THE PERTURBED KLEIN–GORDON EQUATION

RADU PRECUP

ABSTRACT. We establish a general existence result for the Klein–Gordon equation with multivalued perturbations. The approach is based on a new fixed point theorem given in [10].

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The method and the results of this paper are in connection with those in Couchouron–Precup [3], [4], Kamenskii–Obukhovskii–Zecca [5], O'Regan–Precup [8] and Precup [10].

Let Ω be a bounded open subset of $\mathbf{R}^n$ and $0 < T < \infty$. We consider boundary value problems of the form

$$\left\{ \begin{array}{l} \frac{\partial^2 u}{\partial t^2}(x,t) - \Delta u(x,t) + m_0^2 u(x,t) \in \Phi\left[u, \frac{\partial u}{\partial t}\right](x,t) \\ \qquad\qquad\qquad \text{on } \Omega \times (0,T), \\ u(x,0) = u_0(x), \ \frac{\partial u}{\partial t}(x,0) = v_0(x) \quad \text{on } \Omega, \\ u = 0 \quad \text{on } \partial\Omega \times (0,T). \end{array} \right. \tag{0.1}$$

Let $E := L^2(\Omega)$, $H = H_0^1(\Omega)$ with norm $|u|_H = \left(\int_\Omega |\nabla u|^2 dx\right)^{\frac{1}{2}}$, and $F := H_0^1(\Omega) \times L^2(\Omega)$ with norm $|[u,v]|_F = \left(|u|_H^2 + |v|_E^2\right)^{\frac{1}{2}}$. By a solution of (0.1) we mean a weak solution u of the problem

$$\left\{ \begin{array}{l} \frac{\partial^2 u}{\partial t^2} - \Delta u + m_0^2 u = f \quad \text{on } \Omega \times (0,T), \\ u(x,0) = u_0(x), \ \frac{\partial u}{\partial t}(x,0) = v_0(x) \quad \text{on } \Omega, \\ u = 0 \quad \text{on } \partial\Omega \times (0,T), \end{array} \right. \tag{0.2}$$

that is $u \in C^1([0,T];E) \cap C([0,T];H)$, the function $t \mapsto (u'(t), v)_E$ is absolutely continuous on $[0,T]$ for each $v \in H$, $u(0) = u_0$, $u'(0) = v_0$ and

$$\frac{d}{dt}\left(u'(t), v\right)_E + (u(t), v)_H = \left(f(t) - m_0^2 u(t), v\right)_E \tag{0.3}$$

a.e. on $[0,T]$ for all $v \in H$. Here $f \in L^2(0,T;E)$ is a selection of $\Phi[u,u']$, i.e.,

$$f(t) \in \Phi\left[u, u'\right](t)$$

for a.e. $t \in [0,T]$. We shall assume that Φ is a multivalued map from $C([0,T];F)$ to $L^2(0,T;E)$.

Lemma 1. *Let $f \in L^2(0,T;E)$, $u_0 \in H$ and $v_0 \in E$. Then there exists a unique weak solution u of (0.2). In addition*

$$\left(\left|u'(t)\right|_E^2 + |u(t)|_H^2\right)^{\frac{1}{2}} \leq b + \int_0^t |f(s)|_E \, ds, \tag{0.4}$$

where $b^2 = |u_0|_H^2 + |v_0|_E^2 + m_0^2 |u_0|_E^2$.

Proof. The existence, approximation and uniqueness are standard results; see for example Precup [9], p 172. To derive (0.4) put $v = u_k'(t)$ into (0.3), where (u_k) is a sequence of approximation solutions having $u_k' \in H$. Then

$$\frac{1}{2}\frac{d}{dt}\left(\left|u_k'(t)\right|_E^2 + |u_k(t)|_H^2 + m_0^2 |u_k(t)|_E^2\right) = \left(f(t), u_k'(t)\right)_E \tag{0.5}$$

