NAAT 2018

International Conference on Numerical Analysis and Approximation Theory

Fourth Edition

Cluj-Napoca, Romania, September 6–9, 2018

BOOK OF ABSTRACTS AND PROGRAM

This edition is organized jointly with
Tiberiu Popoviciu Institute of Numerical Analysis,
Romanian Academy; www.ictp.acad.ro

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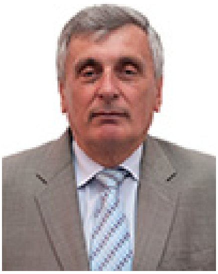
Professor Gheorghe Coman



Professor Petru Blaga

The meeting is devoted to aspects of approximation of functions, integral and differential operators, linear approximation processes, splines, numerical analysis, statistics, stochastic processes, wavelets.

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Invited speakers:

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Carlo Bardaro (*Italy)*
Maria Garrido-Atienza (*Spain*)
Heiner Gonska (*Germany*)
Aaron Melman (*USA*)
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Special thanks to Diana Şotropa.

♦ The organizers would like to acknowledge the substantial support received from **the Rectorate of Babeş-Bolyai University**, for which they are grateful.

♦ This conference was partly funded by grant 39/20.08. 2018 of **the National Authority for Scientific Research/the Ministry of Research and Innovation** and sponsored by **the Association of Mathematicians and Computer Scientists of Cluj (AMIC).**

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Thursday, September 6: 8:30 - 13:30; 14:30 - 16:30
Friday, September 7: 8:30 - 13:30; 14:30 - 16:30
Saturday, September 8: 8:30 - 13:30

CONFERENCE SCHEDULE

Thursday, September 6

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	Chairman: Gradimir Milovanović
9^{10}-10^{00}	Francesco Altomare
10^{00}-10^{50}	Wolfgang Wendland

10^{50}-11^{20} COFFEE BREAK

Notes.
1) All plenary talks are given in Room A.

2) Room A is *N. Iorga* Room and Room B is *T. Popoviciu* Room, both located at the 1st floor of the main building of the Babeş-Bolyai University, 1, M. Kogălniceanu St.

3) Please check daily the updated program posted on the doors of the rooms.

Contributed Talks

	Room A	**Room B**
	Chairman: *Heiner Gonska*	*Chairman:* *Carlo Bardaro*
11^{20}-11^{40}	Margareta Heilmann	Ralf Rigger
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16^{40}-17^{00}	Bjorn Schmalfuss	Mirella Cappelletti Montano
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17^{20}-17^{40}	Flavian Georgescu	Mehmet Unver
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18^{00}-18^{20}	Flavius Pătrulescu	Adonia Opriş

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Grand Hotel Napoca, 1, Octavian Goga St.
web: http://hotelnapoca.ro/

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12^{10}-12^{30}	Diana Otrocol	Vicuta Neagos
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PLENARY TALKS

POSITIVE APPROXIMATION PROCESSES AND INITIAL-BOUNDARY VALUE DIFFERENTIAL PROBLEMS

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MSC 2010: 41A36, 47D06, 47D07, 35K65, 35B65

Keywords: Approximation by positive operators, positive C_0-semigroup of operators, initial boundary-value differential problem.

The talk will be centered about a topic concerning three interrelated subjects: positive approximating operators, positive C_0-semigroups of operators and initial-boundary value evolution problems.

The main aim is to discuss a series of results concerning those sequences $(L_n)_{n\geq 1}$ of bounded linear operators on a Banach space E whose iterates converge to a C_0-semigroup $(T(t))_{t\geq 0}$ of operators on E.

To such a semigroup it is naturally associated its infinitesimal generator $A : D(A) \to E$ which, in turn, gives rise to an abstract Cauchy problem (initial-boundary value problem) whose solutions can be given, al least from a theoretical point of view, by the semigroup itself.

Thus, if it is possible to determine the operator A and its domain $D(A)$, then the initial sequence $(L_n)_{n\geq 1}$ becomes the key tool to approximate and to study (especially, from a qualitative point of view) the solutions of the Cauchy problem.

The principal ideas and some of the more recent results on such functional analytic approach to study these kinds of problems, will be discussed in the context of continuous function spaces by also assuming that the operators $L_n, n \geq 1$, are positive. Moreover, particular attention will be devoted to the important case when the approximating operators are constructively generated by a given positive linear operator $T : C(K) \to C(K)$ which, in turn, allows to determine the differential operator $(A, D(A))$ as well, K being a compact subset of $\mathbb{R}^d$, $d \geq 1$, having non-empty interior.

Initial-boundary value evolution problems corresponding to these particular settings, occur, for instance, in the study of diffusion problems arising from different areas such as biology, mathematical finance and physics.

For more details and for several other aspects related to the above outlined theory, the reader is referred to the monograph [1].

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PALEY-WIENER THEOREMS FOR MELLIN TRANSFORMS AND THE EXPONENTIAL SAMPLING

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MSC 2010: 26D10, 30D20, 44A05

Keywords: Mellin transforms, Paley-Wiener spaces, Bernstein spaces, band-limited functions, polar-analytic functions.

The exponential sampling formula, introduced by a group of physicists and engineers during the end of seventieth, was studied in a rigorous form, through the Mellin transform theory, by P.L. Butzer and S. Jansche during the ninetees. This formula is formally equivalent to the classical Shannon sampling formula of signal analysis, valid for Fourier bandlimited functions. Indeed, by a formal change of variable and change of function it is possible to obtain one formula from the other. However, this equivalence is only formal. Indeed, the structure of the Paley-Wiener space of all continuous functions in $L^2(\mathbb{R})$ which are bandlimited, is characterized by the famous Paley-Wiener theorem of Fourier analysis which states that a (Fourier) bandlimited function has an extension to the complex field as an entire function of exponential type (the Bernstein space). In Mellin transform setting this is not true. Indeed, it is shown that a (non trivial) Mellin bandlimited function cannot be extended as an entire function over $\mathbb{C}$. As proved in [1], it has an extension as an analytic function to the Riemann surface of the (complex) logarithm with certain exponential type conditions (Mellin-Bernstein spaces). In this talk, we discuss some further version of the Paley-Wiener theorem in Mellin setting, which avoid the use of Riemann surfaces and analytic branches, employing a new concept of analyticity (see [2]). Another version characterizes the space of all the functions whose Mellin transform decays exponentially at infinity involving Hardy-type spaces (see [3]). Applications to exponential sampling theory are described.

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STOCHASTIC DIFFERENTIAL EQUATIONS DRIVEN FRACTIONAL BROWNIAN MOTION

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In this talk we are concerned with the study of the existence and uniqueness of solutions of stochastic (partial) differential equations driven by a fractional Brownian motion (fBm), as well as their longtime behavior. We will analyze different approaches and consider both the cases of an fBm with Hurst parameter $H \in (1/2, 1)$ and $H \in (1/3, 1/2]$.

This talk is based on some joint works with Duc Hoang Luu (Max-Planck Institut of Leipzig), A. Neuenkirch (University of Mannheim) and B. Schmalfuss (University of Jena).

