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INSTITUTE OF NUMERICAL ANALYSIS  
ROMANIAN ACADEMY



TIBERIU POPOVICIU INSTITUTE OF NUMERICAL ANALYSIS  
ROMANIAN ACADEMY, CLUJ-NAPOCA BRANCH

# Numerical Analysis, Numerical Modeling, Approximation Theory

NA-NM-AT 2024

## Book of Abstracts

- version of November 3, 2024 -



Cluj Academic Days  
Edition XXXVII  
November 4–7, 2024



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# Linear approximation processes connected with squared fundamental functions

Octavian Agratini

*ICTP, Romanian Academy and Babeş-Bolyai University, Cluj-Napoca*

## Abstract

The aim of this talk is to present a general class of positive linear operators of discrete type related to squared fundamental basis functions. If these operators are expressed by a series, we propose to truncate them by a finite sum while still keeping the property of converging towards the identity operator in a certain weighted space. A relation between the local smoothness of functions and the local approximation is obtained. In the variant in which the operators are described by a finite sum, we establish the limit of their iterates.



# Regional control problems for interacting populations with nonlocal reaction terms

Sebastian Anița

*Al. I. Cuza University, Iași*

## Abstract

We investigate the problem of eradicating an alien predator population in a seasonal environment. It is expressed as a zero-stabilizability problem with state constraints. The dynamics of the predators is described by a prey-predator system with nonlocal reaction terms and time-periodic rates. First we are concerned with a control which acts on the predators, in a subset of the relevant habitat. A necessary condition and a sufficient condition for the eradicability (zero-stabilizability) of predators is given in terms of the sign of the principal eigenvalue of an appropriate operator that is not self-adjoint. Next we investigate the eradicability of the predators via controllers of harvesting type which act on the predator population or on the prey population. The idea of this last approach is to diminish the prey population to a level at which this cannot sustain the predator population. Some principal eigenvalues will be involved in this investigation as well.



# New ideas for discretizing convection dominated differential equations

Constantin Băcuță  
*University of Delaware, USA*

## Abstract

We present recent results regarding discretization for a model convection-diffusion problem. We relate an upwinding Petrov-Galerkin method with a corresponding finite difference upwinding approach and with the classical finite element stream-line diffusion discretization. We compare the resulting linear systems and the error estimates for the proposed discretizations. The ideas can be used in obtaining new and efficient discretizations for the multidimensional convection dominated problems.



# Bi-Lipschitz invariants of unitary group representations and Euclidean embeddings of orbifolds

Radu Bălan

*University of Maryland, USA*

## Abstract

Consider a finite group acting unitarily on a finite dimensional real vector space. This paper discusses Euclidean stable embeddings of the quotient space of orbits. Our embeddings are given by linear transformations of sorted coorbits. Our main result shows that, whenever such embeddings are injective they are automatically bi-Lipschitz. We show that any other Euclidean Lipschitz embedding factors through any such stable embeddings.

This is joint work with Efstratios Tsoukanis and Matthias Wellershoff.



# A computer assisted proof in shape optimization

Beniamin Bogosel

*École Polytechnique, Palaiseau, France*

## Abstract

It has been conjectured by Pólya and Szegő in 1951 that among  $n$ -gons with fixed area the regular one minimizes the first eigenvalue of the Dirichlet-Laplace operator. Despite its apparent simplicity, this result has only been proved for triangles and quadrilaterals. In this work we show that regular pentagons and hexagons are local minimizers.

The local minimality proof requires the non-negativity of the eigenvalues of the associated Hessian matrix. Since theoretical arguments do not allow, for now, to complete the proof, the Hessian eigenvalues are computed using numerical methods. To validate the non-negativity of these eigenvalues, explicit a priori error estimates for approximations of PDEs and interval arithmetic are used to control every error term appearing in the numerical approximations.

Results presented are obtained in collaboration with Dorin Bucur.



# Numerical methods for advection-dominated transport: a comparative study

Imre Boros

*ICTP, Romanian Academy, Cluj-Napoca*

## Abstract

We study numerical methods for one-dimensional, advection-dominated transport problems, focusing on minimizing numerical diffusion. We compare explicit finite difference (FD) schemes, a global random walk (GRW) algorithm, and a method of lines (MOL) approach. Our results show that FD and GRW methods perform similarly at certain Courant numbers, while the GRW—free from numerical diffusion—effectively preserves solution shape across various Courant values. These findings highlight GRW and MOL as efficient methods for controlling numerical diffusion in advection-dominated cases.





