TWO CLASSES OF POSITIVE LINEAR OPERATORS

BY

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1. The purpose of this paper is to introduce two classes of positive linear operators $(V_m^{[\alpha]})$ and $(W_m^{[\alpha]})$, depending on a non-negative parameter α which may depend only on the natural number m, and then to establish some of their approximation properties to real valued functions. The first of these represents a generalization of the Baskakov operators corresponding to a special form of the negative binomial probability distribution, while the second one constitutes a generalization of the operators of Meyer-König and Zeller, coerresponding to the usual form of the negative binomial probability distribution.

We mention that some other generalizations and investigations of the Baskakov operators and of the Meyer-König and Zeller operators can be found in the papers by Schurer [23], [24], Cheney and Sharma [3], Jakimovski and Ramanujan [11], Jakimovski and Leviatan [9], [10], Lupaş [15[, [16], Müller [20], [21], [22], Stancu [28], [31], Iperen and Sikkema [13], Stancu F. [34], Lupaş and Müller [17], Eisenberg [5] Iperen [12], King and Swetits [14], Eisenberg and Wood [6], Wood [35].

2. We define the operator $V_m^{[\alpha]}$, for each f, real valued function bounded on $[0, \infty)$, by

$$(V_m^{[\alpha]}f)(x)=\sum_{k=0}^{\infty} v_{m,k}^{[\alpha]}(x)\, f\left(\frac{k}{m}\right), \tag{1}$$

where for every $x \in [0, \infty)$ we have

$$v_{m,k}^{[\alpha]}(x)= \tag{2}$$

$$\binom{m+k-1}{k}\frac{(1+\alpha)(1+2\alpha)\ldots(1+\overline{m-1}\alpha)\,x(x+\alpha)\ldots(x+\overline{k-1}\alpha)}{(1+x)(1+x+\alpha)\ldots(1+x+\overline{m+k-1}\alpha)},$$

by using the notation: $\overline{n-r}\,\alpha=(n-r)\alpha$.

Since $v_{m,0}^{[\alpha]}(0) = 1$ and $v_{m,k}^{[\alpha]}(0) = 0$ if $k \geq 1$, one observes that we always have: $(V_m^{[\alpha]} f)(0)=f(0)$.

3. We notice that the fundamental polynomials $v_{m,k}^{[\alpha]}$ can be expressed in a more compact form by means of the notion of factorial power.

The factorial power of order n (:natural number) and increment h (:real number) of u is defined by

$$u^{(n,h)}=u\,(u-h)\ldots(u-\overline{n-1}h).$$

If $n=0$ and $u\neq 0$ we set $u^{(0,h)}=1$, while if $h=1$ we shall write, for brevity, $u^{(n)}$, in place of $u^{(n,1)}$. For $h=0$ we have $u^{(n,0)}=u^n$.

In the sequel we shall make use of the following rules for operating with factorial powers:

$$u^{(-n,h)}=\frac{1}{u^{(n,h)}},\quad u^{(-n,-h)}=\frac{1}{u^{(n,-h)}}=\frac{1}{(u+\overline{n-1}h)^{(n,h)}},$$

$$u^{(n,h)}=u^{(j,h)}\cdot(u-jh)^{(n-j,h)}, \tag{3}$$

if, of course, the denominators are different from zero, n and j being natural numbers such that $n>j$.

Evidently, we can write

$$v^{[\alpha]}_{m,k}(x)=\binom{m+k-1}{k}\frac{1^{(m,-\alpha)}x^{(k,-\alpha)}}{(1+x)^{(m+k,-\alpha)}}. \tag{2'}$$

4. The operators $V^{[\alpha]}_m$ include as a special case ($\alpha=0$) the Baskakov operators [2] defined by

$$(Q_m f)(x)=\sum_{k=0}^{\infty}\binom{m+k-1}{k}\frac{x^k}{(1+x)^{m+k}}f\left(\frac{k}{m}\right),$$

which can be constructed if one uses the form considered by Fisher [8] for the negative binomial distribution (see [28]).

