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# DETERMINISTIC COMPONENTS IN THE LIGHT CURVE AMPLITUDE OF Y OPH

Alexandru Pop[1], Călin Vamoş[2], and Vlad Turcu[1]
[1] Astronomical Institute of the Romanian Academy, Astronomical Observatory Cluj-Napoca, Str. Cireşilor 19, 400487 Cluj-Napoca, Romania; andi_pop@yahoo.com, vladturcu@yahoo.com
[2] "T. Popoviciu" Institute of Numerical Analysis, Romanian Academy, P.O. Box 68, 400110 Cluj-Napoca, Romania; cvamos@ictp.acad.ro
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## ABSTRACT

About two decades after the discovery of the amplitude decline of the light curve of the classical Cepheid Y Oph, its study is resumed using an increased amount of homogenized data and an extended time base. In our approach, the investigation of different time series concerning the light curve amplitude of Y Oph is not only the reason for the present study, but also a stimulus for developing a coherent methodology for studying long- and short-term variability phenomena in variable stars, taking into account the details of concrete observing conditions: amount of data, data sampling, time base, and individual errors of observational data. The statistical significance of this decreasing trend was estimated by assuming its linearity. We approached the decision-making process by formulating adequate null and alternative hypotheses, and testing the value of the regression line slope for different data sets via Monte Carlo simulations. A variability analysis, through various methods, of the original data and of the residuals obtained after removing the linear trend was performed. We also proposed a new statistical test, based on amplitude spectrum analysis and Monte Carlo simulations, intended to evaluate how detectible is a given (linear) trend in well-defined observing conditions: the trend detection probability. The main conclusion of our study on Y Oph is that, even if the false alarm probability is low enough to consider the decreasing trend to be statistically significant, the available data do not allow us to obtain a reasonably powerful test. We are able to confirm the light curve amplitude decline, and the order of magnitude of its slope with a better statistical substantiation. According to the obtained values of the trend detection probability, it seems that the trend we are dealing with is marked by a low detectibility. Our attempt to find signs of possible variability phenomena at shorter timescales ended by emphasizing the relative constancy of our data, within their precision limits.

*Key words:* Cepheids – methods: data analysis – methods: statistical – stars: individual (Y Oph)

## 1. INTRODUCTION

Y Ophiuchi (HIP 87495) is a $\delta$ Cephei type pulsating star—a classical Cepheid—having a pulsation period of about 17 days. There are several reasons justifying the interest in this variable star, which emphasize the intriguing character of Y Oph. Although it is a low amplitude Cepheid with a quasi-symmetric light curve, its period is considerably longer than expected (Mérand et al. 2007; see, also, e.g., Lloyd et al. 1987). According to Abt & Levy (1978), Y Oph could be a spectroscopic binary system having an orbital period of $P_{\rm orb} = 2612$ days and an eccentric orbit ($e = 0.60$). Because these authors considered the binarity evidence to be only marginal, the problem of binarity confirmation has remained unsolved. The radial velocity (RV) observations performed by Evans & Lyons (1986) could not supply a confirmation of the orbital period variation reported by Abt & Levy (1978). Usenko (2005) analyzed 299 RV data on Y Oph and found evidence for multiperiodicity and for the presence of at least one companion. According to the recent near-infrared interferometric studies (Mérand et al. 2007), Y Oph is surrounded by a circumstellar envelope.

From the photometric point of view, the interest in the study of Y Oph is related to the decrease of the full amplitude of its light curve during the last century, revealed by Fernie (1990) (see also Fernie et al. 1995). In both quoted papers, we are warned about problems with older data (before 1940). The main problem is related to the effective wavelength at which observations were performed. In addition, their errors are one or two orders of magnitude larger than those associated with photometric photoelectric data after 1940. Another possible pathology could also arise from the sampling features of old data sets. Concerning the reality of this amplitude trend, Fernie et al. (1995) emphasized that it had the same sign and order of magnitude in both $B$ and $V$ bandpasses.

Recently, Pop (2007b) performed an analysis of 33 data sets collected from the literature (see Table 1) grouped together in 27 data sets in order to obtain data sets that are as short as possible, having the best possible phase coverage. The application of the Fourier decomposition technique revealed at first glance some light curve shape variability phenomena occurring at all phases of the pulsation cycle. Besides possible real cycle-to-cycle shape variations of the light curve, another spurious cause—at least in some situations—could be the interplay between the presence of phase gaps and the adopted number of harmonics in the Fourier decomposition.

The present study approaches the investigation of the temporal variability of the light curve amplitude of Y Oph relying on the analysis or reanalysis of 27 photoelectric data sets on this star (25 in the Johnson $V$ filter, one in the Carlsberg Automatic Meridian Circle (CAMC) natural system magnitudes, and one in the $H_P$ passband), covering a time base $\Delta T$ of about 51.6 years (see Pop 2007b and Table 1 below). The transformation relations from the CAMC natural system and $H_p$ magnitudes system to Johnson $V$ magnitudes (Lloyd et al. 1987, Harmanec 1998), which depend on $B - V$ and $U - B$ catalog values, do not influence $V$ amplitude determinations for Y Oph. Two other photoelectric data sets were (re)analyzed, one of them being previously considered by Fernie (1990). We also took into account seven amplitude data converted by Fernie et al. (1995) from RV data (see their Table 4). In addition, we performed a revision of the old data, obtained before 1940, quoted by Fernie (1990),which enabled us to further extend the time base of our data set to about 113.6 years.

**Table 1**
Analyzed, or Reanalyzed Photometric Photoelectric Data (pe) Together with the Sources for the Original Data Sets According to Pop (2007b)