# Cauchy inverse problem with non-smooth boundary data in steady-state anisotropic heat conduction

Mihai Bucătaru

*University of Bucharest, Romania*

## Abstract

We investigate a situation that usually occurs for numerous real life problems in science and engineering, namely the reconstruction of the missing non-smooth thermal boundary conditions (i.e. temperature and normal heat flux) on an inaccessible portion of the boundary of an anisotropic solid from the knowledge of over-prescribed data (i.e. Cauchy data) on the remaining and accessible boundary in the steady-state case. It is well-known that the solution to this inverse Cauchy problem, in a very weak sense in this case, is unstable, provided that such a solution exists. Consequently, a stabilizing method is developed herein based on a priori knowledge on the solution to this Cauchy problem and the smoothing nature of the corresponding direct problems employed. More precisely, this inverse problem is approached by transforming it into a control one which reduces to solving an appropriate minimization problem in a suitable function space. The latter is approached by employing an appropriate variational method which yields a gradient-based iterative algorithm that consists, at each step, of two direct and two corresponding adjoint problems. It is worth mentioning that this algorithm is designed to approximate merely square integrable boundary data, hence the notion of solution and the convergence analysis require special attention. The numerical implementation is realized for various two-dimensional non-homogeneous anisotropic solids using the finite element method (FEM), whilst regularization is achieved by ceasing the iterative process according to three stopping regularizing criteria.

This is joint work with Iulian Cîmpean and Liviu Marin.



# Asymptotic analysis of a sign-changing transmission problem with a rapidly oscillating interface

Renata Bunoiu

*University of Lorraine, France*

## Abstract

We study the asymptotic behavior of a sign-changing transmission problem, stated in a symmetric oscillating domain obtained by gluing together a positive and a negative material, separated by an imperfect and rapidly oscillating interface. The interface separating the two heterogeneous materials has a periodic microstructure and is a small perturbation of a flat interface. The solution of the transmission problem is continuous and its flux has a jump on the oscillating interface. Under certain conditions on the properties of the two materials, we derive the limit problem and we prove the convergence result. The T-coercivity method is used to handle the lack of coercivity for both the microscopic and the macroscopic limit problems.

Based on joint work with K. Ramdani (INRIA Nancy, France) and C. Timofte (University of Bucharest, Romania).



# Sketching techniques for optimization problems

Coralia Cartîş

*University of Oxford, UK*

## Abstract

We discuss random and deterministic subspace methods for nonconvex optimization problems. We are interested in the global optimisation of functions with low effective dimensionality, that vary only along certain important directions or components. We show that the effective subspace of variation can be efficiently learned in advance of the optimization process; we contrast this with random embedding techniques that focus directly on optimization rather than learning. For local optimization, time permitting, we will also discuss efficient choices of subspaces that blend randomisation techniques with expert deterministic choices.



# Characterizing the classical convergence orders $p > 1$

Emil Căținaș

*ICTP, Romanian Academy, Cluj-Napoca*

## Abstract

The well known definition of the classical ( $C$ -)quadratic order of a sequence  $x_k \rightarrow x^* \in \mathbb{R}^N$  is that its errors  $e_k = \|x^* - x_k\|$  satisfy

$$\lim_{k \rightarrow \infty} \frac{e_{k+1}}{e_k^2} = Q_2 \in (0, +\infty),$$

which we prove that is equivalent to:  $\exists \rho \in (0, 1)$  and  $c_k \rightarrow c \in (0, +\infty)$  such that

$$e_k = c_k \cdot \rho^{2^k}, \quad k \geq 0.$$

The result is extended to orders  $p_0 > 1$ .

When the new asymptotic constants  $c$  and  $\rho$  are known in closed form, we get a priori evaluation of the errors and also of the number of steps required to attain a given error; two such examples are the approximation of  $\pi$  by the Brent-Salamin algorithm and the division in floating point arithmetic using the Newton method.

We give some a posteriori formulae for evaluating  $c$  and  $\rho$ , which we illustrate on matrix equations.

[1] E. Catinas, How many steps still left to  $x^*$ ?, *SIAM Rev.*, 63 (2021) no. 3, pp. 585–624, <http://doi.org/10.1137/19M1244858>

[2] E. Catinas, Characterizing the classical convergence orders, manuscript.



