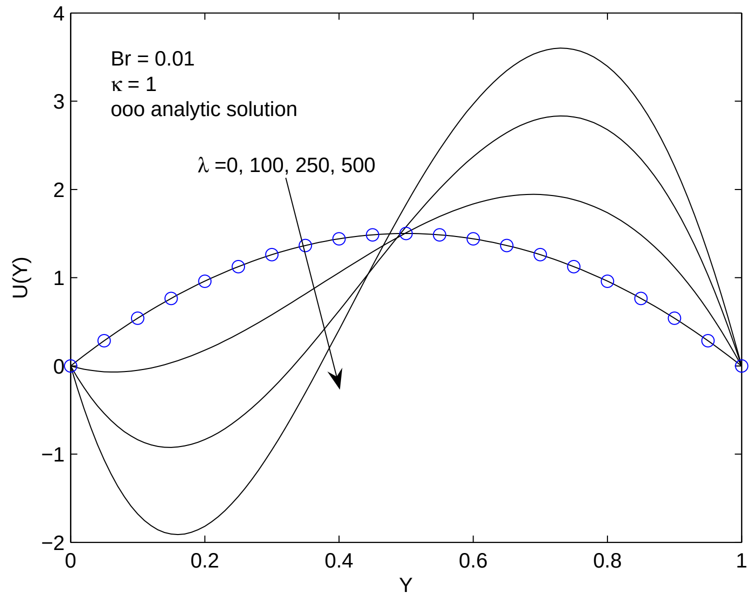

Figure 2: Velocity profiles for different values of λ

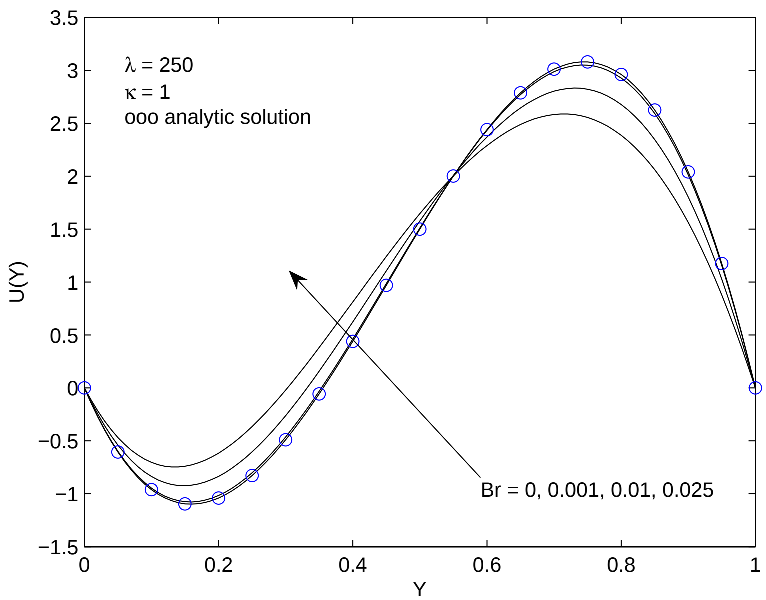

Figure 3: Velocity profiles for different values of Br

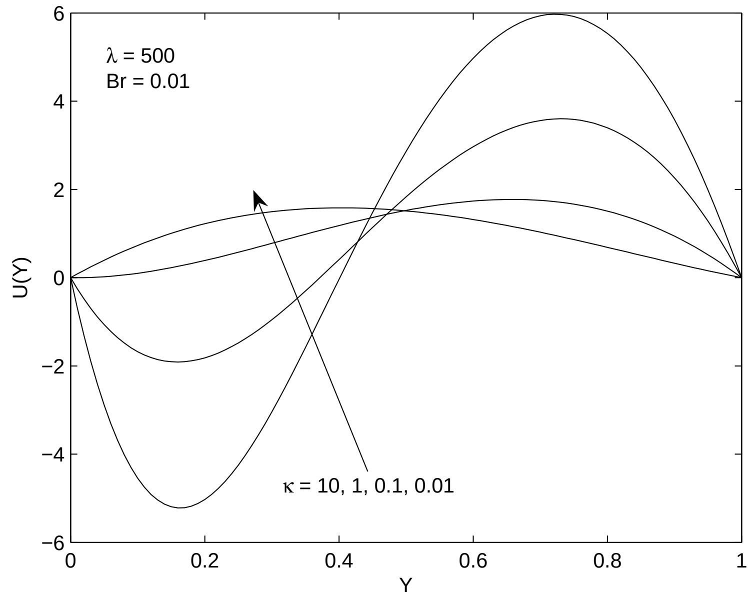

Figure 4: Velocity profiles for different values of κ

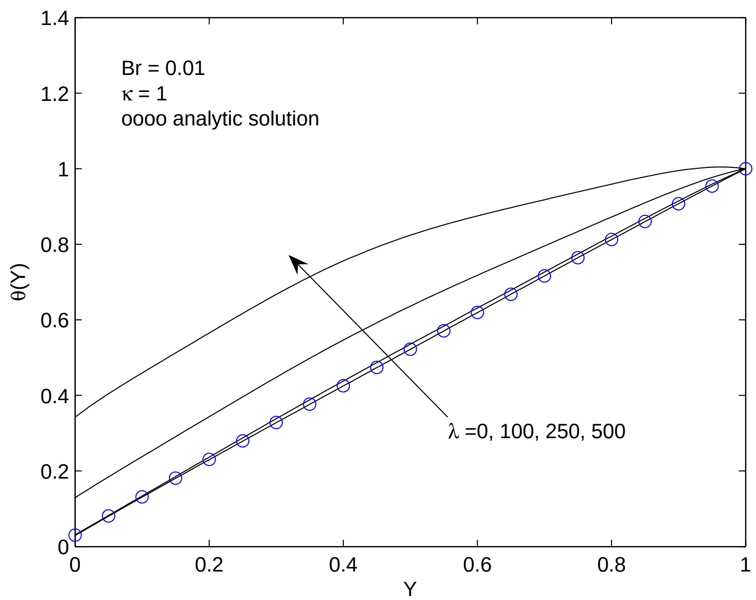

Figure 5: Temperature profiles for different values of λ

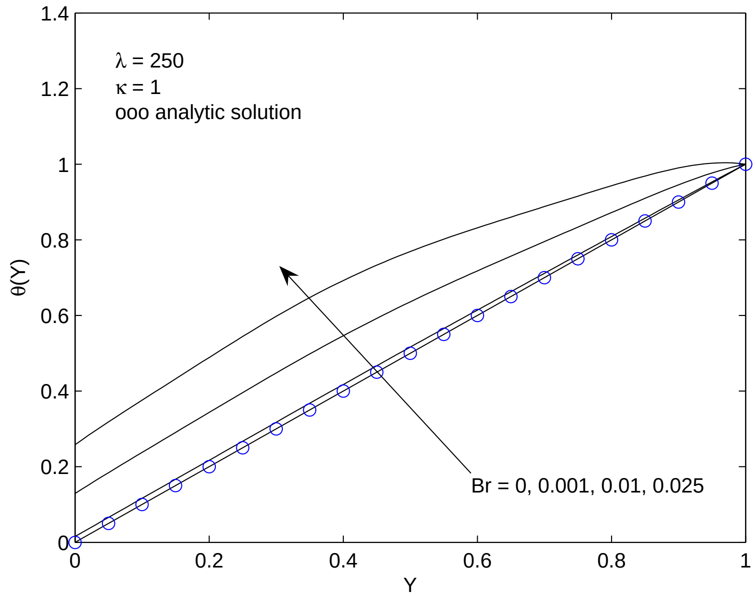

Figure 6: Temperature profiles for different values of Br

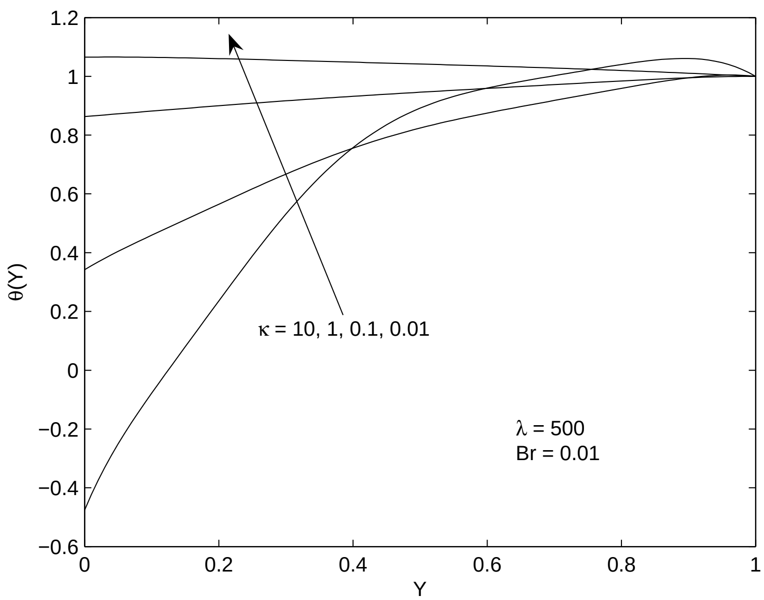

Figure 7: Temperature profiles for different values of κ

References

1. Aung, W.: Fully developed laminar free convection between vertical plates heated asymmetrically. In:Int. J. Heat Mass Transfer, vol. 15, 1972, p. 1577-1580.

2. Aung, W., Fletcher, L.S., Sernas, V.: Developing laminar free convection between vertical flat plates with asymmetric heating. In: Int. J. Heat Mass Transfer, vol.15, 1972), p. 2293-2308.

3. Aung, W., Worku, G.: Developing flow and flow reversal in a vertical channel with asymmetric wall temperatures. In: J. Heat Transfer, vol. 108, 1986, p. 299-304.