CHLODOVSKY-BERNSTEIN POLYNOMIAL OPERATORS - PAST AND PRESENT

Heiner Gonska

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The talk will be on a long-neglected polynomial approximation process for continuous functions defined on the real half-line. It was introduced by the Russian mathematician Igor Nikolaevich Chlodovsky. Although the first publication on it appeared in 1937 already, progress on investigating it was interrupted for more than half a century. As Butzer and Karsli said in 2009, this approximation process is not so easy to handle.

Some progress was made over the last ten years. We will present some of these latest developments including very recent results.

EIGENVALUE LOCALIZATION FOR MATRIX POLYNOMIALS

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MSC 2010: 12D10, 15A18, 15A42, 15A54, 26C10, 30C10, 30C15, 47A56, 65F15

Keywords: Bound, localization, eigenvalue, scalar, polynomial, matrix polynomial.

We survey a number of well-known and less well-known results ([5], [6]) on the location of scalar polynomial zeros and their generalization to localization results for the eigenvalues of matrix polynomials. Such polynomials occur in polynomial eigenvalues problems, which can be found in a wide range of engineering applications. We include results for matrix polynomials expressed in generalized bases, covering all classical orthogonal bases, such as Hermite, Legendre, Chebyshev, etc.

Finally, we show how some of the aforementioned results for scalar polynomials lead to extensions of the Eneström-Kakeya theorem for polynomials with positive coefficients.

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SOME CLASSES OF ORTHOGONAL POLYNOMIALS IN THE COMPLEX PLANE AND APPLICATIONS

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MSC 2010: 33C45, 33C47, 65D25, 65D30, 78A30

Keywords: Complex orthogonal polynomials, recurrence relations, zeros, numerical differentiation, quadrature formula.

We consider a few classes of polynomials orthogonal in the complex plane with respect to the Hermitian and Non-Hermitian inner products, as well as some applications of such polynomials.

The first class of such complex polynomials was introduced and studied in [1], [2] and [3]. The inner product is not Hermitian and defined by

$$(f,g) = \int_{\Gamma} f(z)g(z)w(z)(\mathrm{i}z)^{-1}\mathrm{d}z,$$

where $z \mapsto w(z)$ is a *complex* weight function holomorphic in the half disk $D_+ = \{z \in \mathbb{C} \mid |z| < 1,\ \mathrm{im}\ z > 0\}$. Polynomials orthogonal on the radial rays in the complex plane is the second class of polynomials in our investigation. This class was introduced in [4] (see also [7], [5], [6]).

Beside some analysis of such kinds of orthogonality and an electrostatic interpretation of zeros of the polynomials on the radial rays, we give several applications of these polynomials in numerical integration and numerical differentiation.

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THE SCOTTISH CAFE AND STATISTICAL CONVERGENCE

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Many valuable problems and solutions are originated in a cafe so-called "Scottish Cafe" in Lviv around 1930's. These problems are listed in the well-known book *The Scottish Book, Mathematics from the Scottish Cafe.* The participants of this informal meetings includes well-known mathematicians such as Stefan Banach, Hugo Steinhaus, Stanislaw Mazur, W. Orlicz, J.P. Schauder, M. Kac, S. Kacmarz, S. Saks, S. Ulam etc.

Here is the one problem posed by S. Mazur (July 22, 1935):

"A sequence (x_n) is asymptotically convergent to L if there exits a subsequence of density one convergent to L. In the domain of all sequences this notion of convergence is not equivalent to any Toeplitz (regular method). How is it in the domain of bounded sequences?" Mazur also made some comments that indicates the solution of this problem is negative (see, The Scottish Book, Second Edition, page 55, problem 5).

In our talk we will provide a positive answer to Mazur's problem and his claim has to be false. In order to prove our result we first note that the notion of asymptotic convergence of Mazur is equivalent to the notion of statistical convergence. We show that statistical convergence is always boundedly (as well as over the space of uniformly integrable sequences) equivalent to a nonnegative regular matrix method which is boundedly multiplicative. We also characterize the set of bounded multipliers of multiplicative methods. This talk is mainly based on my joint paper with M.K. Khan (Matrix characterization of A-statistical convergence; J. Math. Anal. Appl. 335 (2007), 406-417).

ON NEUMANN'S METHOD AND DOUBLE LAYER POTENTIALS

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MSC 2010: 31A10, 45F15, 65N30

Keywords: Boundary integral equations, boundary element methods.

Neumann's classical integral equations with the double layer boundary potential is considered on different spaces of boundary charges such as continuous data, L^2 and energy trace spaces on the domain's boundary for interior and exterior boundary value problems of elliptic partial differential equations. Corresponding known results for different classes of boundaries are discussed in view of collocation and Galerkin boundary element methods.

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CONTRIBUTED TALKS

ASYMPTOTIC PROPERTIES AND OPERATOR NORMS OF GAUSS-WEIERSTRASS OPERATORS AND THEIR LEFT QUASI INTERPOLANTS

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MSC 2010: 41A36, 41A45, 47A30

Keywords: Approximation by positive operators, operator norm.

The Gauß–Weierstraß convolution operators W_n $(n = 1, 2, 3, \ldots)$ are defined by

$$\left(W_n f\right)(x) = \sqrt{\tfrac{n}{\pi}} \int_{-\infty}^{\infty} f(t) \exp\left(-n(t-x)^2\right) dt \qquad (1)$$

(see, e.g., [6, SS 5.2.9]). They are positive linear approximation operators which are applicable to the class $L_c(\mathbb{R})$ of all locally integrable real functions f on $\mathbb{R}$ satisfying the growth condition $f(t) = \mathcal{O}(e^{ct^2})$, as $t \to \pm\infty$, for some $c > 0$, provided that $n > c$. If f is continuous we have

$$\lim_{x\to\infty} \left(W_n f\right)(x) = f(x)$$

uniformly on compact subsets of $\mathbb{R}$.
The complete asymptotic expansion of $(W_n f)(x)$ as n tends to infinity appears to be a special case of the results [3] on a more general operator defined by Altomare and Milella [5].
In 2014 Sablonnière [7] defined quasi-interpolants of W_n and studied their basic properties including the operator norm for bounded functions with respect to the sup-norm. In particular, he proved asymptotic expansions for polynomials and functions having a bounded higher order derivative on the whole line.
In the first part we report recent results [2] by presenting the complete asymptotic expansions of $(W_n f)(x)$ and the left quasi-interpolant $(W_n^{[r]} f)(x)$ as n tends to infinity, for functions belonging to the class $L_c(\mathbb{R})$ which are assumed to be only locally smooth. The corresponding results for the Favard operators which are the discrete version of W_n can be found in [4, 1].
Finally, in the second part, we consider the operator norms of W_n and $W_n^{[r]}$ when acting on various function spaces.

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http://dx.doi.org/10.1016/j.jat.2013.12.003

DIFFERENCES OF POSITIVE LINEAR OPERATORS

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MSC 2010: 41A25, 41A36

Keywords: First modulus of continuity, positive linear operators.

The results obtained are motivated by the recent results which give a solution to a problem proposed by A. Lupaş in [1]. One of the questions raised by him was to give an estimate for

$$B_n \circ \overline{\mathbb{B}}_n - \overline{\mathbb{B}}_n \circ B_n =: U_n - S_n,$$

where B_n are the Bernstein operators and $\overline{\mathbb{B}}_n$ are the Beta operators. We introduce new inequalities for such differences of positive linear operators and their derivatives in terms of moduli of continuity. This is a joint work with Ioan Raşa from Technical University of Cluj-Napoca, Romania.