Integrating from 0 to t we obtain

$$\left|u_k'(t)\right|_E^2 + |u_k(t)|_H^2 \leq b^2 + 2\int_0^t |f(s)|_E \left|u_k'(s)\right|_E ds.$$

Let

$$A(t) = \left|u_k'(t)\right|_E^2 + |u_k(t)|_H^2$$

and

$$B(t) = b^2 + 2\int_0^t |f(s)|_E \left|u_k'(s)\right|_E ds.$$

We have $A(t) \leq B(t)$ and

$$B'(t) = 2|f(t)|_E \left|u_k'(t)\right|_E \leq 2|f(t)|_E \sqrt{A(t)}.$$

Hence

$$B'(t) \leq 2|f(t)|_E \sqrt{B(t)}.$$

Integration gives

$$\sqrt{B(t)} \leq \sqrt{B(0)} + \int_0^t |f(s)|_E \, ds,$$

which immediately yields (0.4) for u_k. Finally we let $k \to \infty$. □

Define $\Psi : L^2(0,T;E) \to C([0,T];F)$ by

$$\Psi f = \left[u, u'\right], \text{ where } u \text{ is the weak solution of (0.2).}$$

Clearly, if u is a weak solution of (0.1), then $w := [u,u']$ solves $w \in \Psi\Phi w$. Conversely, if $w = [u,v]$ is a solution of $w \in \Psi\Phi w$, then $v = u'$ and u is a weak solution of (0.1). Let K is the closed convex subset of all functions $[u,v] \in C([0,T];F)$ with $u(0) = u_0$ and $v(0) = v_0$.

Theorem 2. *The above solution operator Ψ satisfies the following conditions: (H1) For every $f_1, f_2 \in L^2(0,T;E)$ and a.e. $t \in [0,T]$, one has*

$$|\Psi f_1(t) - \Psi f_2(t)|_F \leq \int_0^t |f_1(s) - f_2(s)|_E \, ds; \tag{0.6}$$

(H2) For any compact $C \subset E$ and any sequence (f_k) of $L^2(0,T;E)$ with $\{f_k(t) : k \geq 1\} \subset C$ for a.e. $t \in [0,T]$, the weak convergence $f_k \to f$ implies $\Psi f_k \to \Psi f$ strongly in $C([0,T];F)$.

Proof. To check (H1), let $f_1, f_2 \in L^2(0,T;E)$ and let u_1, u_2 be the corresponding weak solutions of (0.2). Clearly $(u_1 - u_2)(0) = (u_1' - u_2')(0) = 0$. Then from (0.4) we find that

$$\left(\left| \left(u_1' - u_2'\right)(t)\right|_E^2 + \left|(u_1 - u_2)(t)\right|_H^2 \right)^{\frac{1}{2}}$$
$$\leq \int_0^t \left|(f_1 - f_2)(s)\right|_E ds.$$

Hence (0.6) holds. To check (H2) assume that $f_k \to f$ weakly in $L^2(0,T;E)$ and $f_k(t) \in C$ a.e. on $[0,T]$ for all k, where C is a compact subset of E. Since by (H1) the map Ψ is linear and bounded, we have $\Psi f_k \to \Psi f$ weakly in $C([0,T];F)$. Using an approximation argument, we can prove that the set $\{\Psi f_k\}$ is relatively compact in $C([0,T];F)$. This implies that $\Psi f_k \to \Psi f$ strongly in $C([0,T];F)$. □

The following result is a consequence of Theorem 3.1 in Precup [10]. Notice (H1), (H2) are respectively conditions (Ψ1) and (Ψ3) in [10].