4. Aung, W., Worku, G.: Theory of fully developed, combined convection including flow reversal. In: J. Heat Transfer, vol. 108, 1986, p. 485-488.

5. Barletta, A.: Analysis of combined forced and free flow in a vertical channel with viscous dissipation and isothermal-isoflux boundary conditions. In: J. Heat Transfer, vol. 121, 1999, p. 349-356.

6. Barletta, A.: Fully developed mixed convection and flow reversal in a vertical rectangular duct with uniform wall heat flux. In: Int. J. Heat Mass Transfer, vol. 45, 2002, p. 641-654.

7. Boulama, K., Galanis, N.: Analytical solution for fully developed mixed convection between parallel vertical plates with heat and mass transfer. In: J. Heat Transfer, vol. 126, 2004, p. 381-388.

8. Bejan, A., 1995. Convection Heat Transfer (2nd edition), Wiley, New York.

9. Barletta, A.: Laminar mixed convection with viscous dissipation in a vertical channel. In: Int. J. Heat Mass Transfer, vol. 41, 1988, p. 3501-3513.

10. Pop, I., Ingham, D.B., 2001. Convective Heat Transfer: Mathematical and Computational Modeling of Viscous Fluids and Porous Media, Pergamon, Oxford.

11. Kohr, M., Pop, I., 2004.Viscous Incompressible Flow for Low Reynolds Numbers, WIT Press, Southamton.