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THE DUNKL GENERALIZATION OF STANCU TYPE Q-SZÁSZ-MIRAKJAN-KANTOROVICH OPERATORS AND SOME APPROXIMATION RESULTS

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MSC 2010: 41A25, 41A36, 33C45

Keywords: q-integers, q-exponential functions, q-hypergeometric functions, Dunkl's analogue, Szász operators, Stancu type q-Szász-Mirakjan-Kantorovich operators, rate of convergence, modulus of continuity and Peetre's K-functional.

In this paper, a Dunkl type generalization of Stancu type q-Szász-Mirakjan-Kantorovich positive linear operators of the exponential function is introduced. With the help of well-known Korovkin's theorem, some approximation properties and also the rate of convergence for these operators in terms of the classical and second-order modulus of continuity, Peetre's K-functional and Lipschitz functions are investigated. Further, some approximation results for bivariate Stancu type q-Szász-Mirakjan-Kantorovich operators are obtained.

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FUNCTIONS WITH CONVEX IMAGES UNDER BERNSTEIN OPERATORS

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MSC 2010: 26A51, 26D05

Keywords: Bernstein operators, convex function, log-convex function.

The preservation of convexity under the Bernstein operators B_n is exhaustively investigated in literature.

In this talk we present some non-convex functions f for which $B_n f$ is convex. Inequalities for such functions are also discussed.

QUANTITATIVE RESULTS FOR THE ITERATES OF SOME KING TYPE OPERATORS

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MSC 2010: 41A25, 41A36

Keywords: Iterates, linear positive operators, convergence.

In this article we define the -q variant of some King type operators which fix the functions e_0 and $e_2 + \alpha e_1$, $\alpha > 0$. We study the rates of convergence for the iterates of these operators using the first and the second order modulus of continuity. We show that the convergence is faster in the case of -q operators ($q < 1$) than in the classical case ($q = 1$).

ON THE POSITIVE SEMIGROUPS GENERATED BY FLEMING-VIOT TYPE DIFFERENTIAL OPERATORS

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MSC 2010: 35K65, 41A36, 47D06, 47D07

Keywords: Bernstein-Durrmeyer operators with Jacobi weights, Fleming-Viot type differential operator, positive semigroup, approximation of semigroups.

Joint work with Francesco Altomare and Vita Leonessa ([1]).

In this talk, we introduce, in the framework of function spaces defined on the d-dimensional hypercube of $\mathbb{R}^d$, $d \geq 1$, a class of polynomial type positive linear operators, which generalize the Bernstein-Durrmeyer operators with Jacobi weights on $[0, 1]$ ([3, 2]). By means of them, we study some degenerate second-order elliptic differential operators, often referred to as Fleming-Viot type operators, showing that their closures generate positive semigroups both in the space of all continuous functions and in weighted L^p-spaces. In addition, we show that those semigroups are approximated by iterates the above mentioned Bernstein-Durrmeyer operators.

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ON A NEWTON-HERMITE-STEFFENSEN TYPE METHOD

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MSC 2010: 65H05

Keywords: Inverse interpolatory iterative methods, Newton method, local convergence, monotone convergence.

We consider the solving of nonlinear equations in $\mathbb{R}$, and we introduce an inverse interpolatory iterative method of Hermite type, for which the nodes are given using the Newton method:

$$\begin{aligned} y_n &= x_n - \tfrac{f(x_n)}{f'(x_n)} \\ x_{n+1} &= y_n - \tfrac{f(y_n)}{f'(y_n)} - \tfrac{[y_n,y_n,x_n;f]f^2(y_n)}{f'(y_n)[y_n,x_n;f]^2}. \end{aligned}$$

The convergence order of the resulted method is 5, and the efficiency index is higher than in the case of the Newton and the Steffensen methods.

Under some natural conditions, the generated iterates converge monotonically to the solution, and one may obtain larger convergence sets than the usual attraction balls.

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ON THE POMPEIU MEAN-VALUE THEOREM

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MSC 2010: 26A24

Keywords: Pompeiu mean-value theorem, compact set, affine support.

We generalize the Pompeiu mean-value theorem by replacing the graph of a continuous function with a compact set.

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A BOSE-EINSTEIN CONDENSATE DARK MATTER MODEL TESTED WITH THE SPARC GALACTIC ROTATION CURVES DATA

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MSC 2010: 83F05, 85A40

Keywords: Cosmology, galactic rotation curves, dark matter models.

In the present study we investigate the properties of the galactic rotation curves in the Bose-Einstein Condensate dark matter model, with quadratic self-interaction, by using more than one hundred galaxies from the recently published Spitzer Photomery & Accurate Rotation Curves (SPARC) data.

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ON A STOCHASTIC ARC FURNACE MODEL

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MSC 2010: 34F05

Keywords: Electric arc furnace, random differential equation, Ornstein-Uhlenbeck process.

One approach of modeling of an electric arc furnace (EAF) is by the power balance equation which results in a nonlinear ordinary differential equation.

In real-world data can be observed that arc-current and arc-voltage vary randomly in time (cf. [1]), for example they oscillate with a randomly time-varying amplitude and a slight shiver. Therefore it is better to model them as stochastic processes and then solve a random differential equation.

Here we want to propose one example for a modulation by using the Ornstein-Uhlenbeck process and present some results which we gained by studying this model.

This is a joint work with Anna Chekhanova and Hans-Jörg Starkloff.

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AN APPROACH TO DEALING WITH UNCERTAINTY IN NUMERICAL MODELS

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Keywords: Valuation of dependences, stability of numerical algorithms, principle of uncertainty, relative metric.

The article discusses a method for valuation of the functional dependencies applied for estimation of some numerical models. The method for valuation of dependencies is applied similar to ”functional stability analysis” method, which is intended to detect instability in numerical algorithms [1].

The uncertainty principle is a fundamental concept [2], [3]. In this paper, we propose a new approach for dealing with such uncertainty, which combines the uncertainties in the values of parameters and transformation uncertainty that describes local degree of correlation at a given point of domain.

For optimal evaluation of the dependencies, we use relative distance in space of input parameters. The used mathematical theory is based on the theory of functional analysis on metric spaces, where the metric gives an estimation of the error [4]. The p-relative distance was introduced by Ren-Cang [5]. In [6] was introduced the main results of M-relative distances.

We use calculations with semi-logarithmic derivative ensuring accordance with the relative distance and we establish relevant properties. A valuation is applied for an analytical expression.

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APPROXIMATION BY MAX-PRODUCT OPERATORS OF KANTOROVICH TYPE

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MSC 2010: 41A35, 41A25, 41A20

Keywords: Max-product operators, max-product operators of Kantorovich kind, uniform approximation, shape preserving properties, localization results, max-product Kantorovich-Choquet operators.

We associate to various linear Kantorovich type approximation operators, nonlinear max-product operators for which we obtain quantitative approximation results in the uniform norm, shape preserving properties and localization results.

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ON A CONVEXITY PROBLEM

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MSC 2010: 26D15, 26D10, 46N30

Keywords: Linear positive operators, convex functions, Bernstein operators.

The work presented here is a continuation of what was done in [2] and it is strongly connected to the work done in [1]. Applications are given for Mirakyan-Favard-Szász, Baskakov and Szász-Schurer type operators.