Theorem 3. *Assume that the following conditions are satisfied: (Φ1) The values of Φ are nonempty, weakly compact, convex and Φ is weakly sequentially upper semi-continuous on any compact convex subset A of K. (Φ2) For every $a > 0$ there exists $\nu_a \in L^2(0,T;\mathbf{R}_+)$ such that $|f(t)|_E \leq \nu_a(t)$ a.e. on $[0,T]$ for all $f \in \Phi w$ and $w \in K$ with $\|w\| \leq a$. (Φ3) For every separable closed subspaces E_0, F_0 of E and F respectively, there exists a constant $m \geq 0$ and a function $\delta \in L^{r'}(0,T;\mathbf{R}_+)$, $r' > 2$, such that*

$$\begin{aligned} \beta_{E_0}\left(\Phi(M)(t) \cap E_0\right) \;\; &\leq \;\; m\,\beta_{F_0}(M(.))(t) \\ &\quad + \int_0^t \delta(s)\; \beta_{F_0}(M(.))(s)\,ds \end{aligned}$$

a.e. on $[0,T]$ for every bounded countable set $M \subset K$ with $M(t) \subset F_0$ a.e. on $[0,T]$, for which there exists $\nu \in L^\infty(0,T)$ with $|w(t)|_F \leq \nu(t)$ a.e. on $[0,T]$ for any $w \in M$. Here for a Banach space X, the symbol β_X stands for the ball measure of noncompactness in X. (L–S) There exists a bounded, convex and open subset U of $C([0,T];F)$ containing the origin such that

$$w \notin \lambda \Psi \Phi w$$

for all $w \in \overline{U} \setminus U$ and $\lambda \in]0,1[$. Then (0.1) has at least one solution in $\overline{U}$.

Remark 1. Assume that

$$|f(t)|_E \leq a(t) + b(t)\,|w(t)|_F + \int_0^t c(t,s)\,|w(s)|_F\,ds \tag{0.7}$$

a.e. on $[0,T]$ for all $w \in K$ and $f \in \Phi w$, where $a \in L^2(0,T;\mathbf{R}_+)$, $b \in L^{r'}(0,T;\mathbf{R}_+)$ and $c \in L^{r'}\left([0,T]^2;\mathbf{R}_+\right)$ for some $r' > 1$. Then condition (L–S) holds for $U = \{w \in K : \|w\| < R\}$, any $R > \left||\Psi(0)|_F\right|_\infty$ and a suitable equivalent norm $\|.\|$ on K. Indeed, if w solves $w \in \lambda\Psi\Phi w$ for some $\lambda \in]0,1[$, then for any $\theta > 0$ we have

$$|w(t)|_F \leq \lambda\,|\Psi(0)(t)|_F + \lambda \int_0^t |\Phi w(s)|_E\,ds$$

$$\leq \lambda \left|\Psi(0)(t)\right|_F$$

$$+\lambda \int_0^t \left(a(s) + b(s) \left|w(s)\right|_F + \int_0^s c(s,\tau) \left|w(\tau)\right|_F d\tau \right) ds$$

$$\leq \lambda \left|\Psi(0)(t)\right|_F$$

$$+\lambda \int_0^t \left(e^{\theta s} \left(a + b \left|w\right|_F e^{-\theta s} \right) + \int_0^s c e^{\theta \tau} e^{-\theta \tau} \left|w\right|_F d\tau \right) ds.$$

Define an equivalent norm on K by

$$\|u\| = \left| \left| e^{-\theta t} u(t) \right|_F \right|_\infty .$$

Then using Hölder's Inequality we obtain

$$\|u\| \leq \left| |\Psi(0)|_F \right|_\infty + \lambda \frac{C}{\theta} \|u\| .$$

Now choose a sufficiently large θ such that

$$\left| |\Psi(0)|_F \right|_\infty + \frac{C}{\theta} R \leq R.$$

Then the above inequality is impossible for $\|u\| = R$ and $\lambda \in]0,1[$.

Examples

1. A class of superlinear hyperbolic inclusions

Let

$$\Phi[u,v](t) = \Phi_0[u,v](t) - \sigma |u(t)|^{p-2} u(t),$$

where $2 < p \leq 2\frac{n-1}{n-2}$ $(n \geq 3)$, $\sigma \in \mathbf{R}_+$ and Φ_0 is a multivalued map from $C([0,T];F)$ to $L^2(0,T;E)$ such that conditions (Φ1)-(Φ3) are satisfied for Φ_0. In addition assume

$$|f(t)|_E \leq g(t) + a|w(t)|_F^\gamma \tag{0.8}$$

a.e. on $[0,T]$ for all $w \in F$ and $f \in \Phi w$.. Here $g \in L^2(0,T;\mathbf{R}_+)$, $a \in \mathbf{R}_+$ and $\gamma \in [0,1[$.