| $t$ (HJD – 2400000) | $A$ (mag) $\sigma_A$ (mag) | $\Delta T$ (d) | $N$ | Reference |
|---|---|---|---|---|
| 32965.6820 | 0.5365 ± 0.0096 | 387.9760 | 33 | Mitchell et al. (1964) **(Eggen 1951)** |
| 34937.4800 | 0.507 ± 0.014 | 745.9120 | 20 | Mitchell et al. (1964)[**Walraven et al. (1958)** + **Irwin (1961)**] |
| 36203.5000 | 0.449 ± 0.030 | ? | ? | Svolopoulos (1960) (present paper) |
| 37269.8420 | 0.4678 ± 0.0053 | 507.6960 | 23 | **Mitchell et al. (1964)** |
| 38944.2683 | 0.431 ± 0.018 | 866.6697 | 29 | **Wisniewski & Johnson (1968)** |
| 39762.9120 | 0.5318 ± 0.0045 | 170.6749 | 35 | **Pel (1976)** |
| 39888.0705 | 0.457 ± 0.014 | 425.8070 | 23 | Schmidt (1971) (present paper) |
| 40776.2204 | 0.543 ± 0.017 | 1474.9894 | 43 | **Feltz & McNamara (1980)** |
| 42939.4360 | 0.499 ± 0.018 | 177.6240 | 14 | **Dean (1977)** |
| 43649.3593 | 0.4804 ± 0.0078 | 800.9340 | 39 | **Moffett & Barnes (1980)** |
| 44103.6270 | 0.464 ± 0.019 | 767.9200 | 17 | **Eggen (1983)** |
| 44545.5635 | 0.4963 ± 0.0031 | 1221.8610 | 34 | **Coulson & Caldwell (1985)** |
| 46292.1502 | 0.4807 ± 0.0059 | 23.9753 | 23 | Berdnikov (1987) |
| 46600.0575 | 0.512 ± 0.042 | 78.7850 | 34 | Lloyd et al. (1987) |
| 46805.7161 | 0.509 ± 0.012 | 398.9250 | 41 | **Berdnikov (1992a)** + Berdnikov (1992b) |
| 47231.0385 | 0.497 ± 0.017 | 414.9270 | 16 | Fernie (1990) |
| 47416.6330 | 0.4922 ± 0.0088 | 34.9549 | 30 | Berdnikov (1992c) |
| 47755.6816 | 0.492 ± 0.013 | 40.9072 | 42 | Berdnikov (1992d) |
| 48312.1561 | 0.453 ± 0.018 | 421.9228 | 38 | Berdnikov (1992e) + **Berdnikov (1992f)** |
| 48511.0634 | 0.4678 ± 0.0025 | 1100.9343 | 56 | ESA (1997) |
| 48882.1344 | 0.451 ± 0.016 | 23.9535 | 19 | Berdnikov (1993) |
| 49112.9463 | 0.484 ± 0.012 | 803.9172 | 32 | Arellano-Ferro et al. (1998) |
| 49163.7355 | 0.4787 ± 0.0050 | 123.7050 | 61 | **Fernie et al. (1995)** (**APT 1993** + DDO 1993) |
| 49723.0207 | 0.4716 ± 0.0070 | 205.7797 | 25 | Berdnikov & Vozyakova (1995) + Berdnikov & Turner (1995) |
| 49977.1384 | 0.451 ± 0.030 | 85.8625 | 26 | Berdnikov et al. (1997) |
| 50444.9254 | 0.4826 ± 0.0065 | 279.4887 | 54 | Berdnikov et al. (1998) + Berdnikov & Turner (1998) |
| 50904.1505 | 0.4897 ± 0.0049 | 25.0211 | 29 | Berdnikov & Turner (2000) |
| 51267.6019 | 0.4653 ± 0.0093 | 37.9715 | 28 | Berdnikov & Turner (2001) |
| 51828.2859 | 0.4669 ± 0.0076 | 427.8846 | 39 | Ignatova & Vozyakova (2000) |

**Notes.** We also included two additional amplitude values estimated within the present study. The data previously taken into account by Fernie et al. (1995) are marked with bold fonts.

Beyond the interest in detecting new phenomena, there are always constraints imposed by the quality of the available data, which could affect the quantitative aspects of the respective investigation. In the case of Y Oph, Pop (2007b) found that the number of data in the 27 analyzed data sets varies between 14 and 61 data points, covering time bases from about 24 days to 4 years, having quite low signal-to-noise ratio (S/N) values, in the range 4.9–42. Moreover, the standard errors of the light curve amplitude vary between 2.5 mmag and 42 mmag. That is why the main goal of our approach is to reevaluate the light curve amplitude decrease phenomenon, and to perform a rigorous statistical analysis by taking into account the involved errors. In the decision-making process, we considered either the null hypothesis only (the presence of Gaussian noise), or this null hypothesis with the alternative one (the presence of the observed trend). We also focused on the search for possible low level modulation of the pulsation amplitude caused by the presence of the hypothetical unseen companion and under the circumstances of the high eccentricity of the binary orbit as inferred by Abt & Levy (1978). Within the frame of amplitude spectrum analysis, we proposed a Monte Carlo approach to the estimation of the detection probability of a linear trend. According to its formulation, it supplied us a statistical test with a high degree of specificity.

## 2. OBSERVATIONAL DATA

Our approach of primary observational data (light curves) on Y Oph was focused on the minimization of the two main types of errors encountered in analyzing data obtained from different sources: conformity and reduction errors (Sterken & Manfroid 1992). The conformity errors are related to differences between standard $V$ Johnson and the other photometric systems used. In our case, the reduction errors resulted from particular methods used by different authors to estimate the light curve amplitude.

The kernel of our amplitude data collection on Y Oph was the 27 photoelectric data sets previously analyzed by Pop (2007b) (see Table 1 of the present paper). In that study, for each data set we performed the Fourier decomposition technique based on nonlinear least squares fitting with a rigorous choice of the considered harmonics number. In the choice of the number of periodic terms in the truncated Fourier series

$$m(t) = m_0 + \sum_{k=1}^{K} A_k \cos(2\pi k f t + \varphi_k),$$

we have taken into account the minimum sampling interval in the phase diagram computed for the pulsation frequency, and the statistical significance of the models' parameters (amplitudes and phases) (see Pop et al. 2004, and references therein). In the above equation, $m_0$ is the zero point of the light curve, $A_k$ and $\varphi_k$ are the amplitude and the phase of the $k^{\text{th}}$ periodic term in the Fourier decomposition, while $f$ is the pulsation frequency. We also have to emphasize that some data sets were split in order to obtain shorter data sets but still have good phase coverage. The light curve full amplitude $A$ was then computed, and the corresponding standard error $\sigma_A$ was estimated using the method given by Rodríguez (1999). The homogenization of the amplitude estimation led to minimization of the reduction errors. The 27 sets of Fourier coefficients of the Y Oph light curve together with different types of light curve structural parameters are given in the previous paper of Pop (2007b). In Table 1, we displayed the 27 values of the light curve amplitude with their standard errors according to Pop (2007b). For each data set, we also gave its time base and the corresponding number of data points. In addition, we included two other amplitude estimates obtained by us using the above methodology on the basis of the data sets published by Svolopoulos (1960) and Schmidt (1971). For the amplitude value determined from the first data set, which consists of a mean light curve, we adopted the standard error previously given by Fernie (1990). Concerning the 29 light curve amplitude values given in Table 1, according to the above quoted paper, we have to make the following remarks: (1) there are 19 new analyzed data sets, (2) we used 14 new amplitude values, and (3) 15 amplitude values were redetermined, and for three of them, extended data sets with better phase coverage were used.

As we already mentioned in Section 1, we also included seven amplitude points of data converted by Fernie et al. (1995) from RV data (see Table 4 therein). We rejected the amplitude value of 0.663 ± 0.192 mag, because of its very large standard error. Thereafter, we denoted the data set containing photometric photoelectric data only, pe data, and the combined data set, pe + RV data.