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ON THE DECOMPOSITION OF SOME LINEAR POSITIVE OPERATORS

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MSC 2010: 41A35, 41A10, 41A25

Keywords: Linear operators, Beta operators, positive operators.

In this talk we consider the decomposition of some discretely linear positive operators. Our results generalize the results obtained in [1].

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INVARIANT MEASURES ASSOCIATED TO φ - MAX - IFSs WITH PROBABILITIES

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MSC 2010: 28A80, 37C70, 54H20

Keywords: φ-max-contraction, comparison function, iterated function system with probabilities, Markov operator, fixed point, invariant measure.

We prove that the Markov operator associated to an iterated function system consisting of φ-max-contractions with probabilities has a unique invariant measure whose support is the attractor of the system.

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SPECTRAL COLLOCATION SOLUTIONS TO A CLASS OF NONLINEAR BVPs ON THE HALF LINE

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MSC 2010: 65L10, 65L60, 65L70, 65N35

Keywords: Collocation, Laguerre-Gauss-Radau, Kidder problem, nonlinear Schrödinger, radially symmetric solutions.

The existence, uniqueness and regularity of solutions to the boundary value problems

$$\begin{array}{c} \frac{1}{p(x)}\left(p\left(x\right)u'\left(x\right)\right)' = q\left(x\right)f\left(x, u\left(x\right), p\left(x\right)u'\left(x\right)\right),\ x \in (0,\infty) \\ \alpha u\left(0\right) - \beta u'\left(0\right) = r,\ \lim_{x\to\infty} u\left(x\right) = 0, \end{array} \tag{1}$$

are established. We assume $\alpha > 0$, $\beta \geq 0$ and r is a given constant and f, p and $\frac{1}{q}$ are continuous. Our aim is to accurately approximate the solutions of these problems, as well as to some PDEs reducible to (1), by a high order Laguerre-Gauss-Radau collocation method (see [1], Ch. 2).

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PARAMETER ESTIMATIONS FOR A LINEAR PARABOLIC FRACTIONAL SPDE WITH JUMPS

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MSC 2010: 60H15, 62F12, 60G22

Keywords: Stochastic partial differential equations, parameter estimations.

A drift parameter estimation problem is studied for a linear parabolic stochastic partial differential equation driven by a multiplicative cylindrical fractional Brownian motion with Hurst index $h \in]1/2, 1[$ and a multiplicative Poisson process with values in a Hilbert space. Equations are introduced for the Galerkin approximations of the mild solution process. A mean square estimation criterion is used for these equations. It is proved that the estimate is unbiased and weakly consistent for the original problem.

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HALF-LINEAR DIFFERENTIAL OPERATORS IN OSCILLATION THEORY

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MSC 2010: 34C10, 34C15

Keywords: Riccati technique, half-linear equation, oscillation theory, oscillation constant, conditional oscillation.

This is a joint work with M. Veselý. We consider half-linear differential equations given by operators with one-dimensional p-Laplacian. The main subject of this talk is to present results concerning the conditional oscillation of such equations, i.e., we find a border value which separates oscillatory equations from non-oscillatory ones and we explicitly determine this borderline (depending on the equations coefficients). For some of the presented results, we refer to [1, 2].

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REPRESENTATIONS FOR K-TH ORDER KANTOROVICH MODIFICATIONS OF LINKING OPERATORS

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MSC 2010: 41A36, 41A10, 41A28

Keywords: Linking operators, Baskakov-Durrmeyer type operators, Kantorovich modifications of operators.

In our talk we investigate linking operators acting on the unbounded interval $[0, \infty)$. Linking operators for the Szász-Mirakjan case were defined by Păltănea in [3] and for Baskakov type operators by Heilmann and Raşa in [2]. If the linking papràmeter ρ is a natural number we present a representation for Kantorovich variants of arbitrary order in terms of the Baskakov and Szász-Mirakjan basis functions, respectively. This leads to a simple proof of convexity properties which also solves an open problem mentioned in [1]. At the end of our talk we present a conjecture concerning a limit to B-splines which is connected to the above mentioned representation.

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SOME APPROXIMATION PROPERTIES OF URYSOHN TYPE OPERATORS

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MSC 2010: 41A25, 41A35, 47G10, 47H30

Keywords: Urysohn operators, linear positive operators, approximation.

In the present work, our aim is generalization and extension of the theory of interpolation of functions to functionals by means of Urysohn type operators. In accordance with this purpose, we introduce and study a new type of Urysohn type operators. In particular, we investigate the convergence problem for operators that approximate the Urysohn type operator. We construct our operators by using a nonlinear forms of the kernels together with the Urysohn type operator values instead of the sampling values of the function.

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A NEW METHOD FOR SOLVING COMPLEX SYMMETRIC SYSTEMS OF LINEAR EQUATIONS

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MSC 2010: 65F10

Keywords: Complex, symmetric, convergence, symmetric positive definite.

We consider the system of linear equations $(W + iT)u = b$, where $W, T \in \mathbb{R}^{n\times n}$ are symmetric positive semidefinite matrices with at least one of them being positive definite and $i = \sqrt{-1}$. Letting $b = p+iq$ and $u = x+iy$, we propose a new iterative method for two-by-two block real equivalent form

$$\begin{bmatrix} W & -T \\ T & W \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix},$$

of the system. Convergence of the proposed method is studied and the numerical results of the method are compared with those of the MHSS [1], the PMHSS [2], the TTSCSP [3], the CRI [4] methods.

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CONVERGENCE RATES FOR AN INERTIAL ALGORITHM OF GRADIENT TYPE ASSOCIATED TO A SMOOTH NONCONVEX MINIMIZATION

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MSC 2010: 90C26, 90C30, 65K10

Keywords: Inertial algorithm, nonconvex optimization, Kurdyka-Łojasiewicz inequality, convergence rate.

We investigate an inertial algorithm of gradient type in connection with the minimization of a nonconvex differentiable function. The algorithm is formulated in the spirit of Nesterov's accelerated convex gradient method. We show that the generated sequences converge to a critical point of the objective function, if a regularization of the objective function satisfies the Kurdyka-Łojasiewicz property. Further, we provide convergence rates for the generated sequences and the objective function values formulated in terms of the Łojasiewicz exponent.

ON THE INTERPLAY BETWEEN MARKOV OPERATORS PRESERVING POLYNOMIALS AND CONVEX COMPACT SUBSETS OF $\mathbb{R}^d$

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MSC 2010: 47B65, 47D07

Keywords: Markov operators, polynomial preserving property, second-order elliptic differential operator, Markov semigroup.

Let K be a convex compact subset of $\mathbb{R}^d$, $d \geq 1$, with non-empty interior. In this talk we are interesting to all positive linear operators T acting on the space $C(K)$ of continuous functions on K which leave invariant the polynomials of degree at most 1 and which, in addition, map polynomials into polynomials of the same degree m, $m \geq 2$.

In particular, we discuss the existence of operators T when K is a non-trivial strictly convex subset of $\mathbb{R}^d$, $d \geq 2$, discovering, among other things, a characterization of ellipsoids and balls of $\mathbb{R}^d$. A discussion of the above polynomial preserving property in the setting of product spaces, as well as for convex convolution products of positive linear operators, is also presented.