# Operator influence for real-time process digital twins: efficient and explainable surrogate modeling

Alexandru Ciobănaş  
*Siemens, Bucharest*

## Abstract

Surrogate modeling is a key technology in many real-time Digital Twin applications that require complex predictions. For highly non-linear processes, classical Reduced Order Modeling techniques, such as Krylov methods, are limited as well as require full access to solver internals. While Machine Learning technologies excel in these cases, their high data demands and lack of explainability limit their practical use. In this presentation, we review an active-learning-based Operator Inference approach to surrogate modeling. This method is built on Proper Orthogonal Decomposition combined with regression techniques. It explicitly leverages the structure of the underlying systems, making it fully explainable. Active learning ensures minimal data requirements, making the approach scalable for industrial applications. We will present the basic concept of this technology and demonstrate its application in a real-world battery thermal management case. The proof of concept was realized within a novel Surrogate Modeling Sandbox Framework for STAR-CCM+. We also quantify the storage and compute requirements, which are significantly smaller compared to classical Deep Learning methods.

## References:

- [1] A Differentiable Solver Approach to Operator Inference (2021)
- [2] Model Order Reduction based on Runge-Kutta Neural Network (2021)
- [3] Active-learning-based nonintrusive model order reduction (2023)
- [4] Operator inference with roll outs for learning reduced models from scarce and low-quality data (2023)



# How much data is needed to validate a mathematical model?

Stanca M. Ciupe  
*Virginia Tech, USA*

## Abstract

Uncertainty in parameter estimates from fitting mathematical models to empirical data limits the model's ability to uncover mechanisms of interaction. Understanding the effect of model structure and data availability on model predictions is important for informing model development and experimental design. To address sources of uncertainty in parameter estimation, I will present several methodologies that can help determine when a model can reveal its parameters. I will apply them in the context of virus infections in animals and humans at within-host, population, and multiscale levels.

Using these approaches, I will provide insight into the sources of uncertainty and provide guidelines for the types of model assumptions, optimal experimental design, and biological information needed for improved predictions.



# Good things come in threes: the interface dynamics of drops impacting onto a different liquid

Radu Cîmpeanu

*University of Warwick, UK*

## Abstract

In this talk we explore the canonical physical framework of a drop impacting onto a pool of another liquid, a scenario of relevance to many applications of interest, from inkjet printing to estimating environmental risks after oil spills. A combination of high-speed photography, direct numerical simulation, and mathematical modelling has been used to disentangle the different roles that physical fluid properties play in determining the detailed dynamics in this three-phase multi-fluid system. Simple mechanistic models and experiments of drops impacting onto a pool of the same fluid have led to estimates of the penetration speed being half the impacting drop speed. However, this is only one small part in a rich and intricate behaviour landscape once fluid properties are no longer identical – we uncover velocities between 10% and 90% of the reference initial drop velocity over a range of three orders of magnitude in density and viscosity ratios between the impacting drop and the pool, while also explaining these findings and summarising them into one key compact predictive formula (*Physical Review E* 104 (6), 065102, 2021). In higher speed contexts we then shed light into changes to the splashing threshold within the more complex landscape of three-phase flows (*Journal of Colloid and Interface Science* 641, 585-594, 2023), before ultimately turning to an intermediate system consisting of a drop impacting a pool covered by a thin liquid film (*Scientific Reports* 14 (1), 13671, 2024), as often found in practical applications of interest. The constructive (and on occasion subtle) interplay between analytical, numerical and experimental methodologies will be the core focus of the presentation.



# Analysis and modelling of some observational data

Maria Crăciun

*ICTP, Romanian Academy, Cluj-Napoca*

## Abstract

In this talk we will present a comparison between the Bose-Einstein Condensate model in the relativistic regime and the observational data from the Spitzer Photometry and Accurate Rotation Curves (SPARC) database. Our results indicate that the Bose-Einstein Condensate dark matter model can give a satisfactory description of the observational data.





# Low resolution hybrid models for geophysical fluid dynamics applications

Dan Crişan

*Imperial College, UK*

## Abstract

I will present a class of deterministic and stochastic energy-aware hybrid models that enable simulations of Geophysical Fluid Dynamics models at low resolutions without compromising on quality compared with high-resolution runs. The use of low resolutions makes hybrid models much faster than high-resolution physics-driven runs. This acceleration can be translated into the use of larger ensembles. The proposed hybrid approach is tested on a three-layer quasi-geostrophic model for a beta-plane Gulf Stream flow configuration. The results show that the low-resolution hybrid model reproduces the reference flow features that are resolved on the coarse grid and also gives a more accurate ensemble forecast than the physics-driven model.