Finally, for a better understanding of the behavior of the light curve amplitude of Y Oph, we also reconsidered the data collected by Fernie (1990) before 1940. Concerning these data strongly affected by conformity errors, the following remarks have to be made:

1. Sawyer (1890) and Luizet (1905, 1913) published visual observations made through Argelander's method. We performed a new reduction process using the following procedure. All three data sets were first fitted by us in terms of light scale (maintaining a fixed frequency value). Next we made a correction of comparison star sequences to actual catalog values (common to both authors) for each data set following the directions of Dumont & Gunther (1994). We fit by means of the least squares method: the light step "$u$," the magnitude "$m_c$" of the comparison star "$c$" as it appeared to the observer, for both authors ("$c$" = BD -7°4487 = HIP 86768), and the second order color index correction term "$k_2$" (Hardie 1962). As a result of taking into account the color index terms, the transformation between these corrected "visual magnitudes" and $V$ magnitude values from Simbad databases for comparison stars is nearly a 1:1 correlation. In the third step, we transformed light determinations for Y Oph into standard $V$ magnitudes using values for $u$, $m_c$, and $k_2$ already adjusted. We have to mention that we also analyzed a fourth data set from Sawyer (1892). Due to a lot of discordances regarding comparison stars and also light step readings for Y Oph, the amplitude value determined from this data set was discarded from our final analysis.
2. Pickering (1904) data, which were obtained through a visual photometer, were reanalyzed in the same way as pe data. Due to the appropriate selection of comparison stars, there was no need for a color correction term.
3. The Ten Bruggencate (1931) data sets from Table 3 of Fernie (1990) were discarded because of a different spectral range of observations (monochromatic observations at effective wavelengths of 4458 Å, 4347 Å, 4241 Å, 4140 Å, and 4048 Å).
4. The Robinson (1940) data set consists of photographic plate observations, and was consequently rejected.
5. Krieger (1932) gave spectrophotometric observations at a maximum wavelength of 4800 Å. Because of a very similar band width and effective wavelength compared with those of Becker & Strohmeier (1942), we could extrapolate the amplitude value for 5570 Å, which is nearly the $V$ standard domain.
6. From spectrophotometric observations of Becker & Strohmeier (1942) in three spectral domains (557 nm, 480 nm, and 410 nm), we reanalyzed (in the same way as pe data) the data obtained at the 557 nm wavelength.

**Table 2**
Revised Amplitude Values of Y Oph Before 1940

| $t$ (HJD – 2400000) | $A$ (mag) | $\sigma_A$ (mag) | Reference |
|---|---|---|---|
| 10345.0500 | 0.618 | ±0.071 | Sawyer (1890) |
| 14509.1500 | 0.533 | ±0.076 | Pickering (1904) |
| 15604.1500 | 0.636 | ±0.030 | Luizet (1905) |
| 17576.0000 | 0.662 | ±0.024 | Luizet (1913) |
| 25831.1500 | 0.58 | ±0.060∗ | Krieger (1932) |
| 29264.0000 | 0.59 | ±0.08∗ | Becker & Strohmeier (1942) |

**Note**. The standard errors marked with ∗ are taken from Fernie (1990).

**Table 3**
Values of the Regression Line Parameters for the Four Considered Amplitude Data Sets on Y Oph

| Data Sets | $\Delta T$ (yr) | $N$ | $a \pm \sigma_a$ (mag) | $b \pm \sigma_b$ (mmag yr$^{-1}$) |
|---|---|---|---|---|
| old | 51.8 | 10 | 0.670 ± 0.065 | −0.95 ± 1.27 |
| pe | 51.6 | 29 | 0.594 ± 0.013 | −0.87 ± 0.10 |
| pe + RV | 95.3 | 35 | 0.594 ± 0.013 | −0.87 ± 0.10 |
| old + pe + RV | 113.6 | 42 | 0.624 ± 0.011 | −1.091 ± 0.091 |

The final revised and homogenized data before 1940 on the light curve amplitude values of Y Oph are listed in Table 2. We also took into account two other amplitude data values (Bemporad's and Zverev's) given in Table 3 from Fernie (1990). All amplitude data before 1940 will be hereafter referred to as "old data."

## 3. AMPLITUDE TREND ESTIMATION AND SIGNIFICANCE

The slope of light curve amplitude decreases for each of the four data sets (old, pe, pe + RV, and old + pe + RV) on Y Oph was estimated through weighted linear regression, $A_V(t) = a + bt$, taking into account statistical weights $w$ defined as $w = 1/\sigma_A^2$. The values of the regression line parameters together with some characteristics of the considered data sets are given in Table 3. In the case of the old data the resulting slope value was statistically insignificant, while in the other three cases the slopes proved to be statistically significant with $p$-values lower than $10^{-9}$. We remark that the values for the intercept and slope for the regression line for pe + RV data given in Table 3 are identical within the limits of their standard errors. The temporal behavior of the light curve amplitude as displayed by the pe and old + pe + RV data is displayed in Figure 1, together with the calculated linear trends.

We have to remark that the run of the earlier data with respect to the newer ones displayed by Fernie (1990) in his Figure 6, or by Fernie et al. (1995) in Figure 3 of their paper, might even

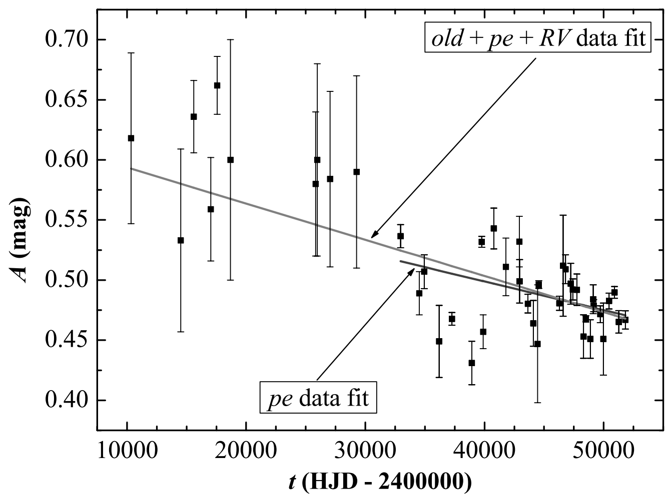


**Figure 1.** All considered data on the light curve amplitude of Y Oph and the pe, together with their weighted linear fits.

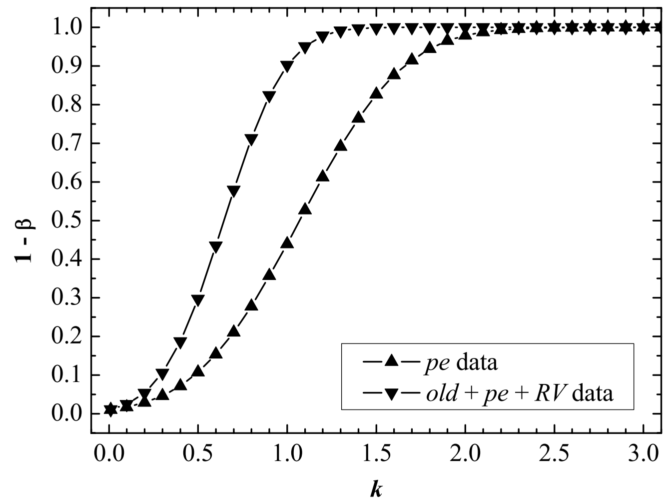


**Figure 2.** Power of the test for pe and old + pe + RV data, assuming a value of the Type I error $\alpha = 0.01$.

suggest a nonlinear trend. The revision of the old data performed by us led to a better fit between the earlier data and the new, photoelectric ones. Thus, the run of the amplitude decline of Y Oph became closer to a linear trend. The overall conclusion is that our estimates of the linear trend slope, obtained using the extended data set, confirm the previous estimates of Fernie et al. (1995): for the analyzed data set we obtained slope values between $-0.87 \pm 0.10$ mmag yr$^{-1}$ and $-1.091 \pm 0.091$ mmag yr$^{-1}$.