Such a polynomial preserving property play a central role in studying the possibility to get a representation/ approximation formula for semigroups generated by certain differential operators, associated with T, in terms of constructively defined linear positive operators associated with the same T (see, e.g. [1, 4]). Anyway, we point out that this is not the only case where a property of this kind is required (see, e.g. [3]). Moreover, in the literature there are many classes of positive linear operators that satisfy it.

All results presented are contained in the joint work [2].

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SOME NEW RESULTS CONCERNING THE CLASSICAL BERNSTEIN QUADRATURE FORMULA

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MSC 2010: 41A36, 41A80.

Keywords: Bernstein operator, convex function, divided difference, Popoviciu theorem, remainder term.

In this talk, we intend to highlight an applicative side of the classical Bernstein polynomials, in contrast to the well-known theory of the uniform approximation of functions. We will present some new results concerning the classical Bernstein quadrature formula

$$\int_a^b F(x)dx \approx \tfrac{b-a}{n+1} \sum_{k=0}^{n} F\left(a + \tfrac{k(b-a)}{n}\right),$$

which can be found in [1] and in another recent paper submitted for publication.

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A NEW ALGORITHM THAT GENERATES THE IMAGE OF THE ATTRACTOR OF A GIFS

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MSC 2010: 28A80, 37C70, 41A65, 65S05, 65P99

Keywords: Generalized infinite iterated function system (GIFS), attractor, deterministic algorithm, grid algorithm.

We provide a new algorithm (called the grid algorithm) designed to generate the image of the attractor of a generalized iterated function system and we compare it with the deterministic algorithm regarding generalized iterated function systems presented by P. Jaros, Ł. Maślanka and F. Strobin in [Algorithms generating images of attractors of generalized iterated function systems, Numer. Algor., 73 (2016), 477-499].

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ON SEVERAL INEQUALITIES IN AN INNER PRODUCT SPACE

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MSC 2010: 46C05, 26D15, 26D10

Keywords: Inner product space, Cauchy-Schwarz inequality.

The aim of this presentation is to prove new results related to several inequalities in an inner product space. Among these inequalities we will mention inequality Cauchy-Schwarz's inequality. Also we obtain some applications of these inequalities.

ON THE UNBOUNDED DIVERGENCE OF SOME INTERPOLATORY PRODUCT INTEGRATION RULES

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MSC 2010: 41A10, 65D32

Keywords: Product integration, Condensation of singularities, superdense set.

Based on some principles of Functional Analysis, this paper emphasizes the topological structure of the set of unbounded divergence for some interpolatory product quadrature formulas on Jacobi and equidistant points of the interval $[-1, 1]$, associated with the Banach space of all s-times continuously differentiable functions and with a weighted Banach space of absolutely integrable functions of order $p > 1$.

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APPROXIMATION PROPERTIES OF CERTAIN BERNSTEIN-STANCU TYPE OPERATORS

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MSC 2010: 41A36, 41A25

Keywords: Bernstein-Stancu operator, q-integers, rate of convergence, moduli of continuity.

In this paper we introduce and investigate a new operator of Bernstein-Stancu type, based on q polynomials. We study approximation properties for these operators based on Korovkin type approximation theorem and study some direct theorems. Also, the study contains numerical considerations regarding the constructed operators based on Maple algorithms.

This is a joint work with Ana-Maria Acu from Lucian Blaga University of Sibiu, Romania, Ogun Dogru from Gazi University, Turkey and Voichita Adriana Radu from Babes-Bolyai University, Romania.

ON THE FIRST MEAN-VALUE THEOREM FOR INTEGRALS

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MSC 2010: 26A24

Keywords: Positive linear functionals, mean-value theorems, integrals of Riemann and Lebesgue type.

We improve the classical First Mean-value Theorem for Integrals and obtain related results.

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MODIFIED KANTOROVICH-STANCU OPERATORS (II)

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MSC 2010: 41A25, 41A36

Keywords: Approximation by linear operators, Kantorovich-Stancu operators.

In this paper we introduce a new kind of Bernstein-Kantorovich-Stancu operators. This operators generalizes the operators introduced in the article *Modified Kantorovich operators with better approximation properties* by V. Gupta, G. Tachev, A. M. Acu, published online 18 may 2018 in Springer, Numer. Algor, https://doi.org/10.1007/s11075-018-0538-7.

ULAM CONSTANTS FOR FUNCTIONAL EQUATIONS

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MSC 2010: 39B82

Keywords: Cauchy, Jensen, Quadratic equations, Hyers-Ulam stability, best constant.

We investigate the best Ulam constants for the Cauchy, Jensen and Quadratic functional equations. Our results are related to those presented in [1, Section 4.2].

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SCHOENBERG SPLINES WITH KNOTS AT THE ROOTS OF CHEBYSHEV POLYNOMIALS

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MSC 2010: 41A36

Keywords: Spline functions, Schoenberg operators, degree of approximation.

The paper studies the degree of approximation of continuous functions using the Schoenberg type operators with knots at the roots of Chebyshev polynomials.

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NATURAL CONVECTION FROM A VERTICAL PLATE EMBEDDED IN A NON-DARCY BIDISPERSE POROUS MEDIUM

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MSC 2010: 76S99, 76R10, 76M25

Keywords: Bidisperse porous medium, free convection, boundary layer, non-Darcy flow.

A mathematical model for the free convection from a vertical plate placed in a non-Darcy BDPM (bidisperse porous medium) containing inertial terms is proposed. The influence of the inertial parameters on the fluid and heat transfer characteristics is studied. The partial differential equations governing the flow and heat transfer in the f-phase and the p-phase are solved numerically using the bvp4c routine from Matlab.

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ON A PARABOLIC PDE

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MSC 2010: 35K20, 34B05, 65M12

Keywords: Parabolic equation and systems, boundary value problems, numerical analysis.

Consider the following parabolic PDE

$$\frac{\partial P}{\partial t} = A\frac{\partial P}{\partial x}\left(P\frac{\partial P}{\partial x}\right),$$

with initial condition

$$P(x,0) = P_0$$

and boundary condition

$$P(0,t) = P_1,\ \text{where } \alpha = 1 - \frac{P_0^2}{P_1^2}.$$

The origin of this problem is in the study of a gas flow through a semi-infinite porous medium (see [1]).

We present two solution for this problem: by reduction to a BVP for ODE and by using line method.

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ON THE BEST ULAM CONSTANT OF THE LINEAR DIFFERENTIAL OPERATOR

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MSC 2010: 34D20, 39B82

Keywords: Ulam stability, Best Ulam constant, linear differential operator.

Let X be a Banach space over $\mathbb{C}$, $a_1, a_2, \ldots, a_n \in \mathbb{C}$ and $D : \mathcal{C}^n(\mathbb{R}, X) \to \mathcal{C}(\mathbb{R}, X)$ given by the relation

$$Dy = y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y, y \in \mathcal{C}^n(\mathbb{R}, X). \qquad (1)$$

Define $\| \cdot \|_\infty : \mathcal{C}^n(\mathbb{R}, X) \to \overline{\mathbb{R}}$ by

$$\|y\|_\infty = \sup_{x \in \mathbb{R}} \|y(x)\|. \qquad (2)$$

The operator D is called Ulam stable if there exists $K \geq 0$ such that for every $\varepsilon > 0$ and every $y \in \mathcal{C}^n(\mathbb{R}, X)$ with

$$\|Dy\|_\infty \leq \varepsilon \qquad (3)$$

there exists $y_0 \in \ker D$ such that

$$\|y - y_0\|_\infty \leq K\varepsilon. \tag{4}$$

Denote by K_D the infimum of all constants K satisfying (4). If K_D is an Ulam constant of D, it is called the best Ulam constant or the Ulam constant of D. In this paper we are looking for the best Ulam constant of D.