Let

$$\Phi_1[u,v](t) = -\sigma |u(t)|^{p-2} u(t).$$

Since $u(t) \in H$ and the embedding of H in $L^{2^*}(\Omega)$ is continuous ($2^* = \frac{2n}{n-2}$), we have that $|u(t)|^{p-1} \in L^{\frac{2^*}{p-1}}(\Omega)$. For $p \leq 2\frac{n-1}{n-2}$ it is easy to check that $2 \leq \frac{2^*}{p-1}$. Hence $L^{\frac{2^*}{p-1}}(\Omega) \subset E$. This shows that $\Phi_1[u,v] \in L^2(0,T;E)$. Also Φ_1 is continuous. Next we show that Φ_1 satisfies (Φ3). Indeed, let $u_1, u_2 \in H$ with $|u_1|_H, |u_2|_H \leq \eta$. We have

$$\begin{aligned}
&\left||u_1|^{p-2} u_1 - |u_2|^{p-2} u_2\right| \\
&\leq \left||u_1|^{p-2}(u_1 - u_2) + \left(|u_1|^{p-2} - |u_2|^{p-2}\right) u_2\right| \\
&\leq [\max(|u_1|, |u_2|)]^{p-2} |u_1 - u_2| \\
&+ (p-2)[\max(|u_1|, |u_2|)]^{p-2} ||u_1| - |u_2|| \\
&\leq (p-1)[\max(|u_1|, |u_2|)]^{p-2} |u_1 - u_2| \\
&\leq (p-1)(|u_1| + |u_2|)^{p-2} |u_1 - u_2| \\
&\leq (p-1)(|u_1| + |u_2|)^{p-1} |u_1 - u_2|^{\frac{1}{2}}.
\end{aligned}$$

Using Hölder's Inequality we deduce that

$$\begin{aligned}
&\left||u_1|^{p-2} u_1 - |u_2|^{p-2} u_2\right|_E \\
&\leq (p-1)\,||u_1| + |u_2||^{p-1}_{L^{2(p-1)}(\Omega)} |u_1 - u_2|_E \\
&\leq (p-1)\left(|u_1|_{L^{2(p-1)}(\Omega)} + |u_2|_{L^{2(p-1)}(\Omega)}\right)^{p-1} |u_1 - u_2|_E \\
&\leq C |u_1 - u_2|_E,
\end{aligned}$$

where the constant C depends only of p, n and η. Here we have used the continuously embedding of H into $L^{2(p-1)}(\Omega)$.

Now we prove that condition (L–S) holds for an open ball U of $C([0,T];F)$, of center 0 and a radius $R > 0$. Let $w = \lambda \Psi \Phi w$ for some $\lambda \in]0,1[$, and let $w = [u, u']$. Then, from (0.5) we have

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\left(\left|u_k'(t)\right|_E^2 + |u_k(t)|_H^2 + m_0^2 |u_k(t)|_E^2\right) \\
&= \lambda\left(f(t) - \sigma |u_k(t)|^{p-2} u_k(t), u_k'(t)\right)_E,
\end{aligned}$$

for some $f \in \Phi_0[u, u']$. Hence

$$\begin{aligned}
&\frac{d}{dt}\left(\left|u_k'(t)\right|_E^2 + |u_k(t)|_H^2 + m_0^2 |u_k(t)|_E^2 + \frac{2\lambda\sigma}{p} |u_k(t)|^p_{L^p(\Omega)}\right) \\
&\leq 2\lambda |f(t)|_E \left|u_k'(t)\right|_E.
\end{aligned}$$