We further note that if $\alpha>0$ and $x>0$ then one verifies directly that the fundamental polynomials given at (2) or at (2′) can also be represented in the following form

$$v^{[\alpha]}_{m,k}(x)=\binom{m+k-1}{k}\frac{B\left(\frac{1}{\alpha}+m,\ \frac{x}{\alpha}+k\right)}{B\left(\frac{1}{\alpha},\ \frac{x}{\alpha}\right)}, \tag{2'}$$

where B is the Euler beta function.

5. We shall now proceed to introduce the second class of operators ($W^{[\alpha]}_m$), defined by

$$(W^{[\alpha]}_m f)(x)=\sum_{k=0}^{\infty}w^{[\alpha]}_{m,k}(x)\,f\left(\frac{k}{m+k}\right), \tag{4}$$

where $0\leq x<1$, f is bounded on $[0,1]$ and

$$w^{[\alpha]}_{m,k}(x)= \tag{5}$$

$$\binom{m+k}{k}\frac{x\,(x+\alpha)\ldots(x+\overline{k-1}\alpha)\,(1-x)\,(1-x+\alpha)\ldots(1-x+m\alpha)}{(1+\alpha)\,(1+2\alpha)\ldots(1+\overline{m+k}\alpha)},$$

employing the factorial powers:

$$\omega_{m,k}^{[\alpha]}(x)=\binom{m+k}{k}\frac{x^{(k,-\alpha)}\,(1-x)^{(m+1,-\alpha)}}{1^{(m+k+1,-\alpha)}}. \qquad (5')$$

6. The operators $W_m^{[\alpha]}$ contain as a special case ($\alpha=0$) the operators of Meyer-König and Zeller [19]:

$$(M_m f)(x)=\sum_{k=0}^{\infty}\binom{m+k}{k}x^k(1-x)^{m+1}f\left(\frac{k}{m+k}\right),$$

obtained by these authors by starting from the usual negative binomial (Pascal) probability distribution.

7. One sees at once that if $\alpha>0$ and $0<x<1$, then the following representation is true

$$\omega_{m,k}^{[\alpha]}(x)=\binom{m+k}{k}\frac{B\left(\frac{x}{\alpha}+k,\ \frac{1-x}{\alpha}+m+1\right)}{B\left(\frac{x}{\alpha},\ \frac{1-x}{\alpha}\right)}, \qquad (6)$$

so that we can write

$$(W_m^{[\alpha]}f)(x)=\frac{1}{B\left(\frac{x}{\alpha},\ \frac{1-x}{\alpha}\right)}\int_0^1 t^{\frac{x}{\alpha}-1}(1-t)^{\frac{1-x}{\alpha}-1}(M_m f)(t)\,dt, \qquad (7)$$

for $\alpha>0$, if $0<x<1$, while if $x=0$ we have $(W_m^{[\alpha]}f)(0)=f(0)$.

8. It should be observed that if the parameter α is non-negative then $(V_m^{[\alpha]})$ and $(W_m^{[\alpha]})$ are sequences of *positive* linear operators.

It may be of interest to note that in [26] and [27] we have introduced, in a similar manner, a class of positive linear operators which represent a generalization of the Bernstein operator (see also the related papers [30], [28], [31], [32], [7] and [33]).

9. We shall now state and prove a theorem concerning the uniform approximation of functions by means of the operators $V_m^{[\alpha]}$.

THEOREM 1. *Let f be a real valued function defined and bounded on* $[0,\infty)$, *continuous on an interval* $[0,A]$, *where* $A>0$, *and continuous to the right on A.*

Assuming that $\alpha=\alpha_m$ *depends on m so that for* $m=1,2,\ldots$ *we have* $0\le\alpha_m\le\beta$ $\left(0<\beta<\frac{1}{2}\right)$ *and* $\alpha_m\to 0$ *as* $m\to\infty$, *then the sequence* $(V_m^{[\alpha]}f)$ *converges to f uniformly on* $[0,A]$.

Proof. Consider the hypergeometric series

$$F(a,b,c;y)=\sum_{k=0}^{\infty}\frac{a^{(k,-1)}\,b^{(k,-1)}}{c^{(k,-1)}}\cdot\frac{y^k}{k!},$$

where the parameters a, b and c satisfy the conditions: $a>0$, $b>0$ and $a+b<c$.

It is known (see, e. g. [1]) that, under these conditions, by taking $y=1$ one can give the following representation in terms of gamma function:

$$F(a, b, c, ;1)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}. \tag{8}$$

In order to prove our theorem we shall make use of a well known theorem of Korovkin on approximation of functions by means of positive linear operators.