In order to go deeply into the evaluation of the statistical significance of the amplitude decreasing trend of Y Oph, we built up statistical tests which demanded the use of a Monte Carlo method because of the two difficulties concerning our data: (1) the unequal precision of individual data, and (2) the low number of data which does not allow us to apply the conclusions formulated by using asymptotic distributions of different statistical tests on the significance of the regression line coefficients. In the following, we took into account two tests. We shall consider as the test statistic the slope of the regression line. We performed 100,000 numerical experiments in order to obtain the distribution of the test statistic under the considered hypotheses.

### 3.1. First Statistical Test: $H_0$—No Trend

The first test considered a null hypothesis ($H_0$) according to which there is no deterministic component in our data on Y Oph. We have no information about the actual shape of this trend. For the "observed" value of the slope, we can estimate only the Type I error ($\alpha$) associated with the chosen null hypothesis. Obviously, $\alpha$ represents the probability of obtaining, under the null hypothesis, a synthetic time series having a regression line slope greater than the observed one. Here the null hypothesis refers to a noise component consisting of a superposition of two independent and identically distributed Gaussian noises: (1) the intrinsic scatter of the original data, featured by the unweighted standard deviation of the original data (considering the same sampling), and (2) the individual errors of each data point, described by the standard error of each amplitude value. Although the use of the unweighted standard deviation of the input data, instead of the weighted one, may overestimate the real intrinsic scatter of the data, we took it into account as a precautionary measure in our evaluation. Applying this test, we found that in the case of old data the null hypothesis cannot be rejected. For pe data we can reject $H_0$, but only at quite large significance levels of 0.01528, while for the old + pe + RV data, $H_0$ can be rejected at lower significance levels of about 0.00148.

### 3.2. Second Statistical Test: $H_0$—No Linear Trend

In the second test, we shall consider the linear shape of the observed trend in the light curve amplitude of Y Oph as a working hypothesis. Obviously, the linear trend is the simplest possible trend, which does not exhaust the large variety of trends that are able to fit the observational data. The null hypothesis ($H_0$) assumes the lack of any linear deterministic component in the "observed" time series, and the presence of a similar noise, i.e., the slope of the trend is null. We assumed that the intrinsic scatter of the original data is featured in this case by the standard deviation of the residuals of the unweighted linear fit to these data. The second noise component is identical with that in the first test. Then, the alternative hypothesis ($H_1$) is that there exists a linear trend with a given slope $b$ and the same intercept $a$ as estimated from a weighted linear regression ($A_V(t) = a_{\rm obs} + bt$, with the particular case $b = b_{\rm obs}$) to the original data with the noise component of the null hypothesis. Consequently, under these circumstances, for each simulation, and for a given value of the "observed" slope $b$, we estimated the corresponding value of the Type I error ($\alpha$) and that of the associated Type II error ($\beta$) for the chosen pair of null and alternative hypotheses

The power of our test ($1 - \beta$) was estimated by considering different slope values ($A_V(t) = a_{\rm obs} + kb_{\rm obs}t$, $k$ taking integer values) for pe and old + pe + RV data sets, and a significance level $\alpha = 0.01$ (see Figure 2). According to the results presented in this figure, our test is more powerful in the case of old + pe + RV data, than for pe ones. In the case of old data, the power of the test is very low ($1 - \beta = 0.044$). Even in the case of old + pe + RV data, which is the richest data set, with the longest time base, the power of the test is relatively low, i.e., 0.902. This indicates that for the available data the hypothesis of a linear trend cannot be tested at an acceptable value of the significance level.

For the slope values of the pe and old + pe + RV data sets (see the above $A_V(t)$ expression with $k = 1$), we investigated the relation between Type I and II errors, by estimating the value of $\beta$ for different values of $\alpha$ (see Figure 3). We can conclude that either for pe or for old + pe + RV data, there is no reasonable

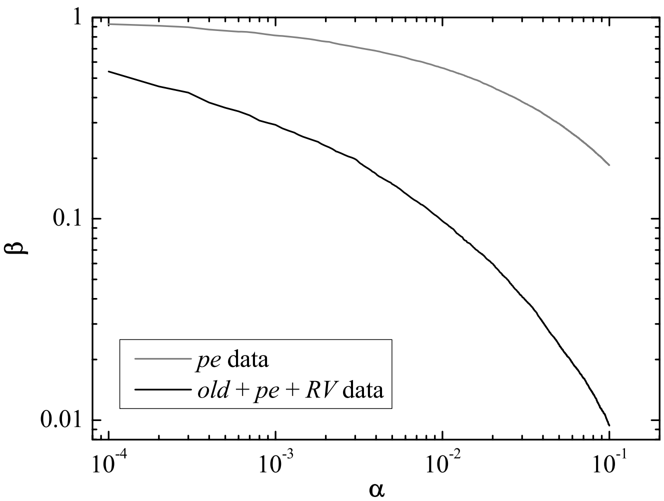


**Figure 3.** Value of the type II error ($\beta$) as a function of that of type I error ($\alpha$), for pe and old + pe + RV data.

low value for the probability of Type I error $\alpha$, for which there corresponds enough low value for the probability of Type II error $\beta$. Thus, we found the trend to be statistically significant at significance levels of 0.01495 for pe data, and 0.00012 for old + pe + RV data, the corresponding $\beta$ values being 0.49860 and 0.50083, respectively. In the case of old + pe + RV amplitude data, an optimal choice, according to Figure 3, is that for a risk $\alpha = 0.0354$ to erroneously accept the presence of a linear trend, the risk to erroneously reject the presence of a linear trend with the above estimated slope value is $\beta = 0.0353$.

## 4. SEARCHING FOR VARIABILITY—CLASSICAL APPROACH

The variability of the Y Oph amplitude before and after removing the linear trend was analyzed through two classical statistical tests. The first step was to investigate the influence of the unequal precision of the amplitude estimates on their apparent variability using the $F$ test (see Pop et al. 2004) in order to compare the weighted and unweighted values of the data variance ($\sigma_w^2$ and $\sigma_{uw}^2$). We applied this test to both the original amplitude data and the corresponding detrended residuals. We computed the values of the statistic $F = \max(\sigma_{uw}^2, \sigma_w^2)/\min(\sigma_{uw}^2, \sigma_w^2)$, and those of the corresponding upper-tail $p$-values of the statistic.

