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# Biperiodic orbital period modulation of the W UMa binary system CK Bootis

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ABSTRACT

A study of the orbital period variation of the W UMa system CK Bootis is made using an extended observational time base. The biperiodicity of the orbital period modulation is emphasized. Both detected periodicities (24.14 yr and 10.62 yr) cannot be explained through the light-time effect unless the companion would be a white dwarf as suggested by other authors, too. Moreover, we also argue that, nowadays at least, it seems that there is no causal relation between the orbital period variation and the recently discovered visual companion. Consequently, we infer that at least one of the two periodicities may be related to the magnetic activity cycles in the component stars of CK Boo, while the other periodicity could be related to the presence of a fourth companion in the system.



## 1. Introduction

Usually, a first approach of the analysis of astronomical time series obtained through stellar variability studies (light and radial velocity curves), implicitly assumes that we are dealing with the superposition of a *deterministic component* (secular trend or (multi) periodic signal) and a *random component* (observational noise or possible correlated variability physically related to the observed object). Very often, the secular trend can be modelled by low order polynomials. The residuals obtained after removing the polynomial trend have to be further analysed in order to investigate the presence of possible (multi)periodic variability phenomena. The methodology suitable for such studies has been developed alongside the techniques for analysis of the pulsating stars light curves (e.g. Barning, 1963; Wehlau and Leung, 1964; López de Coca et al., 1984; Breger et al., 1987). This methodology relies on the successive prewhitening in the time domain.

In the case of period variability phenomena, we are dealing with *O–C* curves relying on timing data of light curves extrema of the observed variable star. It is well-known that concurrently with the gradual increase of the observing time base, the shape of many variable stars *O–C* curves became more and more intricate. Correspondingly, more complex ephemerides were needed to describe the distribution of the observed timing data. Thus, from low order polynomial ephemerides (e.g. Wood and Forbes, 1963), one evolved either to superposition of (multi)periodic term(s) on low order polynomial models (e.g. Kreiner, 1971; Panchatsaram and Abhyankar, 1981; Rafert, 1982; Abhyankar and Panchatsaram, 1984; Demircan and Selam, 1993; Pop, 1996; Pop et al., 1996, 2000, 2003; Borkovits et al., 2002, 2005; Li and Qian, 2005), or to high order polynomial ephemerides (e.g. Derman and Demircan, 1992; Kalimeris et al., 1994a,b).

CK Bootis (BD + 9°2916, HD 128141, HIP 71319) is an A type W UMa binary system with a spectral type of F7/F8 V (Rucinski and Lu, 1999), and an orbital period of about 0.355 d. Examining the run of the *O–C* curve of CK Boo, Aslan and Derman (1986) hypothesized that a period variation could occur in this binary system, but without assuming a given mathematical law. Demircan (1987) also noted some change in the run of the *O–C* residuals, but he expressed his doubt (like Aslan and Derman, 1986) concerning the reality of the corresponding period change. Jia et al. (1992) emphasized for the first time a linear increase of the orbital period of CK Boo. Later, Qian and Liu (2000) confirmed the presence of a parabolic trend in the *O–C* residuals. Moreover, they found evidence for a small amplitude period modulation phenomenon superposed on this trend. They estimated the involved periodicity at about 14 yr. On the other hand, they also found that the luminosity of the secondary component varies in phase, approximately with the same period. They concluded that the observed orbital period modulation in CK Boo is related to cyclic magnetic activity of the secondary component through Applegate's (1992) mechanism. Kalci and Derman (2005) confirmed the previous obtained results. The period of the orbital modulation was found to be about 15.75 yr. They also concluded that the amplitude of this period variation might increase with time and, for this reason, they rejected the hypothesis of a third body in the system CK Boo. Consequently,