It is clear that we may assume that $0<\alpha_m\leq\beta$ since for $\alpha_m=0$ our operator defined at (1)—(2) reduces to the Baskakov operator Q_m and for it the theorem is true (see [2]).

Consider the functions g_j, where $g_j(t)=t^j$ ($j=0, 1, 2$) for all $t\geq 0$. If we apply our operator to the function g_0 and we take into account (3) and (8) we can write successively

$$(V_m^{[\alpha]} g_0)(x) = \sum_{k=0}^{\infty}\binom{m+k-1}{k}\frac{1^{(m,-\alpha)}x^{(k,-\alpha)}}{(1+x)^{(m+k,-\alpha)}} \tag{9}$$

$$=\frac{1^{(m,-\alpha)}}{(1+x)^{(m,-\alpha)}}\sum_{k=0}^{\infty}\binom{m+k-1}{k}\frac{x^{(k,-\alpha)}}{(1+x+m\alpha)^{(k,-\alpha)}}$$

$$=\frac{1^{(m,-\alpha)}}{(1+x)^{(m,-\alpha)}}F\left(m, \frac{x}{\alpha}, \frac{1+x}{\alpha}+m; 1\right)$$

$$=\frac{1^{(m,-\alpha)}}{(1+x)^{(m,-\alpha)}}\frac{\Gamma\left(\frac{1+x}{\alpha}+m\right)\Gamma\left(\frac{1}{\alpha}\right)}{\Gamma\left(\frac{1+x}{\alpha}\right)\Gamma\left(\frac{1}{\alpha}+m\right)}=1,$$

since

$$\Gamma\left(\frac{1+x}{\alpha}+m\right)=\frac{(1+x)^{(m,-\alpha)}}{\alpha^m}\Gamma\left(\frac{1+x}{\alpha}\right),\quad \Gamma\left(\frac{1}{\alpha}+m\right)=\frac{1^{(m,-\alpha)}}{\alpha^m}\Gamma\left(\frac{1}{\alpha}\right).$$

Going on to g_1 we get

$$(V_m^{[\alpha]}g_1)(x)=\sum_{k=1}^{\infty}\binom{m+k-1}{k}\frac{k}{m}\cdot\frac{1^{(m,-\alpha)}x^{(k,-\alpha)}}{(1+x)^{(m+k,-\alpha)}}=$$

$$=\sum_{k=1}^{\infty}\binom{m+k-1}{k-1}\frac{1^{(m,-\alpha)}x^{(k,-\alpha)}}{(1+x)^{(m+k,-\alpha)}}, \tag{10}$$

because we have

$$\binom{m+k-1}{k}\frac{k}{m}=\binom{m+k-1}{k-1}.$$

If we use the change $k-1=j$ we obtain

$$(V_m^{[\alpha]}g_1)(x)=\sum_{j=0}^{\infty}\binom{m+j}{j}\frac{1^{(m,-\alpha)}x^{(j+1,-\alpha)}}{(1+x)^{(m+j+1,-\alpha)}}=$$

$$= \frac{1^{(m,-\alpha)}x}{(1+x)^{(m+1,-\alpha)}} \sum_{j=0}^{\infty} \binom{m+j}{j} \frac{(x+\alpha)^{(j,-\alpha)}}{(1+x+\overline{m+1}\alpha)^{(j,-\alpha)}} =$$

$$= \frac{1^{(m,-\alpha)}x}{(1+x)^{(m+1,-\alpha)}} F\left(m+1,\ \frac{x}{\alpha}+1,\ \frac{1+x}{\alpha}+m+1;\ 1\right) =$$

$$= \frac{1^{(m,-\alpha)}x}{(1+x)^{(m+1,-\alpha)}} \cdot \frac{\Gamma\left(\frac{1+x}{\alpha}+m+1\right)\Gamma\left(\frac{1}{\alpha}-1\right)}{\Gamma\left(\frac{1+x}{\alpha}\right)\Gamma\left(\frac{1}{\alpha}+m\right)} = x, \tag{11}$$

since

$$\Gamma\left(\frac{1+x}{\alpha}+m+1\right) = \frac{(1+x)^{(m+1,-\alpha)}}{\alpha^{m+1}} \Gamma\left(\frac{1-x}{\alpha}\right), \quad \Gamma\left(\frac{1}{\alpha}+m\right) =$$

$$= \frac{(1-\alpha)\cdot 1^{(m,-\alpha)}}{\alpha^{m+1}} \Gamma\left(\frac{1}{\alpha}-1\right).$$