For old and pe data (amplitude values, and residuals obtained after detrending), within the precision limits, and adopting a significance level of 0.01, the apparent variances (quantified by $\sigma_{uw}^2$) are not an effect of the unequal precision of observations (which has been taken into account in $\sigma_w^2$). In the case of pe + RV data, at a significance level lower than 0.007, and for old + pe + RV data, at a significance level of 0.0002, we obtained $\sigma_{uw}^2 > \sigma_w^2$. Thus, we can conclude that in these cases the data weights reduce the data variance, and consequently the apparent variance $\sigma_{uw}^2$ is not entirely real, but is at least partly an effect of the unequal precision of our data. We note that these results are valid only by assuming the Gaussian character of the errors in the analyzed time series.

The second step in order to investigate the amplitude variability was to apply the $\chi^2$ test (e.g., Saha & Hoessel 1990; Caldwell et al. 1991; Saha et al. 1996). For each of the considered data sets we computed the values of the reduced $\chi_v^2$ statistic, where $v = N - 1$ is the number of degrees of freedom, and those of the corresponding upper-tail $p$-value.

Apparently, we may conclude that all the analyzed data sets on the light curve amplitude of Y Oph, except the old ones, display variability phenomena at significant levels. However, it is well known that the application of the $\chi^2$ test assumes the Gaussianity of the input data. Any deviation from this underlying hypothesis could supply false variability detection (e.g., Saha & Hoessel 1990).

## 5. SEARCHING FOR VARIABILITY AND/OR PERIODICITY

A complementary and more focused approach to the general problem of variability detection in the light curve amplitude data on Y Oph was to perform an analysis of the amplitude spectrum of the residuals obtained after detrending, in order to detect the presence of a periodicity, possibly related to a hypothetical orbital motion. For this analysis, we considered the detrended pe, and old + pe + RV data, the first data set having the highest precision and being the most homogeneous from a photometric viewpoint, as well as that of applied mathematical treatment, while the second has the largest time base and is described by the linear trend with the highest statistical significance. The amplitude spectrum of the standardized residuals was computed in the frequency range from the lowest resolvable frequency $f_{\rm low} = 1/\Delta T$ to $f_{\rm high} = f_{Ny} = 1/(2\Delta t_{\rm min})$, with a step $\Delta f \leqslant 1/(20\Delta T)$, where $\Delta T$ is the time base of the data set, and $\Delta t_{\rm min}$ is the minimum sampling interval. The data weights were taken into account with the normalization indicated by Scargle (1989), i.e., $\sum_{j=1}^{N} w_j = N$, where $w_j = (\bar{S}_w\ \sigma_{Aj}^2)^{-1}$, and $\bar{S}_w = (1/N)\sum_{j=1}^{N} \sigma_{Aj}^{-2}$. Thus, for pe residuals we obtained $f_{\rm low} = 5.3 \times 10^{-5}$ cd$^{-1}$, and $f_{\rm high} = 9.845 \times 10^{-3}$ cd$^{-1}$, while for the old + pe + RV residuals $f_{\rm low} = 2.4 \times 10^{-5}$ cd$^{-1}$, and $f_{\rm high} = 3.1574 \times 10^{-2}$ cd$^{-1}$, with $\Delta f = 10^{-6}$ cd$^{-1}$.

Within the considered frequency range, the values of the mean amplitude $(A_{\rm mean})_{\rm obs}$ and the amplitude of the maximum peak $(A_{\rm max})_{\rm obs}$ were determined. The statistical significance of these spectral features was investigated through the method of Pop (2005) derived from that of Kuschnig et al. (1997; see also Pop 2007a). Both of them are Monte Carlo type methods relying on the hypothesis of observational noise Gaussianity (G). In order to remove this restrictive hypothesis, we replaced it by using the bootstrap (B) and permutation (P) tests (see Pop & Vamoş 2007, and references therein). The advantage of this approach is that it allows us to perform a diagnosis of the observed time series with the same computational effort as Kuschnig's et al. (1997) method.

Let us consider a given experiment, and the bivariate distribution of resulted $n_{\rm ex}$ pairs ($A_{\rm mean}, A_{\rm max}$; in the present study we took $n_{\rm ex} = 50{,}000$). Let us denote

1. $n_v$—the number of simulated data for which $A_{\rm mean} > (A_{\rm mean})_{\rm obs}$ and $A_{\rm max} > (A_{\rm max})_{\rm obs}$,
2. $n_p$—the number of simulated data for which $A_{\rm mean} \leqslant (A_{\rm mean})_{\rm obs}$ and $A_{\rm max} > (A_{\rm max})_{\rm obs}$,
3. $n_n$—the number of simulated data for which $A_{\rm mean} > (A_{\rm mean})_{\rm obs}$ and $A_{\rm max} \leqslant (A_{\rm max})_{\rm obs}$,
4. $n_c$—the number of simulated data for which $A_{\rm mean} \leqslant (A_{\rm mean})_{\rm obs}$ and $A_{\rm max} \leqslant (A_{\rm max})_{\rm obs}$.

We define the probabilities of accepting the following hypotheses concerning the observed phenomenon (expressed in

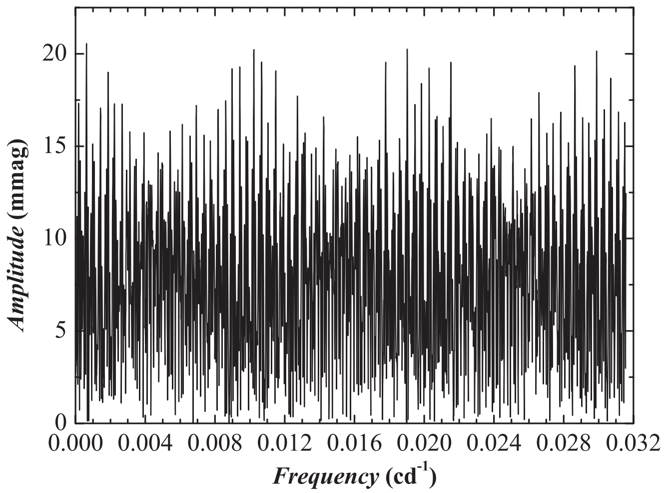


**Figure 4.** Amplitude spectrum of the residuals obtained after removing the linear trend (weighted fit) from the old + pe + RV data set.

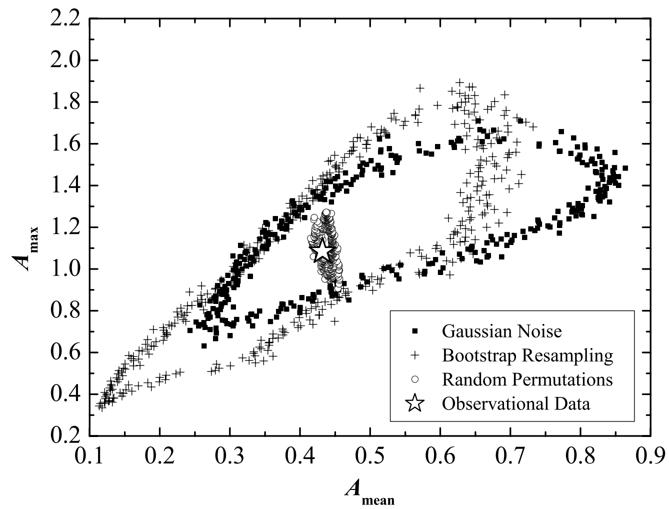


**Figure 5.** Position of the "observed point" ($(A_{mean})_{obs}, (A_{max})_{obs}$) with respect to the clusters of the different simulated data points obtained on the basis of old + pe + RV amplitude residuals (weighted fit).