This is joint work with Igor Shevchenko (National Oceanography Centre, UK) and it is based on the paper: I Shevchenko, D Crisan, On energy-aware hybrid models, *Journal of Advances in Modeling Earth Systems* 16 (8), 2024.



# Closed-loop control systems

Robert Csetnek

*Vienna University, Austria*

## Abstract

In order to develop fast optimization methods, we analyze the asymptotic behavior of a large class of autonomous dissipative inertial continuous dynamics. The function to be minimized (not necessarily convex) enters the dynamic via its gradient, which is assumed to be Lipschitz continuous on bounded subsets. This results in autonomous dynamical systems with nonlinear damping and nonlinear driving force. Stabilization of the trajectories and convergence rates are discussed.



# A framework for statistical inference via randomized algorithms

Edgar Dobriban

*University of Pennsylvania, USA*

## Abstract

Randomized algorithms, such as randomized sketching or stochastic optimization, offer a promising approach to alleviate the computational burden associated with analyzing large datasets. However, these algorithms produce non-deterministic outputs, which presents challenges in evaluating their accuracy. In this paper, we propose a statistical inference framework designed to quantify the uncertainty inherent in the outputs of randomized algorithms. Our primary conclusion is that statistical inference can be performed for the target of a sequence of randomized algorithms as long as, in the limit, their outputs fluctuate around the target according to any (possibly unknown) probability distribution. In this context, we introduce appropriate statistical inference methods—sub-randomization, multi-run plug-in, and multi-run aggregation—by estimating the unknown parameters of the limiting distribution through multiple runs of the randomized algorithm or tailored estimation techniques. To illustrate our framework, we develop methods for statistical inference of least squares parameters using random sketching, including sketch-and-solve, partial, and iterative sketching, while characterizing their limiting distribution in potentially high-dimensional scenarios. Furthermore, we apply our inference framework to stochastic optimization, encompassing stochastic gradient descent and stochastic optimization with momentum. The effectiveness of our results is validated through a comprehensive range of simulations.



# Loew-rank gradient flow – a first order algorithm for non-convex optimization

Andrei Drăgănescu

*University of Maryland Baltimore County, USA*

## Abstract

We introduce a novel class of first-order methods for unconstrained optimization, called low-rank gradient flows (LRGFs). The idea behind these methods is to construct at every optimization step a low-rank quadratic surrogate for the cost function, followed by an analytic solve for the gradient flow on the surrogate model; the optimization step concludes with a line search on the curve representing the gradient flow. It is shown that the above steps are condensed in a very simple formula for the gradient flow, at a cost per step that is comparable to that of a nonlinear conjugate gradient algorithm. The fact that the line search is conducted along a curve distinguishes LRGF from other first order optimization methods, where the line search is conducted along a search direction, that is, a straight line. This may also help LRGF better navigate the geometry of the cost function, allowing the method to avoid local minima more often than other first-order methods, as shown by numerical experiments. For higher dimensional problems the convergence can be accelerated using a multilevel strategy based on reduced order models.



# Distributed computing for physics-based data-driven reduced modeling at scale

Ionuț Farcaș

*University of Texas at Austin, USA*

## Abstract

High-performance computing (HPC) has revolutionized our ability to perform detailed simulations of complex real-world processes. A prominent contemporary example is from aerospace propulsion, where HPC is used for rotating detonation rocket engine (RDRE) simulations in support of the design of next-generation rocket engines; however, these simulations take millions of core hours even on powerful supercomputers, which makes them impractical for engineering tasks like design exploration and risk assessment. Reduced-order models (ROMs) address this limitation by constructing computationally cheap yet sufficiently accurate approximations that serve as surrogates for the high-fidelity model. We present a new distributed algorithm that achieves fast and scalable construction of predictive physics-based ROMs trained from sparse datasets of extremely large state dimension. The algorithm learns structured physics-based ROMs that approximate the dynamical systems underlying those datasets. This enables model reduction for problems at a scale and complexity that exceeds the capabilities of existing approaches. We demonstrate our algorithm's scalability using up to 2,048 cores on the Frontera supercomputer at the Texas Advanced Computing Center. We focus on a real-world three-dimensional RDRE for which one millisecond of simulated physical time requires one million core hours on a supercomputer. Using a training dataset of 2,536 snapshots each of state dimension 76 million, our distributed algorithm enables the construction of a predictive data-driven reduced model in just 13 seconds on 2,048 cores on Frontera. Distribution Statement A: Approved for Public Release; Distribution is Unlimited. AFRL-2024-1411