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taking into account the spectral type of this system, they adopted the mechanism of magnetic activity cycles occurring in both component stars as a cause for the observed orbital period variation. A year later, Yüce et al. (2006) resumed the study of orbital period variation of CK Boo. They supplied a new confirmation of the parabolic trend and of the periodic modulation in the *O–C* curve. They assumed that the last is caused by the presence of a third body in the system, with a mass of 0.48 $M_{Sun}$, revolving on an eccentric orbit ($e$ = 0.55) with a period of 17.41 yr. Concerning the nature of this companion, more massive than the secondary component, Yüce et al. (2006) considered that it could be either a compact object, or a close binary. The light-time effect in case of CK Boo was also approached by Pribulla and Rucinski (2006). The involved periodicity – considered by the authors to be very uncertain – is 21.3 yr, while the corresponding orbit was found to be highly eccentric $e$ = 0.96. The minimum mass of the hypothetical companion was estimated to be extremely high with respect to that of the secondary star of the system, i.e., 1.50 $M_{Sun}$. Taking into account the small UV excess detected by Krzesiński et al. (1991), Pribulla and Rucinski (2006) concluded that it could be a white dwarf.

Valuable complementary information has been supplied by Pribulla and Rucinski (2006), D'Angelo et al. (2006), and Rucinski et al. (2007) through both spectroscopic and adaptive optics observations. They found that CK Boo has a close visual companion at a separation of 0.12 arcsec, with a late spectral type M0/M1 V. According to Rucinski et al. (2007) its mass might be 0.45 $M_{Sun}$. They estimated the projected separation between the close binary and companion at 19 AU, and the orbital period of the wide system at 54 yr.

From methodological viewpoint, we have to note that both Qian and Liu's (2000) and Kalci and Derman's (2005) approaches, which emphasized the presence of a wavelike component in the run of the *O–C* residuals, relies on the method of Kalimeris et al. (1994a). This makes this binary system an interesting target for a study using a complementary approach of (multi)periodicity analysis namely that described in the papers of Pop (1996, 1999, 2005) and Pop et al. (2000, 2003, 2011) (see also Borkovits et al., 2005). We reanalysed the period variability of the W UMa binary system CK Boo, relying on an extended time base: from $\Delta T \approx 29$ yr (Kalci and Derman, 2005), to $\Delta T \approx 36$ yr. This enabled us to emphasize the biperiodic character of the orbital period variation, superposed on its linear increase, previously detected by Qian and Liu (2000), and Kalci and Derman (2005). The interpretation of these results and their astrophysical implications are investigated.

## 2. Observational data

For the present study of the contact binary CK Boo, we collected 70 times of primary minimum light and 52 times of secondary minimum light from the Lichtenknecker – Database of the BAV (www.bav-astro.de/LKDB) and "*O–C* Gateway Database of times of minima (E) and maxima (RR)" of Variable Star Section of Czech Astronomical Society (http://var.astro.cz/ocgate/index.php), as well as from the papers of Kalci and Derman (2005), Diethelm (2011), and Samolyk (2011). All timing data have been checked according to their original sources. The observed minima corresponding to the same event (eclipse) have been averaged. The final data set containing both primary and secondary minima consists in 101 data points which cover a time base of about 36 yr.

## 3. Analysis methods

The methodology used for the analysis and modelling of the timing data of CK Boo was already applied in the recent paper of Pop et al. (2011) (see also Pop, 1996, 1999 and Pop et al., 2000, 2003). It relies on the classical *O–C* method and takes into account the nonlinear fitting of an ephemeris consisting in the superposition of a $K$ order *polynomial term* ($P[K]$) and a *multiperiodic term* ($F[L];[M_1],[M_2],\ldots,[M_L]$)

$$t_n = t_0 + \sum_{k=1}^{K}\tau_k n^k + \sum_{l=1}^{L}\sum_{m=1}^{M_l}\tau_{lm}\sin(\Omega_{lm}n + \Phi_{lm}), \tag{3.1}$$

where $f_{0l} = P_p/P_{sl}$, $P_p \equiv \tau_1$ being the orbital (*primary*) period, while $P_{sl}$ ($l = 1,2,\ldots,L$) being the *secondary* periodicities previously detected in the *O–C* curve. The periodicity detection was performed using the Monte Carlo method used by Pop et al. (2010) (see also Pop, 2005, 2007 and Pop and Vamoş, 2007).