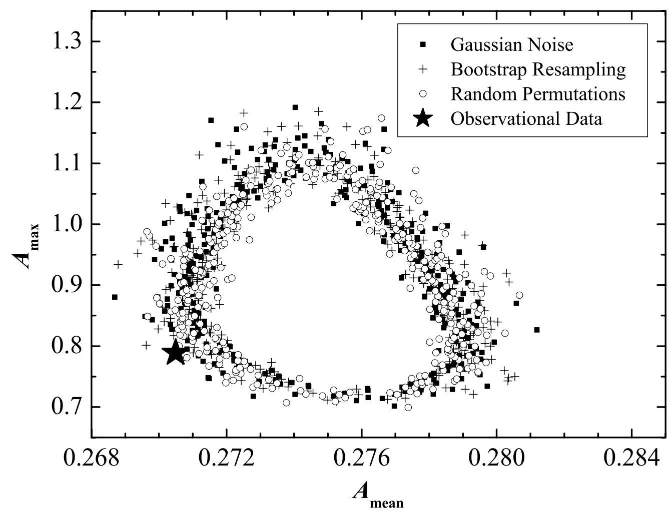


**Figure 6.** Position of the "observed point" ($(A_{mean})_{obs}, (A_{max})_{obs}$) with respect to the clusters of the different simulated data points obtained on the basis of old + pe + RV amplitude residuals (unweighted fit).

percents): constancy, noise, periodicity, variability as

$$P_c = 100 \cdot (1 - n_c/n_{ex}),$$
$$P_n = 100 \cdot (1 - (n_n + n_v)/n_{ex}),$$
$$P_p = 100 \cdot (1 - (n_p + n_v)/n_{ex}),$$
$$P_v = 100 \cdot (1 - (n_p + n_v + n_n)/n_{ex}).$$

The present definition of $P_v$ was modified with respect to that previously given by Pop & Vamoş (2007). Note that $P_c$ and $P_v$ refer to complementary events. These probabilities supply us a simple and intuitive way for diagnosing the analyzed data on the basis of the position of the observed point with respect to the null hypothesis, defined by the cluster of simulated data points through one of the above-mentioned procedures (G, B, or P). Obviously, the decision is only partial: it allows us only to reject a well-defined null hypothesis along a given direction (variability versus constancy, (multi)periodicity versus lack of deterministic components, noisy data versus lack of noise). Thus, a significant value of $A_{mean}$ may be related to a noisy spectrum, a significant value of $A_{max}$ may be caused by an amplitude peak associated with a periodic signal, while simultaneously high values of the two quantities indicate significant contributions of noise and/or (multi)periodicity, defining a variability phenomenon.

The amplitude spectra of the Y Oph amplitude residuals (see Figure 4 for the case of old + pe + RV amplitude residuals) seem to be rather noisy. They do not display any prominent peak. In the low frequency domain, around the value of the hypothetical orbital frequency ($f_{orb} = 0.000383$ cd$^{-1}$), there is no evidence for a modulation of the pulsation amplitude due to the orbital motion. In both cases, the highest peak appeared at close frequencies, of 0.000624, and 0.000627 cd$^{-1}$, i.e. at periods of 4.39 and 4.37 years, respectively.

Figures 5 and 6 display the two sets of contours of the bivariate distributions resulted from Monte Carlo simulations for the old + pe + RV amplitude residuals obtained from weighted and unweighted linear regression. Table 4 summarized the results of the Y Oph amplitude variability diagnosis for the pe (upper row) and old + pe + RV (lower row) residuals, corresponding to the three Monte Carlo simulations using Gaussian noise (G), bootstrap resampling (B), and random permutations (P).

In Figures 5 and 6 and Table 4, we immediately note the strong influence of the unequal observational errors on the variability diagnosis. The use of the statistical weights in the

**Table 4**
Variability Diagnosis for the Two Data Sets of pe (Upper Row) and old + pe + RV (Lower Row) Amplitude Residuals for Weighted and Unweighted (Marked in Italic Fonts) Fits

| Method | $P_c$ (%) | $P_n$ (%) | $P_p$ (%) | $P_v$ (%) |
|---|---|---|---|---|
| G | 74.302 | 32.300 | 33.640 | 25.698 |
| | *99.926* | 3.248 | 7.350 | 0.074 |
| | 84.380 | 21.564 | 19.918 | 15.620 |
| | *100.000* | 0.036 | 5.836 | 0.000 |
| B | 46.344 | 61.026 | 60.648 | 53.656 |
| | *99.918* | 3.364 | 7.116 | 0.082 |
| | 48.414 | 61.572 | 56.734 | 51.586 |
| | *100.000* | 0.040 | 7.576 | 0.000 |
| P | 98.712 | 4.132 | 35.624 | 1.288 |
| | *99.938* | 3.264 | 7.022 | 0.062 |
| | 99.864 | 0.660 | 21.686 | 0.136 |
| | *100.000* | 0.022 | 7.188 | 0.000 |

computation of the amplitude spectrum has several effects. Thus, in the case of the weighted data, (1) the contours of the three clusters (G, B, P) display significant shape differences, (2) the shape of the G and B clusters reveals a correlation between the amplitude of the highest peak and the mean amplitude of the amplitude spectrum, and (3) the area covered by the P cluster is significantly smaller than those covered by the G and B ones. The case of the unweighted data revealed the strong resemblance among the three clusters from both the viewpoint of their extent and the shape of their contours. Comparing the two sets of clusters we have to note that (1) in the case of the weighted data, all three clusters are obviously not only shifted toward higher $A_{\text{mean}}$ and $A_{\text{max}}$ values, but also expanded in different ratios, with respect to the case of unweighted data, proving the presence of a higher degree of variability in these data, and (2) in the case of weighted data, the "observed" point is situated inside the clusters associated with the null hypothesis, more or less close to their centroids, while in the case of unweighted data it is shifted toward the constancy border of the null hypotheses.

According to the obtained results, we can conclude that there is no variability phenomenon in the analyzed data on Y Oph, neither deterministic components, nor some excess in the amount of noise with respect to the considered null hypotheses. Obviously, we cannot discuss in terms of an absolute constancy of our data, but we may place ourselves in the frame of a relative constancy, defined within the limits of a given null hypothesis, maybe not necessarily concerning the data Gaussianity.

## 6. ESTIMATION OF THE TREND DETECTION PROBABILITY

In this section, we tried to give an answer to the following question: "How detectible is the trend of Y Oph light curve amplitude, assuming that (1) the decreasing trend is real, (2) it may be considered to be linear and featured by the estimated regression line parameters, and also (3) the Gaussianity of the observational errors (with unequal precision), under conditions of the available data sampling"?