# Mixed Dirichlet–Robin problem for coupled anisotropic Darcy–Forchheimer–Brinkman systems

Andrei Gasparovici

*ICTP, Romanian Academy and Babeş-Bolyai University, Cluj-Napoca*

## Abstract

We study a mixed Dirichlet–Robin boundary value problem for a system of coupled anisotropic Darcy–Forchheimer–Brinkman equations as a general model of viscous flows in multidisperse porous media.

First, we consider the mixed Dirichlet–Robin problem for the anisotropic Darcy–Forchheimer–Brinkman system, a generalization of the Darcy–Forchheimer–Brinkman system for fluids with an anisotropic viscosity tensor. Regarding this problem, we present a well-posedness result for sufficiently small data obtained using a variational technique combined with the Banach fixed-point theorem.

Second, we consider the mixed Dirichlet–Robin problem for multiple coupled anisotropic Darcy–Forchheimer–Brinkman equations. Such a system generalizes models proposed in the literature for describing fluid flows in multidisperse porous media. Concerning this problem, we obtain an existence and uniqueness result by employing the Perov fixed-point theorem, a generalization of the Banach fixed-point theorem for operators defined on spaces endowed with vector-valued metrics.

Finally, as applications of the previously mentioned boundary value problems, we present some numerical results related to viscous flows in bidisperse and tridisperse porous media.



# Chebfun approximation to ground state solutions of some semilinear Laplace equations

Călin-Ioan Gheorghiu

*ICTP, Romanian Academy, Cluj-Napoca*

## Abstract

We use the Chebfun package to approximate numerically the structure of the set of positive radial solutions for a class of supercritical semilinear elliptic Dirichlet boundary value problems. This structure (bifurcation diagram) is plotted only at the heuristic level in many important works.

In this communication, we try to specify this structure as accurately as possible for the class of problems mentioned above.



# Multivariate rational approximation: Loewner matrix approaches and recent advances

Ion Victor Goșea

*Max Planck Institute, Magdeburg, Germany*

## Abstract

Multivariate rational model interpolation addresses the problem of constructing a reduced-order model that captures accurately the behavior of a potentially large-scale model depending on several variables. In the context of dynamical systems governed by differential and algebraic equations, the multivariate nature mainly comes from the parametric dependency of the underlying system or model. These parameters account for physical characteristics such as mass, length, or material properties (in mechanical systems), flow velocity, temperature (in fluid cases), chemical properties (in biological systems), etc. In engineering applications, the parameters are embedded within the model as tuning variables for the output of interest. The challenges and motivations for dynamical multivariate/parametric reduced order model (pROM) construction stem from three inevitable facts about modern computing and engineers' concerns: (i) First, accurate modeling often leads to large-scale dynamical systems with complex dynamics, for which simulation times and data storage needs become prohibitive, or at least impractical for engineers and practitioners; (ii) Second, the explicit mathematical model describing the underlying phenomena may not be always accessible while input-output data may be measured either from a computer-based (black-box) simulator or directly from a physical experiment; as a consequence, the internal variables of the dynamical phenomena are usually too large to be stored or simply inaccessible; (iii) Third, a potentially large number of parameters modifying the physical phenomena response may be necessary to be used in the next steps of the process.

We develop the so-called Loewner matrix framework, which is an interpolatory approach for the approximation of linear and nonlinear systems. The purpose is to extend this framework to linear parametric systems with an arbitrary number  $n$  of parameters. One main innovation established is the construction of data-based realizations for any number of parameters. Equally importantly, we show how to alleviate the computational burden, by avoiding the explicit construction of large-scale  $n$ -dimensional Loewner matrices. Several numerical test cases show the applicability of the proposed approach in various scenarios.





# Relaxed-inertial proximal point algorithms for problems involving strongly quasiconvex functions

Sorin Grad

*Institute Polytechnique de Paris, France*

## Abstract

Introduced in the 1970's by Martinet for minimizing convex functions and extended shortly afterwards by Rockafellar towards monotone inclusion problems, the proximal point algorithm turned out to be a viable computational method for solving various classes of optimization problems even beyond the convex framework.