EXPLICIT ALGEBRAIC SOLUTION TO ZOLOTAREV'S "FIRST PROBLEM" FOR POLYNOMIALS OF DEGREE $n \in \{6, 7, 8\}$

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MSC 2010: 41A10, 41A29, 41A50

Keywords: Algebraic solution, approximation, explicit, first problem, least deviation from zero, polynomial, proper, Zolotarev.

For Zolotarev's so-called "First Problem", see [1], we provide an explicit algebraic solution for the polynomial degree $n \in \{6, 7, 8\}$. We have announced this result on a poster presented at the Poster Session of the IX Jaen Conference on Approximation Theory, see pp. 59-60 at URL http://www.ujaen.es/revista/jja/jca/principal.pdf

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APPROXIMATION BY MODIFIED U_n^ρ OPERATORS

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MSC 2010: 41A10, 41A25, 41A36

Keywords: λ-Bernstein operators, U_n^ρ operators, moduli of continuity, rate of convergence, Voronovskaya type theorem.

The U_n^ρ operators was introduced in 2007 by R. Păltănea and since then, many researchers extend the study of them in vary directions. In our paper we consider a generalization of these operators using a new Bezier bases $\tilde{b}_{n,k}(\lambda, x)$ with shape parameter λ. Some approximation properties are given, including local approximation, error estimation in terms of moduli of continuity and Voronovskaja-type asymptotic formulas.

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BOUNDS AND CLOSED FORMS FOR SPECIAL FUNCTIONS

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MSC 2010: 33E30, 33C05, 94A17

Keywords: Heun function, entropy, hypergeometric function.

We obtain new bounds and closed forms for some hypergeometric/Heun functions. The results are related to those presented in [1] and [2]. We also recall the following
CONJECTURE (I. Raşa, University of Bielsko-Biala, November 15, 2017)
$f \in C[0,1]$ log-concave $\Rightarrow$

$$\sum_{\substack{i+j=h\\0\le i\le n-1\\0\le j\le n}} \binom{n-1}{i}\binom{n}{j}\Big((n-1-i)f\left(\tfrac{j}{n}\right)\Delta^2_{\frac{1}{n}}f\left(\tfrac{i}{n}\right)-$$

$$-(n-j)\Delta^1_{\frac{1}{n}}f\left(\tfrac{i}{n}\right)\Delta^1_{\frac{1}{n}}f\left(\tfrac{j}{n}\right)\Big)\le 0,$$

for all $n\ge 1,\ h\in\{0,1,\ldots,2n-2\}$.

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BETTER CONVERGENCE PROPERTIES OF CERTAIN POSITIVE LINEAR OPERATORS

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MSC 2010: 41A10, 41A25, 41A36.

Keywords: Linear positive operators, Bernstein operators.

In this paper there are presented the modified positive linear operators which depend on a certain function ϱ defined on $[0, 1]$, and they have a better degree of approximation than classical ones. To illustrate this, some examples are given for Bernstein operators, Lupas operators and genuine Bernstein-Durrmeyer operators.

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NUMERICAL OPTIMAL CONTROL FOR SATELLITE ATTITUDE PROFILES

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MSC 2010: 49J15, 90C20, 90C30

Keywords: Satellite attitude profile, slew, optimal control, numerical simulation, NMPC.

Many modern science satellites are 3-axis stabilized. Attitude profiles play a central role in satellite control. Besides the dynamical properties numerous constraints need to be fulfilled. These additional constraints are usually of elementary geometric nature and can often be described as affine-linear functions. Several examples of problems of this type are stated and solutions with different optimal control methods [BH81] and respective numerical simulations will be presented. The generation of valid attitudes has often to consider the alignment of the solar arrays with sun line. In [RYMC04] a generic way for calculating such attitudes is given. By regarding the problem form an 'euler angle point of view', the remaining two degrees of freedom form a flat torus and allow the search for attitudes that obey further geometric constraints. Another way to approach this problem is to use optimal control methods. A method based on a simple cost function is proposed. Further some analytic background for understanding the choice of the cost function is given. Besides the attitude profile also the dynamics has to be considered. Using numerical model predictive control [W09], [D15] an approximative filter to reduce spacecraft rates to a given level and the corresponding numerical results are presented.

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MASTER SLAVE SYNCHRONIZATION FOR STOCHASTIC LATTICE SYSTEMS

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MSC 2010: 60H15

Keywords: Random dynamical system, random inertial manifold, synchronization.

We consider two stochastic lattice systems. The nodes of each of these systems are linearly coupled. In addition there is a coupling for any node between these systems. We state the existence of an invariant random manifold in the phase space such that we have synchronization in the following sense. Any solution trajectory of the systems converges exponentially fast to some some solution trajectory "living" on the random manifold.

APPROXIMATION WITH LOCAL ITERATED FUNCTION SYSTEMS

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MSC 2010: 65D05

Keywords: Fractal functions, attractors, interpolation.

The set of real data can be approximated with fractal interpolation functions. These fractal interpolation functions can be constructed with the so-called iterated function systems. Local iterated function systems are important generalization of iterated function systems. In order to obtain new approximation methods we can combine this methods with the classical interpolation methods. In this article using the fact that graphs of piecewise polynomial functions can be written as the fixed points of local iterated function system we give some delimitation of the error for this new approximation methods.

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STONE-WEIERSTRASS THEOREMS FOR RANDOM FUNCTIONS

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MSC 2010: 41A10, 41A65, 60G99

Keywords: Random function, Stone-Weierstrass theorem, stochastic convergence.

Due to an increased use of random models and an intensive investigation of random ordinary and partial differential equations a more systematic treatment of approximation methods for random functions and random variables in function spaces seems to be appropriate. Basic results in deterministic approximation theory are related to different versions of Stone-Weierstrass theorems. In the talk some stochastic versions of Stone-Weierstrass theorems are presented, generalizing corresponding results of, e.g., [1] and [2].

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A HEURISTIC INTRODUCTION TO NUMERICAL MODELING OF DIFFUSION PROCESSES

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MSC 2010: 60

Keywords: Diffusion, independent identically distributed (i.i.d.) random variables.

Sums of sequences of independent identically distributed (i.i.d.) random variables can be formulated by the Law of Large Numbers or the Central Limit Theorem as diffusion processes. This property of the i.i.d. random variables, which lies at the core of numerical modeling of diffusion processes, will be illustrated by generating various diffusion processes and further used to model diffusion at both microscopic and macroscopic level by numerical solutions of Ito and Fokker-Planck equations, respectively.

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QUANTITATIVE ESTIMATES FOR q-BERNSTEIN OPERATORS IN MOBILE INTERVALS

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MSC 2010: 41A25, 41A36

Keywords: q-Bernstein operators, mobile interval.