In order to find the answer to the above question, we took into account a previous idea of Zwintz et al. (2000). They tackled the problem of classification of the data sets supplied by *Hubble Space Telescope* Fine Guidance Sensor (*HST* FGS) and then analyzed through Kuschnig's et al. (1997) method. There, the authors considered the case of "trend data sets," for which the highest significant peak in the amplitude spectrum (computed in the frequency range from 0 to the respective Nyquist limit) occurs at the lowest resolvable frequency.

An interesting, and useful by-product of the application of the method of Kuschnig et al. (1997) and of that developed by Pop (2005, 2007a; see also Section 5) is the number of occurrences of the highest peak in the amplitude spectrum at a given frequency (see also Kallinger & Weiss 2002). It supplies information concerning the interaction between the input signal (e.g., Gaussian noise) and the data sampling.

Taking into account the idea of Zwintz et al. (2000) and the above way to extract information from Monte Carlo simulations, we propose the following approach: we perform numerical simulations as described in Section 5, but with two differences: (1) the input signal consists of a linear trend (T) (with the same parameters as estimated through linear regression) superposed over a Gaussian noise (G) (see Section 3.2 above) and (2) the amplitude spectrum is computed in the range $[0, f_{\text{high}} \cong f_{Ny}]$. Considering the lowest resolvable frequency in the amplitude spectrum $f_{\text{low}} = 1/\Delta T$, let us denote with $n_{\text{peak}}^{G+T}$ the number of occurrences of the highest peak—which is not necessarily statistically significant, too—in the frequency range $[0, f_{\text{low}}[$. We define the **t**rend **d**etection probability by means of the following expression

$$\text{Prob}_{\text{td}}^{G+T} = 100 \cdot n_{\text{peak}}^{G+T} / n_{\text{ex}}.$$

It quantifies our chance to detect the signature of a linear trend (as resulting from linear regression) in the amplitude spectrum, being given the observed data scatter, their unequal precision, and their sampling pathology. In a similar way, we compute the trend detection probability for the case in which the input data consist only of Gaussian noise

$$\text{Prob}_{\text{td}}^{G} = 100 \cdot n_{\text{peak}}^{G} / n_{\text{ex}}.$$

Obviously, the meaning of $\text{Prob}_{\text{td}}^{G}$ is that of a false-alarm probability, i.e., the probability of detecting the signature of a trend when the input data consist only of Gaussian noise with the given unequal precision and sampling, while the quantity $1 - \text{Prob}_{\text{td}}^{G}$ is the corresponding reliability of the test.

Taking into account the classification of Zwintz et al. (2000) in the case of the S/N, we propose the following classification of the decision levels featuring our chance to detect the assumed trend in the amplitude spectrum:

1. detectible trend: $\text{Prob}_{\text{td}}^{G+T} > 99.9\%$,
2. possible detectible trend: $85\% \leqslant \text{Prob}_{\text{td}}^{G+T} \leqslant 99.9\%$,
3. trend difficult to detect: $\text{Prob}_{\text{td}}^{G} < \text{Prob}_{\text{td}}^{G+T} < 85\%$,
4. no trend: $\text{Prob}_{\text{td}}^{G+T} \leqslant \text{Prob}_{\text{td}}^{G}$.

The test proposed in this section is more specific than the previous one based on $A_{\text{max}}$ (see Section 5) or $A_{\text{max}}/A_{\text{mean}}$ (Kuschnig et al. 1997), because the occurrence of the event of interest is related to a given frequency range of the amplitude spectrum. Obviously, it can also be applied by taking into account more complicated models such as periodic ones, in order to estimate the detection probability in the amplitude spectrum of a periodic signal with given frequency and amplitude values. We also have to remark that the trend detection probability is conceptually related to the notion of the power of a statistical test.

We applied the test proposed by Zwintz et al. (2000) to the pe and old + pe + RV data sets on the light curve amplitude of Y Oph for which we estimated significant linear trends, at least with respect to the null hypothesis. The values of the lowest resolvable frequency for these data sets were already given in the beginning of Section 5. The values of the frequency corresponding to the highest peak are 0.008978 cd$^{-1}$ for pe data, and 0.023838 cd$^{-1}$ for old + pe + RV data. In both cases, this one is situated at higher frequencies than $f_{\text{low}}$, and consequently, these data sets could not be classified as "trend data sets," according to the criterion given by Zwintz et al. (2000). It is interesting to mention that ignoring the individual statistical weights for the considered data sets, the old + pe + RV data set proved to be the only "trend data set". The highest and also the most significant peak in the amplitude spectrum appeared at a frequency of 0.000015 cd$^{-1}$, i.e., below the lowest resolvable frequency (see Section 5).

We performed the test proposed by us in this section using $n_{\text{ex}} = 100{,}000$ simulated data sets generated in the same way as described in the statistical test in Section 3.2 (the observational data scatter is assumed to be featured by the unweighted standard deviation of the residuals obtained after removing the linear trend, $\sigma_{\text{res}}$, and the observational noise is assumed to be Gaussian), considering the pairs of values of the regression line parameters for the pe and old + pe + RV data (see Table 3). The results are displayed in Table 5. Figures 7 and 8 illustrate

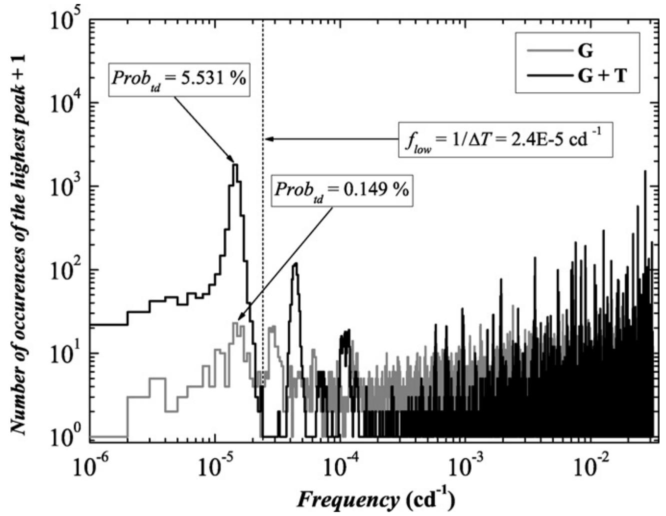


**Figure 7.** Histograms of the incidence of the highest peak in the amplitude spectrum for the case of old + pe + RV amplitude data, taking into account the data weights.