## 4. Data analysis and results

The following linear ephemeris has been established for the analyzed data set of CK Boo

$$t_n = 2449135.2816 + 0.35515426n \\ \pm 0.0013 \pm 0.00000013 \tag{4.1}$$

Within the limits of the available data, the *O–C* residuals of this contact binary may be well described by the superposition of two modulating signals with periodicities of 24.14 and 10.62 yr, on a parabolic trend corresponding to a linear increase of the orbital period (Fig. 1). The values of the parameters of the established ephemeris obtained through nonlinear least squares fitting, are given in Table 1, while the steps of prewhitening process, in the amplitude spectra of the *O–C* residuals, are displayed in Fig. 2. The statistical significance of the highest peak appearing in the amplitude spectra of the successive obtained residuals has been estimated using the Monte Carlo method mentioned in Section 3. The involved numerical experiments relied on null hypotheses built up using Gaussian noise, bootstrap resampling, and random permutations. In all these cases, the two above periodicities have been detected at confidence levels of 100%. The remaining *O–C* residuals proved to consist of observational noise with Gaussian character only.

### 4.1. The secular trend in the orbital period

The presence of a secular parabolic trend in the *O–C* curve of CK Boo was already emphasized by Qian and Liu (2000), Kalci and Derman (2005), and Yüce et al. (2006). Our present study confirmed the linear increase of the orbital period (Table 1). Like the above mentioned authors, we interpreted this period variation as an effect of the conservative mass transfer occurring in the system (e.g. Huang, 1963; Pringle, 1975). Accordingly, we found that the primary component receives matter from the secondary one at a rate of $\dot{M}_1 = 1.62 \times 10^{-8} M_{Sun}\mathrm{yr}^{-1}$, i.e., a slower rate than estimated by Kalci and Derman (2005) ($\dot{M}_1 = 5.73 \times 10^{-8} M_{Sun}\mathrm{yr}^{-1}$), and Yüce et al. (2006) ($\dot{M}_1 = 3.92 \times 10^{-8} M_{Sun}\mathrm{yr}^{-1}$). Our estimation relies on the mass values of the two components given by these last authors.

### 4.2. The magnetic activity cycles hypothesis

As we already mentioned, according to its spectral type (F7/F8), the contact system CK Boo should display an orbital period modulation phenomenon induced by the cyclic magnetic activity of the component stars (e.g. Hall, 1989, 1990; Applegate, 1992). Interesting, complementary information, which is useful for interpretation of the detected periodicities from the viewpoint of magnetic activity cycles may be obtained from the correlation between the length of the activity cycle ($P_{cyc}$) and the angular velocity ($\Omega$) for active stars found by Lanza and Rodonò (1999), which relies on the dynamo theory. They assumed: (i) that the length of the modulation cy-

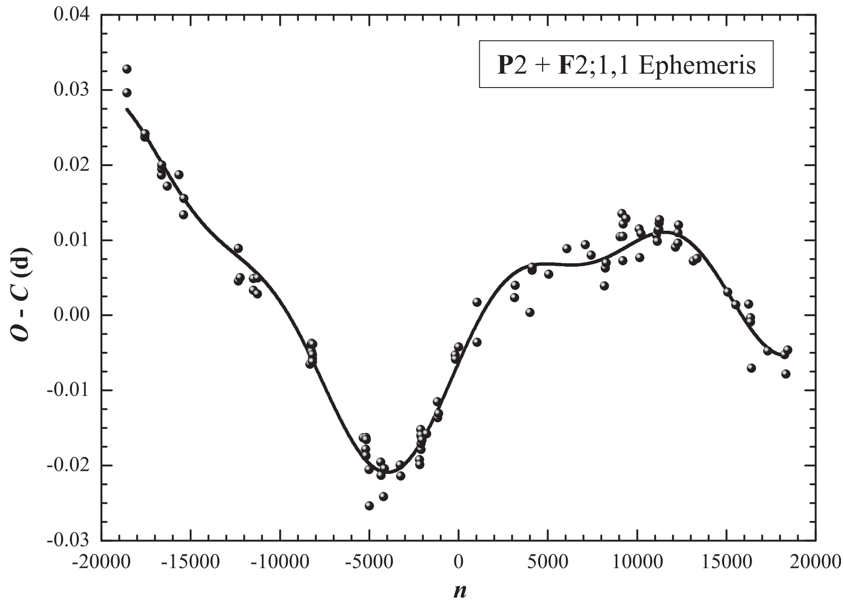


**Fig. 1.** The parabolic+biperiodic model of the *O–C* curve of CK Boo.

**Table 1**
Parameters of the *polynomial+multiperiodic* (**P2** + **F2**; 1, 1) ephemeris for the *O–C* curve of CK Boo.