In this paper we introduce q-Bernstein type operators which is an extension of classical case since they convert q-Bernstein operators if n is sufficiently large. We also study some approximation properties and obtain direct and inverse theorems.
This work was supported by the Ahi Evran University Scientific Research Projects Coordination Unit. Project Number: FEF.A4.18.004

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AN APPLICATION OF INVERSE PADÉ INTERPOLATION

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MSC 2010: 65H05,65Y99, 65-0

Keywords: Nonlinear equations, Padé approximation, Maple.

We use inverse Padé interpolation and Maple to find the following fourth order method for the solution of a scalar nonlinear equation

$$\Phi(x) = x - \frac{f(x)}{f'(x)} \left\{ 1 + \frac{\frac{1}{2}}{\frac{f'(x)}{f''(x)} \left[\frac{f'(x)}{f(x)} + \frac{f'''(x)}{3f'(x)} \right] - 1} \right\}.$$

Also a result on convergence and a MATLAB implementation are given.

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SOME APPROXIMATION PROPERTIES OF GENERALIZED MEYER-KÖNIG AND ZELLER OPERATORS VIA ABEL CONVERGENCE

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MSC 2010: 41A36, 40A35, 40G10

Keywords: Abel convergence, Korovkin type theorem, modulus of continuity.

Korovkin type approximation theory mainly deals with the convergence of a sequence of positive linear operators defined on the space of real-valued continuous functions on a closed interval. If the ordinary convergence does not work then we may consider a summability method that is consistent with and stronger than ordinary convergence. In this talk, using the Abel convergence method we study some approximation properties of generalized Meyer-König and Zeller operators. Moreover, we obtain the rate of convergence by means of modulus of continuity.

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POSSIBLY INFINITE GENERALIZED ITERATED FUNCTION SYSTEMS COMPRISING φ -MAX CONTRACTIONS

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Keywords: Possibly infinite generalized iterated function system, φ-max contraction, attractor, canonical projection.

One way to generalize the concept of iterated function system was proposed by R. Miculescu and A. Mihail (see [3] and [4]) under the name of generalized iterated function system (for short GIFS). More precisely, given $m \in \mathbb{N}^*$ and a metric space (X, d), a generalized iterated function system of order m is a finite family of functions $f_1, ..., f_n : X^m \to X$ satisfying certain contractive conditions. Another generalization of the notion of iterated function system in given by those systems consisting of φ-max contractions (see [1]).

Combining these two lines of research, we prove that the fractal operator associated to a possibly infinite generalized iterated function system comprising φ-max contractions is a Picard operator (whose fixed point is called the attractor of the system). Paper [2] inspired us to associate to each possibly infinite generalized iterated function system comprising φ-max contractions $\mathcal{F}$ (of order m) an operator

$H_{\mathcal{F}} : \mathcal{C}^m \to \mathcal{C}$, where $\mathcal{C}$ stands for the space of continuous and bounded functions from the shift space on the metric space corresponding to the system. We prove that $H_{\mathcal{F}}$ is a Picard operator whose fixed point is the canonical projection associated to $\mathcal{F}$.

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APPROXIMATION BY BIVARIATE NONLINEAR SINGULAR INTEGRAL OPERATORS IN MOBILE RECTANGLE

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MSC 2010: 41A35, 41A25, 47G10

Keywords: μ-generalized Lebesgue point, rate of convergence, Lipschitz condition, nonlinear analysis.

Let $\Lambda \subset [0,\infty;0,\infty]$ be a two-dimensional index set consisting of $\sigma = (\xi,\eta)$ indices with accumulation point $\sigma_0 = (\xi,\eta)$. We assume that the function $K_\sigma : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}$ is Lebesgue integrable on $\mathbb{R}^2$ and satisfies some conditions including Lipschitz condition with respect to its third variable for any $\sigma \in \Lambda$. The main purpose of this work is to investigate the conditions under which Fatou type pointwise convergence is obtained for the operators in the following setting:

$$(T_\sigma f)(x,y) = \int_{a-\xi}^{b+\xi}\int_{c-\eta}^{d+\eta} K_\sigma(t-x, s-y, f(t,s))\, ds dt, \quad (x,y) \in \mathbf{R},$$

where a, b, c and d are arbitrary real numbers and $\mathbf{R} = (a-\xi, b+\xi; c-\eta, d+\eta)$ is a mobile rectangle, at μ-generalized Lebesgue point of measurable function $f \in L_1^\omega(\mathbf{R})$, as $(x, y, \sigma) \to (x_0, y_0, \sigma_0)$. Here, $L_1^{\omega}(\mathbf{R})$ is the space of all measurable functions f for which $\left|\frac{f}{\omega}\right|$ is integrable over $\mathbf{R}$, where $\omega : \mathbb{R}^2 \to \mathbb{R}^+$ is a weight function satisfying some extra conditions. The obtained results are used for presenting some theorems for the rate of convergences.

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EXPLICIT ALGEBRAIC SOLUTION OF THE ZOLOTAREV PROBLEM VIA GROEBNER BASIS

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Keywords: Differential equation, equioscillation, equiripple, explicit form, Groebner basis, least deviation from zero, Mathematica, polynomial, proper, symbolic computation, Zolotarev.

The exact general solution to Zolotarev's First Problem was known only in terms of elliptic integrals and functions [1]. Interests in giving explicit algebraic decriptions to small degree Zolotarev polynomials revived in some recent work [2, 3]. This talk investigates the power of symbolic computational tools (Groebner basis and quantifier elimination) to solve the problem. It turns out that generic polynomial expressions of a symbolic parameter can be obtained via Groebner basis method using the defining differential equation of the proper Zolotarev polynomials at least up to degree eight. The knowledge of explicit algebraic solutions can be used for solving varoius related computational problems in approximation theory. Most of the computation is done with aid of the Wolfram Mathematica computer algebra system.

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FINITE DIFFERENCE SCHEME FOR THE HIGH ORDER NONLINEAR SCHRÖDINGER EQUATION WITH LOCALIZED DISSIPATION.

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Keywords: HNLS, damping, dissipation, finite difference method.

A Finite Difference scheme for the High Order Nonlinear Schrödinger (HNLS) equation in 1D, with localized damping, will be presented. The equation can model superluminal optical solitons in atomic systems, as well as femtosecond solitons travelling in optical fibers made by different materials. The method can preserve the numerical energy, and can almost preserve the numerical charge. Numerical results will be shown.

This is a joint work with Marcelo M. Cavalcanti (UEM-Brazil), Wellington J. Corrêa (UTFPR-Brazil), and Mauricio Sepúlveda C. (CI^2MA - DIM, UdeC, Chile).

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NON-ALMOST PERIODIC SOLUTIONS OF DIFFERENCE SYSTEMS VIA ITERATIVE METHODS

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MSC 2010: 12H10, 39A06, 39A24, 42A75, 43A60

Keywords: Limit periodicity, almost periodicity, limit periodic sequences, asymptotically almost periodic solutions, difference equations.

This talk is based on the joint works [1, 2, 3] with P. Hasil. Using iterative methods, we study the solution spaces of limit periodic homogeneous linear difference systems, where the coefficient matrices of the considered systems are taken from a given commutative group which does not need to be bounded. In particular, we study such systems whose fundamental matrices are not asymptotically almost periodic or which have solutions vanishing at infinity. We identify conditions on the matrix group which guarantee that these systems form a dense subset in the space of all considered systems. Note that the elements of the coefficient matrices are taken from an infinite field with an absolute value and that the corresponding almost periodic case is treated as well.