We now proceed to the function g_2 and we get

$$(V_m^{[\alpha]} g_2)(x) = \sum_{k=1}^{\infty} \binom{m+k-1}{k} \left(\frac{k}{m}\right)^2 \frac{1^{(m,-\alpha)} x^{(k,-\alpha)}}{(1+x)^{(m+k,-\alpha)}}.$$

By using the identity $k^2 = k(k-1)+k$ we obtain

$$(V_m^{[\alpha]} g_2)(x) \sum_{k=2}^{\infty} \binom{m+k-1}{k} \frac{k(k-1)}{m^2} \cdot \frac{1^{(m,-\alpha)} x^{(k,-\alpha)}}{(1+x)^{(m+k,-\alpha)}} +$$

$$+ \frac{1}{m} \sum_{k=1}^{\infty} \binom{m+k-1}{k} \frac{k}{m} \cdot \frac{1^{(m,-\alpha)} x^{(k,-\alpha)}}{(1+x)^{(m+k,-\alpha)}}.$$

But in accordance with (10) and (11) the last sum gives $\frac{x}{1-\alpha}$, so that if we use the relation

$$\binom{m+k-1}{k} \frac{k(k-1)}{m^2} = \frac{m+1}{m} \binom{m+k-1}{k-2}$$

and make the change $k-2=j$ we find that

$$(V_m^{[\alpha]} g_2)(x) = \frac{m+1}{m} \sum_{j=0}^{\infty} \binom{m+1+j}{j} \frac{1^{(m,-\alpha)} x^{(j+2,-\alpha)}}{(1+x)^{(m+2+j,-\alpha)}} + \frac{x}{m(1-\alpha)} =$$

$$= \frac{m+1}{m} \cdot \frac{1^{(m,-\alpha)} x(x+\alpha)}{(1+x)^{(m+2,-\alpha)}} \sum_{j=0}^{\infty} \binom{m+1+j}{j} \frac{(x+2\alpha)^{(j,-\alpha)}}{(1+x+\overline{m+2}\alpha)^{(j,-\alpha)}} + \frac{x}{m(1-\alpha)} =$$

$$= \frac{(m+1)\, 1^{(m,-\alpha)} x(x+\alpha)}{m(1+x)^{(m+2,-\alpha)}} F\left(m+2,\ \frac{x}{\alpha}+2,\ \frac{1+x}{\alpha}+m+2;\ 1\right) + \frac{x}{m(1-\alpha)} =$$

$$= \frac{(m+1)\, 1^{(m,\,-\alpha)}\, x\,(x+\alpha)}{m\,(1+x)^{(m+2,\,-\alpha)}} \cdot \frac{\Gamma\left(\frac{1+x}{\alpha}+m+2\right)\Gamma\left(\frac{1}{\alpha}-2\right)}{\Gamma\left(\frac{1+x}{\alpha}\right)\Gamma\left(\frac{1}{\alpha}+m\right)} +$$

$$+ \frac{x}{m\,(1-\alpha)} = \frac{(m+1)\,x\,(x+\alpha)}{m\,(1-\alpha)\,(1-2\alpha)} + \frac{x}{m\,(1-\alpha)},$$

because

$$\Gamma\left(\frac{1+x}{\alpha}+m+2\right) = \frac{(1+x)^{(m+2,\,-\alpha)}}{\alpha^{m+2}}\,\Gamma\left(\frac{1+x}{\alpha}\right),\ \ \Gamma\left(\frac{1}{\alpha}+m\right) =$$

$$= \frac{(1-\alpha)\,(1-2\alpha)\cdot 1^{(m,\,-\alpha)}}{\alpha^{m+2}}\,\Gamma\left(\frac{1}{\alpha}-2\right).$$

Thus we obtained the following results

$$(V_m^{[\alpha]} g_0)\,(x)=1,\ \ (V_m^{[\alpha]} g_1)\,(x)=\frac{x}{1-\alpha}$$

$$(V_m^{[\alpha]} g_2)\,(x) = \frac{1}{(1-\alpha)\,(1-2\alpha)}\left[x^2+\frac{x\,(x+1)}{m}+\alpha\left(1-\frac{1}{m}\right)x\right]. \tag{12}$$

Now we can conclude that we have uniformly on $[0, A]$:

$$\lim_{m\to\infty} V_m^{[\alpha]} g_j = g_j \quad (j=0,\,1,\,2),$$

so that the assertion of our theorem follows according to Korovkin's theorem.