| | | | |
|---|---|---|---|
| $t_0 = HJD2449135.27712$ $\pm 0.00035$ | | | |
| $\tau_1 \equiv P_p = 0.355154216 dc^{-1}$ $\pm 0.000000042$ | | | |
| $\tau_2 = 4.87 \cdot 10^{-11}\,\mathrm{dc}^{-2}$ $\pm 0.31 \cdot 10^{-11}$ | | $(1/P_p)(dP_p/dt) = 2.82 \cdot 10^{-7}\,\mathrm{yr}^{-1}$ $\pm 0.27 \cdot 10^{-7}$ | |
| $f_{01} = 0.00004029$ $\pm 0.00000080$ | $P_{S1} = 24.14$ yr $\pm 0.48$ | $\tau_{11} = 0.01303$ d $\pm 0.00035$ | $\Phi_{11} = 5.953$ rad $\pm 0.033$ |
| $f_{02} = 0.0000916$ $\pm 0.0000012$ | $P_{S2} = 10.62$ yr $\pm 0.14$ | $\tau_{21} = 0.00449$ d $\pm 0.00034$ | $\Phi_{21} = 0.561$ rad $\pm 0.069$ |
| $\sigma_{res} = 0.00206\ d$ | | | |

cle in the *O–C* curve ($P_{\mathrm{mod}} \equiv P_S$ in our notations) coincides with $P_{cyc}$, (ii) the synchronism between the orbital motion and the rotation of the component stars ($\Omega = 2\pi/P_{orb}$, with $P_{orb} \equiv P_p$ in our notations). Pop et al. (2011) reconsidered this correlation through linear least squares regression and through a robust linear regression method (*the mean median method*) proposed by Sârbu (1997).

Here, in order to find out the implications of the above correlation in case of the orbital period modulation in CK Boo, we further reconsider it by using recently estimated $P_p$ and $P_S$ values for one Algol system, two RS CVn systems and three W UMa systems taken into account by Lanza and Rodonò (1999). We also included two additional systems: one of Algol type (ST Per), and one of W UMa type (OO Aql) (see Table 2). Both considered Algol systems (see Table 2), display longer periods previously attributed to the light-time effect related to the presence of an unseen companion (see Panchatsaram, 1981; Qian, 2001 for RT Per, and Demircan and Selam, 1993; Borkovits and Hegedüs, 1996; Pop et al., 2000 for ST Per). In case of RT Per, Pop et al., 1996 emphasized the existence of a secondary, shorter periodicity, while in case of ST Per an additional, shorter periodicity has been considered by Demircan and Selam (1993) (see also Borkovits and Hegedüs, 1996) and re-evaluated by Pop et al. (2000). In case of the W UMa systems investigated by Borkovits et al. (2005), these authors were able to prove the coexistence of a light-time effect and an additional short period modulation in case of AB And, OO Aql, and U Peg. Using the actualized and extended sample (48 binary systems) of Lanza and Rodonò (1999), we found the following regression relationships

$$\log P_{\mathrm{mod}}[yr] = -0.051 - 0.376 \log \Omega[s^{-1}] \\ \pm 0.246 \pm 0.061, \quad (4.1)$$

in case of ordinary least squares method (OLS), and

$$\log P_{\mathrm{mod}}[yr] = 0.27 - 0.29 \log \Omega[s^{-1}] \\ \pm 0.47 \pm 0.12, \quad (4.2)$$

in case of robust regression method of Sârbu (1997). Unlike OLS, the robust method is less sensitive to the outlying points existing in the considered sample, and therefore the slopes estimated through these methods may differ from one another. Our changes in the data sample led to a slight increase of the correlation coefficient (in modulus) from $r = -0.616 \pm 0.093$, to $r = -0.672 \pm 0.080$.