Integration gives

$$\left|u_k'(t)\right|_E^2 + |u_k(t)|_H^2 \leq \widehat{b} + 2\lambda \int_0^t |f(s)|_E \left|u_k'(s)\right|_E ds,$$

where

$$\widehat{b}^2 = |u_0|_H^2 + |v_0|_E^2 + m_0^2 |u_0|_E^2 + \frac{2\sigma}{p} |u_0|^p_{L^p(\Omega)}.$$

Then

$$\left|u'(t)\right|_E^2 + |u(t)|_H^2 \leq \widehat{b} + 2\lambda \int_0^t |f(s)|_E \left|u'(s)\right|_E ds$$

$$\leq \widehat{b}+2\lambda\int_0^t\left(|g(s)|_E+a\left|\left[u(s),u'(s)\right]\right|_F^{\gamma}\right)\left|u'(s)\right|_E ds.$$

Recall the notation $w(t)=[u(t,u'(t))]$. Then

$$|w(t)|_F^2\leq \widehat{b}+2\lambda\int_0^t\left(|g(s)|_E+a\right)\max\left(1,\ |w(s)|_F^{\gamma+1}\right)ds$$

As above we obtain

$$|w(t)|_F\leq\left(1+\lambda(1-\gamma)\int_0^T\left(|g(s)|_E+a\right)ds\right)^{\frac{1}{1-\gamma}}<R,$$

where

$$R=\left(1+(1-\gamma)\int_0^T\left(|g(s)|_E+a\right)ds\right)^{\frac{1}{1-\gamma}}.$$

Hence

$$\left|\left[u(t),u'(t)\right]\right|_F<R,$$

so (L–S) holds with

$$U=\{w:\ ||w(.)|_F|_\infty<R\}.$$

Therefore, in particular, Theorem 2 gives the following generalization of a result in Lions [6].

Corollary 4. *Let Φ_0 satisfies (Φ1)-(Φ3) and (0.8). Let $u_0\in H$, $v_0\in E$, $m_0\in\mathbf{R}$, $\sigma\in\mathbf{R}_+$ and $p\in]2,\frac{2(n-1)}{n-2}]$ if $n\geq 3$, $p>2$ for $n=2$. Then there exists a weak solution u of the problem*

$$\begin{cases}\frac{\partial^2 u}{\partial t^2}-\Delta u+m_0^2u+\sigma|u|^{p-2}u\in\Phi_0\left[u,\frac{\partial u}{\partial t}\right] & \text{on } \Omega\times(0,T),\\ u(x,0)=u_0(x),\ \frac{\partial u}{\partial t}(x,0)=v_0(x) & \text{on } \Omega,\\ u=0 \quad \text{on } \partial\Omega\times(0,T).\end{cases}$$

Remark 2. The result in Lions [6] is obtained when $\Phi_0 w\equiv\{f\}$, where $f\in L^2(0,T;E)$.

2. A class of nonlinear integro-differential inclusions

Let us now establish an existence result for the problem

$$\begin{cases}\frac{\partial^2 u}{\partial t^2}-\Delta u+m_0^2u+\int_0^t\kappa(t-s)\,S\left[u,\frac{\partial u}{\partial t}\right]ds\in\Phi_0\left[u,\frac{\partial u}{\partial t}\right]\\ \qquad\qquad\qquad\qquad\text{on } \Omega\times(0,T),\\ u(x,0)=u_0(x),\ \frac{\partial u}{\partial t}(x,0)=v_0(x) \quad \text{on } \Omega,\\ u=0 \quad \text{on } \partial\Omega\times(0,T).\end{cases} \tag{0.9}$$

Theorem 5. *Let Φ_0 be a multivalued map from $C([0,T];F)$ to $L^2(0,T;E)$ such that conditions (Φ1)-(Φ3) are satisfied for Φ_0, $\kappa\in L^{r'}(0,T;\mathcal{L}(E))$ for some $r'>1$, and let $S:H\times E\to E$ be continuous such that*

$$|Sw|_E\leq a_0+b_0|w|_F$$

for all $w\in F$ and some $a_0,b_0\in\mathbf{R}_+$, and

$$\alpha_E(S(D))\leq c_0\,\alpha_F(D)$$

for any bounded set $D\subset F$ and some $c_0\in\mathbf{R}_+$. Then (0.9) has at least one weak solution.