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OPTIMIZATION ALGORITHMS BASED ON OPERATOR SPLITTING AND ENERGY-PRESERVING NUMERICAL INTEGRATORS

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MSC 2010: 65J08, 90C59, 90C30

Keywords: Optimization algorithm, dynamical system, Strang splitting, conservative numerical method.

The present contribution deals with nonconvex optimization from a dynamical systems perspective. More precisely, we consider the second order evolution equation

$$\ddot{u} + \dot{u} = -\nabla G\,(u) \tag{1}$$

which can be understood (cf. [1]) as describing a damped mechanical system with the nonconvex objective function $G : \mathbb{R}^n \to \mathbb{R}$ as the system's potential energy. Its asymptotic behavior has been analyzed quite recently by Bégout et. al. who show the convergence of solutions, at precise rates, to minimizers of G.

A similar approach applies to the vanishing damping model

$$\ddot{u} + \frac{\gamma}{t}\dot{u} = -\nabla G\left(u\right), \tag{2}$$

which is the continuous counterpart of the celebrated Nesterov optimization algorithm (cf. [5]).

In this context, our aim is to investigate discrete dynamical systems which faithfully reproduce the properties of (1) or (2). These discrete dynamical systems are obtained by means of a two-stage discretization of (1) or (2), consisting in a Strang splitting semidiscretization that separates the conservative and dissipative parts of the system followed by an energy-preserving numerical integration of the conservative part.

The Strang splitting approximation (see [2] and references therein)

$$e^{h(A+B)} \approx e^{\frac{h}{2}A}e^{hB}e^{\frac{h}{2}A}, \quad e^{t(A+B)} \approx \left(e^{\frac{h}{2}A}e^{hB}e^{\frac{h}{2}A}\right)^n, \ t = nh$$

is known to have an improved order of approximation when compared to the simpler Lie spitting

$$e^{h(A+B)} \approx e^{hA}e^{hB}, \quad e^{t(A+B)} \approx \left(e^{hA}e^{hB}\right)^n, \quad t = nh,$$

as both are dealing with the linear problem $\dot{z} = Az + Bz$ by iteratively solving two subproblems: $\dot{z} = Az$ and $\dot{z} = Bz$. Recent progress [3], however, has shown that the semilinear problem $\dot{z} = Az + f\left(z\right)$ can be treated in exactly the same manner by only replacing e^{hB} with the solution operator (nonlinear semigroup) $\mathcal{U}\left(h\right)$ of $\dot{z} = f\left(z\right)$.

The evolution equations (1), (2) both fit nicely in this semilinear framework if rewritten as systems of first order equations

$$\left\{ \begin{array}{rcl} \dot{u} & = & v, \\ \dot{v} & = & -c(t)v - \nabla G\left(u\right), \end{array} \right. \tag{3}$$

with either $c(t) = 1$ or $c(t) = \gamma/t$. Furthermore, the splitting method can be applied in such a way that the conservative and dissipative parts of the system are separated, the subproblems that need to be solved iteratively being

$$\text{(Cons)} \left\{ \begin{array}{rcl} \dot{u} & = & v, \\ \dot{v} & = & -\nabla G\left(u\right) \end{array} \right. \quad \text{and} \quad \text{(Dissip)} \left\{ \begin{array}{rcl} \dot{u} & = & 0, \\ \dot{v} & = & -c(t)v. \end{array} \right.$$

The resulting semidiscretization generates a discrete dynamical system which has

(i) the same equilibria $(u^*, 0)$, with $\nabla G\left(u^*\right) = 0$, as (3);

(ii) the same Lyapunov function $H\left(u, v\right) = \frac{1}{2}\left\|v\right\|^2 + G\left(u\right)$ as (3);

(iii) an $O\left(h^2\right)$ accuracy on finite time intervals.

While the dissipative subproblem is explicitly solvable, the conservative one can be integrated using an energy-preserving numerical scheme (see [4]) such that all the properties (i), (ii) and (iii) remain valid.

Further, the infinite-dimensional case is also discussed.

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DUAL BERNSTEIN POLYNOMIALS: NEW PROPERTIES AND FAST COMPUTATION

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MSC 2010: 33C45, 65Q30

Keywords: Differential equations, recurrence relations, Bernstein basis polynomials, dual Bernstein polynomials, Jacobi polynomials, Hahn polynomials.

Dual Bernstein polynomials introduced in [2], which are associated with the shifted Jacobi inner product, have recently found many applications in numerical analysis and computer graphics (curve intersection using Bézier clipping, degree reduction and merging of Bézier curves, polynomial approximation of rational Bézier curves, numerical solving of boundary value problems, numerical solving of fractional partial differential equations, etc.). Note that skillful use of these polynomials often results in less costly algorithms which solve some computational problems.

In the talk, we give new differential-recurrence properties of dual Bernstein polynomials which follow from relations between dual Bernstein and orthogonal Hahn and Jacobi polynomials. Using these results, the fourth-order differential equation satisfied by dual Bernstein polynomials has been constructed. Next, we obtain a fourth-order recurrence relation for these polynomials. The latter result may be useful in fast evaluation of dual Bernstein polynomials and their linear combinations or integrals which are related to the least-square approximation in Bézier form. See also [1].

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HIGH-FREQUENCY EXPERT OPINIONS AND POWER UTILITY MAXIMIZATION IN A MARKET WITH GAUSSIAN DRIFT

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We consider a continuous-time financial market with partial information on the drift and solve utility maximization problems which include expert opinions on the unobservable drift. Stock returns are driven by a Brownian motion and the drift depends on a factor process which is an Ornstein Uhlenbeck process. Thus the drift is hidden and has to be estimated from observable quantities. If the investor only observes stock prices then the best estimate is the Kalman filter.

However, to improve the estimate, an investor may also rely on expert opinions providing a noisy estimate of the current state of the drift. This reduces the variance of the filter and thus improves expected utility. That procedure can be seen as a continuous-time version of the classical Black-Litterman approach.

For the associated portfolio problem with logarithmic utility explicit solutions are available in the literature. In this talk we consider the case of power utility. Here, we apply dynamic programming techniques and solve the corresponding dynamic programming equation for the value function. In particular we investigate the asymptotic behavior of the filter for high-frequency experts and derive limit theorems for two different asymptotic regimes. In the first, variances of the expert opinions grow linearly with the arrival frequency while in the second they are constant. The derived limit theorems allow for simplified approximate solutions of utility maximization problems since the convergence of the filter carries over to the convergence of the value function. Numerical results are presented.

The talk is based on joint work with A. Gabih, H. Kondakji, J. Sass and D. Westphal.

KOROVKIN TYPE APPROXIMATION THEOREMS IN WEIGHTED SPACES VIA POWER SERIES METHOD

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MSC 2010: 41A36, 41A25

Keywords: Power series method, Korovkin type theorem, weighted space, rates of convergence.

In this paper we consider power series method which is also member of the class of all continuous summability methods. We study a Korovkin type approximation theorem for a sequence of positive linear operators acting from a weighted space C_{ρ_1} into a weighted space B_{ρ_2} with the use of the power series method which includes both Abel and Borel methods. We also consider the rates of convergence of these operators.
This paper was supported by the department of Scientific Research Projects of Hitit University. Project No: 19008

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