According to the Eqs. (4.1) and (4.2) we got the following predictions for the case of CK Boo: (i) $P_{\mathrm{mod}} = 21.68$ yr in case of ordinary least squares method, and (ii) $P_{\mathrm{mod}} = 21.86$ yr in case of robust regression method. The first periodicity detected in the *O–C* curve of CK Boo is $P_{S1} = 24.14$ yr, i.e., higher, but close to the above two predictions. This result could support the hypothesis that magnetic activity cycles are occurring in this system with a length of about 24.14 yr. This hypothesis may also be considered as a cause for the second detected periodicity ($P_{S2} = 10.62$ yr).

Further information concerning the features related to the mechanism of the cyclic magnetic activity would be obtained through Applegate's (1992) model. However, we have to take into account that this model (see also Lanza et al., 1998) approaches the case on which just one component star is magnetically active (in

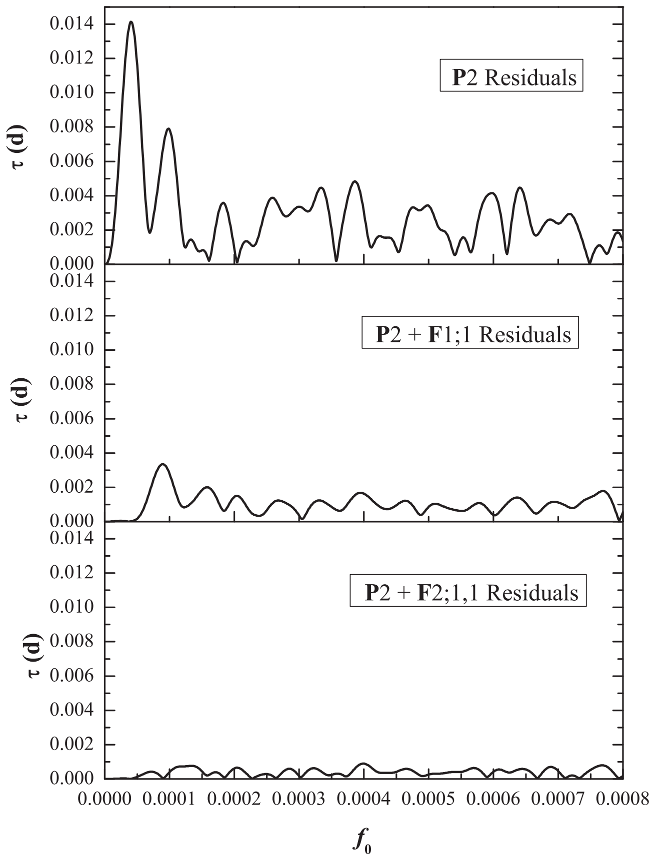


**Fig. 2.** Amplitude spectra corresponding to different stages of prewhitening of the *O–C* curve.

the case of CK Boo both stars are expected to be magnetically active). Other complications could arise from the complex geometry of the contact binary systems. Consequently, we restricted ourselves to the estimation of some overall parameters. We shall use the following notations according to the papers of Applegate (1992), Lanza et al. (1998), Lanza and Rodonò (1999): $A_{O-C}$ – the semi-amplitude of the *O–C* curve, $\Delta P_p/P_p$ – the relative semi-amplitude of the orbital period modulation, $\Delta Q$ – the variation of the quadrupole moment of the active star which would be related to the observed $\Delta P_p/P_p$ value. Estimating the semi-major axis of the system's relative orbit the value $a = 2.468\ R_{Sun}$ (see Gazeas et al., 2006), we got:

$$P_{S1} = 24.14\,\text{yr} : A_{O-C} = 0.02606\,\text{d}, \quad \Delta P_p/P_p = 9.29 \times 10^{-6},$$
$$\Delta Q_1 = -8.72 \times 10^{42}\ \text{kg m}^2, \quad \Delta Q_2 = -9.37 \times 10^{41}\ \text{kg m}^2,$$
$$P_{S2} = 10.62\,\text{yr} : A_{O-C} = 0.00898\,\text{d}, \quad \Delta P_p/P_p = 7.27 \times 10^{-6},$$
$$\Delta Q_1 = -6.83 \times 10^{42}\ \text{kg m}^2, \quad \Delta Q_2 = -7.34 \times 10^{41}\ \text{kg m}^2,$$