Proof. Let

$$\Phi_1 w\,(t) = -\int_0^t \kappa\,(t-s)\,Sw\,(s)\,ds$$

Then

$$|\Phi_1 w\,(t)|_E \leq \int_0^t |\kappa\,(t-s)|_{\mathcal{L}(E)}\,(a_0 + b_0\,|w\,(s)|_F)\,ds. \tag{0.10}$$

It follows that

$$\begin{aligned} |f\,(t)|_E &\leq |a_0 + b_0\,|w\,(.)|_F|_\infty \int_0^t |\kappa\,(t-s)|_{\mathcal{L}(E)}\,ds \\ &\leq |a_0 + b_0\,|w\,(.)|_F|_\infty \int_0^T |\kappa\,(\tau)|_{\mathcal{L}(E)}\,d\tau. \end{aligned}$$

Hence $\Phi := \Phi_0 + \Phi_1$ satisfies (Φ2). Inequality (0.10) also implies that (0.7) holds. To check (Φ3) for Φ_1 let E_0, F_0 be separable closed subspaces of E and F, respectively, and let M be a bounded countable subset of K with $M\,(t) \subset F_0$ a.e. on $[0,T]$, for which there exists $\nu \in L^\infty\,(0,T)$ with $|w\,(t)| \leq \nu\,(t)$ a.e. on $[0,T]$ for any $w \in M$. Then

$$\begin{aligned} \beta_{E_0}\,(\Phi_1\,(M)\,(t) \cap E_0) &\leq 2\beta_E\,(\Phi_1\,(M)\,(t) \cap E_0) \\ &\leq 2\beta_E\,(\Phi_1\,(M)\,(t)) \\ &\leq 2\alpha_E\,(\Phi_1\,(M\,(t))) \\ &\leq 4\int_0^t \alpha_E\,(\kappa\,(t-s)\,S\,(M\,(s)))\,ds \\ &\leq 4\int_0^t |\kappa\,(t-s)|_{\mathcal{L}(E)}\,\alpha_E\,(S\,(M\,(s)))\,ds \\ &\leq 4c_0\int_0^t |\kappa\,(t-s)|_{\mathcal{L}(E)}\,\alpha_F\,(M\,(s))\,ds \\ &\leq 8c_0\int_0^t |\kappa\,(s)|_{\mathcal{L}(E)}\,\beta_{F_0}\,(M\,(s))\,ds. \end{aligned}$$

Hence Φ_1 also satisfies (Φ3). Here the function $\delta\,(t) := 8c_0\,|\kappa\,(t)|_{\mathcal{L}(E)}$ belongs to $L^{r'}\,(0,T)$ and $r' > 2$. Now Theorem 2 finishes the proof. □

Example. Let $k \in L^{r'}\,(0,T;\mathbf{R})$ for some $r' > 1$ and $a, b \in \mathbf{R}$. Then the problem

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} - \Delta u + m_0^2 u = \int_0^t k\,(t-s)\left(au + b\frac{\partial u}{\partial t}\right)ds \;\text{ on }\; \Omega \times (0,T)\,, \\ u\,(x,0) = u_0\,(x)\,,\; \frac{\partial u}{\partial t}\,(x,0) = v_0\,(x) \;\text{ on }\; \Omega, \\ u = 0 \;\text{ on }\; \partial\Omega \times (0,T) \end{cases}$$

has a weak solution.

Remark 3. Similar results can be established for hyperbolic equations and inclusions with infinite delays. Also the above arguments can be extended to discuss perturbations of the nonlinear wave equation involving monotone maps (see Barbu [1] and Brezis [2]).

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