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On the Palais-Smale condition for Hammerstein integral equations in Hilbert spaces

Radu Precup

Babeş-Bolyai University, Department of Applied Mathematics, 3400 Cluj-Napoca, Romania

Abstract

In this paper we deal with nontrivial solvability in balls of Hammerstein integral equations in Hilbert spaces for nonlinearities of potential type. We use a variational approach based on variants of the mountain pass theorem which are due to Guo-Sun-Qi and Schechter. Our main contribution is a new technique to verify compactness conditions of Palais-Smale type. This technique combines the compactness criterium for countable sets in L^p with basic properties of the measures of noncompactness and integral inequalities.

Keywords: Compactness; Hammerstein integral equation; Mountain pass theory

1 Introduction

The fixed point theory and the critical point theory are two domains of nonlinear functional analysis which provide us with general existence principles, particularly useful in establishing existence of solutions to nonlinear boundary value problems and integral equations. Both theories use compactness conditions. Thus, the researchers in critical point theory (see [5, 7, 14, 15]) use the classical Palais-Smale condition or variants in order to find a critical point of a C^1 functional E on a Banach space X, that is a point x where $E'(x) = 0$. The classical Palais-Smale condition requires the sequences $\{x_n\}$ satisfying $E(x_n) \to \mu \in \mathbf{R}$ and $E'(x_n) \to 0$ to have convergent subsequences. An interesting variant, see (2.8) below, is due to Schechter [14]. In applications, such conditions are most frequently fulfilled due to some particular results of compact embedding (Ascoli-Arzèla, Rellich-Kondrachov). Such results hold for several spaces of functions with values in $\mathbf{R}^n$, but fail for the corresponding spaces of functions with values in an infinite dimensional space. In such cases, the compactness conditions have to

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