where the subscripts 1 and 2 refers to the primary and secondary components, respectively. The following remarks can be formulated: (i) the values of the modulation periods are of the same order of magnitude as other W UMa systems (e.g. Lanza and Rodonò, 1999); (ii) the values of the quadrupole moment estimated for the two component stars and for both modulation periods are of the order of $10^{41}$ – $10^{42}$ kg m$^2$ (see also Borkovits et al., 2005), i.e., of the same order of magnitude as cataclysmic variables ($10^{42}$ kg m$^2$, see Lanza and Rodonò, 1999).

### 4.3. The third body hypothesis

Other possible mechanism which may be involved in the orbital period modulation is that of the light-time effect due to the presence of one or two unseen companions revolving on circular orbit(s) around the barycentre of the system. We did not take into account the case of an arbitrary periodic modulation, which corresponds to an eccentric orbit (e.g. Kopal, 1978), for the following two reasons: (i) the nonlinear fitting of a **P**2 + **F**1;2 model was not convergent, and (ii) in case of the **P**2 + **F**2;1,1 model, we found $f_{02}/f_{01} = 2.274 \pm 0.054$, which means that, within the limits of observational errors, $f_{02} \neq 2f_{01}$, i.e. $f_{02}$ is not the first harmonic of $f_{01}$. For the two detected periodicities we estimated the values of the corresponding parameters: $a_{12}\sin i$ – the projection of the semi-major axis of the close binary absolute orbit on the line of sight, $i$ – inclination of the normal to the orbit plane on the line of sight, $f(M)$ – the mass function, $M_3$ – the mass of the hypothetical companion, and $K$ – the semi-amplitude of the radial velocity curve. Thus, taking into account the values of the masses of the binary component stars estimated by Gazeas et al. (2006, see Table 6 therein), the binary system's mass is $M_{12} = 1.597\ M_{Sun}$, and for zero eccentricity of the system, we obtained:

$$P_{S1} = 24.14\,\text{yr} : a_{12}\sin i = 2.256\,\text{AU}, \ f(M) = 0.0197 M_{Sun}, \ K = 2.78\,\text{km s}^{-1};$$
$$P_{S2} = 10.62\,\text{yr} : a_{12}\sin i = 0.777\,\text{AU}, \ f(M) = 0.0042 M_{Sun}, \ K = 2.18\,\text{km s}^{-1}.$$

We present in Fig. 3 the values for the mass ($M_3$) and the orbit radius ($a_3$) of the companion as functions of the inclination value, computed for the two periodicities. Adopting from the same authors the inclination value estimated for photometric solution with a cool spot on the primary star (see Table 4 of Gazeas et al., 2006) $i = 63.7°$, we got:

$$P_{S1} = 24.14\ \text{yr} : M_3 = 0.492 M_{Sun}, \quad a_3 = 8.16\,\text{AU};$$
$$P_{S2} = 10.62\ \text{yr} : M_3 = 0.272 M_{Sun}, \quad a_3 = 5.08\,\text{AU},$$

assuming the coplanarity of orbital planes.

In both cases, the respective systems may be considered as hierarchical triple systems: an inner close binary, and a distant companion, the semimajor axis of their relative orbit being 2296.645 $R_{Sun}$ ($P_{S1}$ = 24.14 yr), and 1280.090 $R_{Sun}$ ($P_{S2}$ = 10.62 yr), respectively. Having in view a two-body approximation, we investigated the stability of these hypothetical triplets using the simple stability criterion of Szebehely and Zare (1977). It resulted that both of