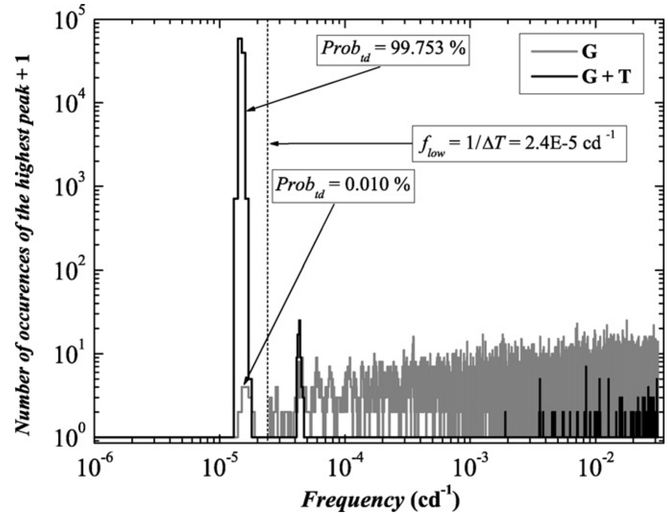


**Figure 8.** Histograms of the incidence of the highest peak in the amplitude spectrum for the case of old + pe + RV amplitude data, without taking into account the data weights.

**Table 5**
Estimation of the Trend Detection Probability for pe and old + pe + RV Amplitude Data Sets on Y Oph[a]

| | pe data | | old + pe + RV data | |
|---|---|---|---|---|
| $\sigma_{\rm res}$ **(mag)** | **0.0** | **0.02631** | **0.0** | **0.03282** |
| $Prob_{\rm td}^{G+T}$ (%) | 49.034 | 7.957 | 15.441 | 5.531 |
| $Prob_{\rm td}^{G}$ (%) | 0.325 | 0.405 | 0.018 | 0.149 |

**Notes.**
[a] Taking into account the estimated values of the regression line parameters, with or without including the observed scatter quantified by the standard deviation of the residuals $\sigma_{\rm res}$ obtained from the unweighted fit.

the concept of trend detection probability and exemplify how this test works, with and without taking into account the data weights, for old + pe + RV data.

According to Table 5, whatever case we consider, we obtain the same result: the descending trend of Y Oph pulsation amplitude, assumed to be linear and featured by the estimated parameters, as it is perceived through the available data sampling and precision, is in fact a trend difficult to detect (see the above case3). It means that the chance of finding the highest peak in the amplitude spectrum in the frequency range [0, $f_{\rm low}$[ is low. Even in the most optimistic case, for about 51% of the simulated data sets, the highest peak in the amplitude spectrum occurred in the frequency domain [$f_{\rm low}$, $f_{\rm high}$]. A remarkable result is that without taking into account the data weights, the trend detection probability increased to about $Prob_{\rm td}^{G+T} = 99.753\%$ (with respect to $Prob_{\rm td}^{G} = 0.010\%$), indicating a highly detectible trend (see Figure 8). As in the previous section, the results of the above numerical experiments revealed the important effect of the individual and unequal data errors: they are able to drastically reduce the trend detectibility. Finally, we have to note an obvious effect of the presence of the linear trend in the data: the occurrence of the highest peak in some frequency regions of the amplitude spectrum becomes highly probable with respect to both the case of the presence only of Gaussian noise and other frequency regions.

## 7. SUMMARY

In this paper, we reevaluated the secular decrease of the $V$ light curve amplitude of Y Oph previously emphasized by Fernie (1990) and Fernie et al. (1995). With this aim in view we reconsidered the data available before 1940. Other photoelectric data sets after this epoch were carefully reanalyzed. Finally, taking into account different data sets, more or less homogeneous from a photometric viewpoint, and assuming the linear character of the amplitude decline, we obtained slope values between $-0.87 \pm 0.10$ mmag yr$^{-1}$ and $-1.091 \pm 0.091$ mmag yr$^{-1}$. Consequently, the previous results of Fernie et al. (1995) were confirmed, but using an increased amount of data, under the circumstances of a better agreement between the amplitude data before and after 1940.

We also performed a detailed investigation of the statistical significance of this linear trend. Thus, if we took into account the null hypothesis according to which there is no deterministic component in the observational data, we could reject it, but at quite low confidence levels, i.e., 98.47% for pe data, and 99.85% for old + pe + RV data. If we consider a more restrictive null hypothesis which assumes the lack of any linear trend in the analyzed data, then we could reject it at slightly higher confidence levels, i.e., 98.51% for pe data, and 99.99% for old + pe + RV data. Unfortunately, although the slope of this trend might be considered statistically significant with respect to the considered null hypothesis (especially for old + pe + RV data), at least because of the low quality of the available data, the corresponding risk of erroneously rejecting the presence of the linear trend is unacceptably high, i.e., about 0.5. In the case of the most homogeneous pe data set, the power of the test (assuming a Type I error $\alpha = 0.01$) is about 0.44, while for old + pe + RV data we obtained a value of about 0.90, which is also too low. Thus, the hypothesis of the decreasing linear trend needs both more accurate data and a longer time base in order to be tested.

The traditional approach of variability analysis through $F$ and $\chi^2$ tests supplied us some general information. Thus, in the case of pe + RV and old + pe + RV data sets the unequal precision of individual data seems to have some contribution to their apparent variability. All data sets except the old ones display different levels of variability. On the other hand, taking into account the Gaussianity frame required by these tests, it is obvious that the above results may not be definitive ones. Consequently, we performed additional variability analyses relying on the analysis of the amplitude spectrum of the pe and old + pe + RV residuals through Monte Carlo simulations, taking or not taking into account the Gaussianity hypothesis. All three considered approaches (Gaussian noise, bootstrap resampling, and random permutations) revealed only a relative constancy of the detrended amplitude data, within the limits of a well-defined null hypothesis. We may conclude that the available data do not allow the detection of possible fine effects (especially a possible modulation due to a hypothetical binarity) in the light curve amplitude.

Assuming the slope value previously estimated, we introduced the notion of the trend detection probability, which gives a measure of how detectible a given linear trend is in well-defined observational conditions (data sampling, individual errors, amount of data). This notion rounds out the test proposed by Zwintz et al. (2000). While the latter is focused on the analysis of a given data set, i.e., the realization of a (unknown) random process, the test based on the trend detection probability proposed by us quantifies our ability to detect a given trend in well-defined observing conditions. The estimation of this quantity is based on Monte Carlo simulations and amplitude spectrum analysis. Taking into account the pe and old + pe + RV data sets, for different amounts of Gaussian noise, we found values of the trend detection probability between 5.531% and 49.034%. Therefore, we concluded that we are dealing with a "trend difficult to detect," if we take into account the individual errors for amplitude data. Indeed, none of the analyzed data sets displayed maxima in their amplitude spectra between 0 $cd^{-1}$ and the corresponding lowest resolvable frequency.

As a final conclusion of our investigation, we have to emphasize the need of high precision and high sampling rate observational data on classical Cepheids with good phase coverage extending over an as small as possible number of pulsation cycles. The above improvements are essential within the frame of approaching the investigation of the interaction between binarity and pulsation, and that of the nonlinear effects appearing in the light curves of these variables. Maybe it is time to reconsider the strategy of observing these stars from the ground and to organize multisite observational campaigns.

This research has made use of NASA's Astrophysics Data System Bibliographic Services and McMaster Cepheid Photometry and Radial Velocity Data Archive.

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