10. We now proceed to establish a similar result concerning the approximation of functions by means of the operators $W_m^{[\alpha]}$.

THEOREM 2. *Let $0<a<1$. If f is continuous on $[0,1]$ and $0\leq\alpha=\alpha_m\to 0$ as $m\to\infty$, then the sequence $(W_m^{[\alpha]} f)$ converges to f uniformly on $[0, a]$.*

Proof. For $\alpha=0$ this theorem was proved by Meyer-König and Zeller [19]. Consequently we may consider $\alpha>0$.

Since $(M_m f)\,(0)=f\,(0)$ we assume that $0<x\leq a<1$.

In order to give a short proof of this theorem we shall make use of the representation (7).

Since we have (see [20], [17]):

$$(M_m g_0)\,(t)=1,\ \ (M_m g_1)\,(t)=t,$$

$$(M_m g_2)\,(t)=t^2+\frac{t\,(1-t)^2}{m}+\beta_m\,(t)\ \ \left(|\,\beta_m\,(t)\,|\leq\frac{8}{27m\,(m-1)}\right)$$

we see at once that

$$(W_m^{[\alpha]} g_0)\,(x)=1\ \ (W_m^{[\alpha]} g_1)\,(x)=\frac{B\left(\frac{x}{\alpha}+1,\ \frac{1-x}{\alpha}\right)}{B\left(\frac{x}{\alpha},\ \frac{1-x}{\alpha}\right)}=x, \tag{13}$$

$$(W_m^{[\alpha]} g_2)\,(x)=\frac{x\,(x+\alpha)}{1+\alpha}+\frac{x\,(1-x)\,(1-x+\alpha)}{m(1+\alpha)(1+2\alpha)}+o\left(\frac{1}{m}\right)\quad (m\to\infty).$$

Hence the operator $W_m^{[a]}$ *preserves the linear functions* and we have uniformly on $[0, a]$:

$$\lim_{m\to\infty} W_m^{[a]} g_j = g_j \quad (j=0, 1, 2).$$

Appealing again to the theorem of Korovkin we see that the assertion of our theorem is true

11. Finally, we wish to indicate that according to the results given by Mamedov [18] and by Shisha and Mond [25], the orders of approximation of a continuous function f by means of the operators introduced in this paper are given respectively by the following inequalities

$$\| f - V_m^{[a]} f \| \leqq 2\omega(\mu_m), \quad \mu_m = \| V_m^{[a]} \varphi_x \|^{1/2},$$

$$\| f - W_m^{[a]} f \| \leq 2\omega(\gamma_m), \quad \gamma_m = \| W_m^{[a]} \varphi_x \|^{1/2},$$

where ω is the modulus of continuity of f, $\| \cdot \|$ stands for the supnorm over $[0, A]$ $(A>0)$, respectively over $[0, a]$ $(0<a<1)$, while the function φ_x is defined as follows: $\varphi_x(t) = (t-x)^2$ for all t in the basic interval $[0, A]$, respectively $[0, a]$, x being any fixed point in these intervals. Employing the results given at (12) and (13) one can obtain effective values for μ_m and γ_m.

In a forthcoming paper we shall establish some new properties of the linear positive operators introduced here and we shall give a new extension of them.

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DOUĂ CLASE DE OPERATORI POZITIVI LINIARI

În această lucrare se introduc două clase de operatori pozitivi liniari, care depind de un parametru, utili în teoria aproximării funcţiilor. Aceştia generalizează operatorii binecunoscuţi ai lui Baskakov şi ai lui Meyer-König şi Zeller. Autorul studiază proprietăţile de convergenţă ale şirurilor de operatori introduşi în cazul cînd ei se aplică unor anumite clase de funcţii.