**Table 2**
Eclipsing binary systems used to bring up-to-date and to extend the sample of Lanza and Rodonò (1999).

| Name | Class | Spectrum | $\Omega$ ($10^{-4}$ s$^{-1}$) | $P_{mod}$ (yr) | Ref. |
|---|---|---|---|---|---|
| RT Per | Algol | F5 + G0 | 0.86 | 26.5 | Pop et al. (1996) |
| ST Per | | A3 V + K1–2 IV | 0.27 | 23.8 | Pop et al. (2000) |
| RT And | RS CVn | F8 V + G0 – K1–3 V | 1.16 | 29.4 | Pop and Roman (2005) |
| WW Dra | | G2 + K0 | 0.16 | 112.2 | Liao and Qian (2010) |
| AB And | W UMa | G5 V | 2.19 | 18.3 | Borkovits et al. (2005) |
| OO Aql | | G5 V | 1.43 | 20.4 | |
| V566 Oph | | F5 V | 1.78 | 19.8 | |
| U Peg | | G2 V | 1.94 | 17.9 | |

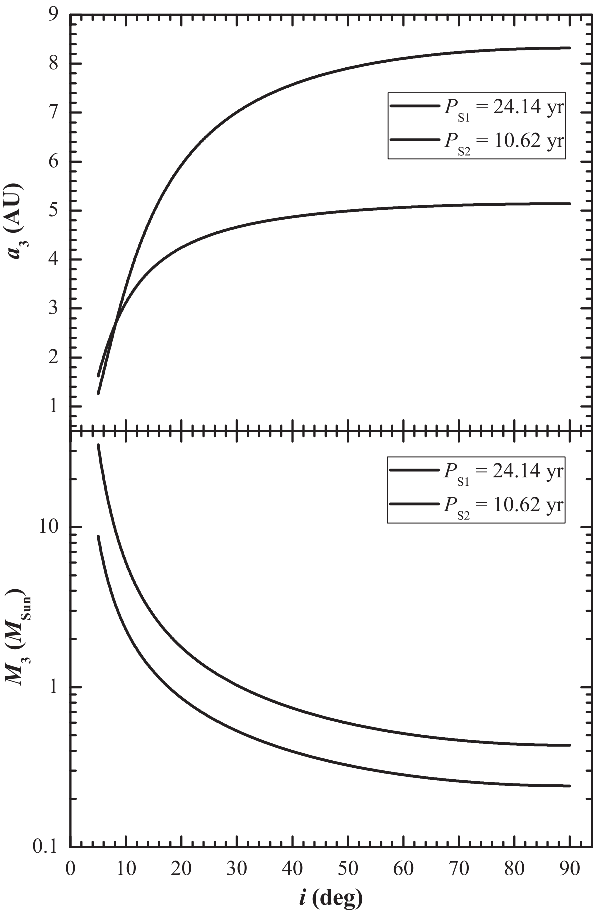


**Fig. 3.** The mass (lower panel) and the semimajor axis of the absolute orbit (upper panel) values of the hypothetical "third body" corresponding to the two detected periodicities, for different inclination values.

them would be stable either for direct or retrograde motion, for all inclination angles, but "more stable", in case of direct motion. It also resulted that the configuration corresponding to the longer periodicity would be "more stable".

Our attempt to investigate the orbital period variability through the light-time effect also has to take into account the possible involvement of the recently discovered visual companion. The projected separation of the two visual components of the system (expressed in AU) is $d_{obs} = \rho/\pi$, where $\rho = 0.12''$ is the angular separation (see Pribulla and Rucinski, 2006; Rucinski et al., 2007), and $\pi_{Hp} = 0.00696''$ is the system parallax as determined by Hipparcos. According to the orientation of the relative orbit of the wide system, there are two extreme situations for the value of $d_{obs}$: (i) *maximum apparent separation*, when the semimajor axis is perpendicular on the line of sight, i.e., $d_{obs} = d_{\max} = a_w\,(1 + e)$, and herefrom $a_w = d_{obs}/(1 + e)$; and (ii) *minimum apparent separation*, when the semimajor axis coincides with the line of sight, i.e., $d_{obs} = d_{\min} = a_w\ (1 - e) \cos i$, and thus $a_w = d_{obs}/((1 - e) \cos i)$. We denoted: $a_w = a_{12} + a_3$ – the semimajor axis of the relative orbit of the wide system, $e$ – the eccentricity of the relative orbit of the wide system, and $i$ – the wide orbit inclination with respect to the line of sight. Using Kepler's third law one can estimate the value of the orbital period of the visual binary system: $P_w = 2\pi(a_w^3/(G(M_{12} + M_3)))^{1/2}$, where $M_{12} = 1.597 M_{\rm Sun}$ (Gazeas et al., 2006), and $M_3 = 0.45\ M_{\rm Sun}$ (Rucinski et al., 2007). Within the hypothesis $d_{obs} = d_{\max}$, for $e = 0$, we got $a_w = 17.24\ AU$, $P_w = 50.04$ yr, and for $e = 0.99$, $a_w = 8.66$ AU, $P_w = 17.82$ yr. Taking into account the other hypothesis, i.e., $d_{obs} = d_{\min}$, then for $i \to 90^\circ$, we obviously have $a_w$, $P_w \to \infty$. As we previously argued, presently, the light-time effect can be taken into account only for circular orbits. In this case ($e = 0$), for $i = 0^\circ$ we have: $a_w = 17.24$ AU, $P_w = 50.04$ yr, while for $i = 63.7^\circ$ (the orbits of the close pair and that of the wide system are coplanar): $a_w = 38.91$ AU, $P_w = 169.66$ yr. Comparing these estimates with the values of $P_{S1}$, $P_{S2}$, as well as with those of $a_3$ displayed in Fig. 3, it becomes obvious that the observed orbital period variability of CK Boo is not an effect of the gravitational coupling with its visual companion.

## 5. Concluding remarks

The main result of our study on the orbital period variability of the contact binary CK Boo is that it is featured by a biperiodic modulation superposed on an already known linear increase. The involved periodicities are $P_{S1} = 24.14$ yr and $P_{S2} = 10.62$ yr. We have to remark that the actual time base covers only 1.5 cycles of $P_{S1}$. In case of $P_{S2}$ the coverage is better, about 3.4 cycles. Consequently, $P_{S1}$ has to be confirmed by future observations.

According to the prediction supplied by the correlation found by Lanza and Rodonò (1999) and updated here by us, the longest periodicity detected in the *O–C* curve of CK Boo could be that of magnetic activity cycles occurring in this system. However, as Lanza and Rodonò (1999) mentioned, this correlation did not take account the situation of multiperiodicity existing in the *O–C* curves of some eclipsing binary systems. They also could not exclude a possible contamination of their sample with some systems displaying an orbital period modulation caused by the light-time effect. Therefore, neither their regression relationship nor those established by us (Eqs. (4.1) and (4.2)) can be considered more than preliminary and approximate ones. Consequently, for the moment, through this approach, it is not possible to decide whether $P_{S1}$, or $P_{S2}$, or even both of them are involved in the magnetic cycles occurring in the system CK Boo.

The interpretation of the two periodic modulations of the orbital period as being caused by the light-time effect allowed us to estimate the masses and the semimajor axes of the orbits of the hypothetical companions. According to Szebehely and Zare (1977) stability criterion, stable triple systems involving low-mass companions corresponding either to $P_{S1}$ or to $P_{S2}$, would be allowed (see Section 4.3). However, for $i = 63.7^\circ$, in the hypothesis of orbits coplanarity, the masses of the hypothetical companions ($M_3 = 0.492\ M_{Sun}$, and $M_3 = 0.272\ M_{Sun}$, respectively) would be larger than the mass of the secondary component of CK Boo ($M_2 = 0.155\ M_{Sun}$). Moreover, according to Fig. 3, even for $i = 90^\circ$, the lowest mass of the companion would be about $0.433\ M_{Sun}$, or $0.242\ M_{Sun}$, respectively. Especially in case of the hypothetical companion corresponding to $P_{S1}$, the corresponding mass would be very close to that estimated for the visual companion, and consequently they would have quite similar evolutionary stages, and relatively close luminosities. The hypothesis of a white dwarf companion, which would be in agreement with the small ultraviolet excess discovered by Krzesiński et al. (1991), may also be taken into account. Concerning the visual companion of CK Boo, it appears not to be involved in the orbital period modulation of this binary system.

Taking into account the above considerations, it seems that at least one of the periodicities detected in the *O–C* curve of CK Boo (we recall that $P_{S1}$ needs future observational confirmation) could be related to the development of magnetic activity cycles in the component stars of this system. However, the possibility that CK Boo is a quadruple system cannot be ruled out.

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