In this talk we propose a relaxed-inertial proximal point type algorithm for solving optimization problems consisting in minimizing strongly quasiconvex functions whose variables lie in closed convex sets. The method is then extended for equilibrium functions involving strongly quasiconvex functions. Computational results confirm the theoretical advances.

Joint work with Felipe Lara (University of Tarapacá) and Raúl Tintaya Marcavillaca (Center for Mathematical Modeling, University of Chile).

## References

- [1] S. M. Grad, F. Lara, R. T. Marcavillaca, Relaxed-inertial proximal point algorithms for nonconvex equilibrium problems with applications, *Journal of Optimization Theory and Applications*, DOI: 10.1007/s10957-023-02375-1.
- [2] S. M. Grad, F. Lara, R. T. Marcavillaca, Relaxed-inertial proximal point type algorithms for quasiconvex minimization, *Journal of Global Optimization* 85:615-635, 2023.



# Numerical spectral analysis of Cauchy-type inverse problems: A probabilistic approach

Andreea Grecu

*ISMMA, Romanian Academy, Bucharest*

## Abstract

We introduce a probabilistic numerical approach for the reconstruction of the unknown boundary data of the steady state heat equation in a bounded domain in  $\mathbb{R}^d$ , having discrete measurements inside the domain and on a part of the boundary. We shall provide theoretical results which reveal that our approach is designed to spectrally approximate the inverse operator that we deal with. Finally, a parallel algorithm shall be presented together with numerical experiments.

This is based on joint work with Iulian Cîmpean and Liviu Marin.



# A Stancu type extension of the Cheney-Sharma-Chlodovsky operators

Eduard Grigoriuc

*ICTP, Romanian Academy and Babeş-Bolyai University, Cluj-Napoca*

## Abstract

In this presentation we discuss a Stancu type extension of the Cheney-Sharma Chlodovsky operators based on the ideas presented by Cătiņaş and Buda, Bostanci and Başcanbaz-Tunca, respectively Söylemez and Taşdelen. For this new operators we study some approximation and convexity properties and the preservation of the Lipschitz constant and order. Finally, we study approximation properties of the new operators with the help of Korovkin type theorems.



# A priori error bounds for reduced order models for fluids

Traian Iliescu  
*Virginia Tech, USA*

## Abstract

Galerkin reduced order models (ROMs), e.g., based on proper orthogonal decomposition (POD) or reduced basis methods, have achieved significant success in the numerical simulation of fluid flows. The ROM numerical analysis, however, is still being developed. In this talk, we take a step in this direction and present a survey of a priori error bounds, with a particular focus on POD-based ROMs. Specifically, we outline the main components of ROM a priori error bounds, emphasize their practical importance, and discuss significant contributions to a priori error bounds for ROMs for fluids.



# Surface reconstruction from noisy data

Stelian Ion

*ISSMA, Romanian Academy, Bucharest*

## Abstract

Surface reconstruction from scattered data is a 2D interpolation problem. There are many theoretical results and more many numerical algorithms related to the construction of a surface from sampled points. If the measurements are affected by random errors, the constructed surface exhibits oscillations and the numerical algorithms must cure the errors to produce a smooth surface. If one is only interested in the shape of the surface, then the penalized least squares method generally furnishes good results. The problem becomes more difficult if one wishes to generate surfaces with smooth curvatures. In this talk we will analyze a class of interpolation methods able to solve such problem.



# On signal calibration using generative models in data assimilation

Oana Lang

*Babeş-Bolyai University, Cluj-Napoca*

## Abstract

Data assimilation is fundamental to numerical modeling, enabling the incorporation of observational data into mathematical models to improve the accuracy and predictive capacity of simulations. This technique is extensively applied across meteorology, oceanography, and environmental science, where dynamic systems require continuous updates. However, calibrating high-dimensional, nonlinear models remains a significant challenge. In this study, we introduce a novel calibration approach using diffusion generative models to produce synthetic data that statistically matches observational increments derived from a numerical approximation of a partial differential equation. This method facilitates efficient model reduction, enabling the assimilation of data from a high-resolution reference system, modeled by a rotating shallow water equation with approximately  $10410^{4104}$  degrees of freedom, into a stochastic system with a substantially reduced dimensionality. The synthetic data is integrated using an enhanced particle filtering method that incorporates tempering and jittering, effectively addressing complex and multimodal distributions. Our findings illustrate how generative models can improve the predictive accuracy of particle filters, offering a more computationally efficient framework for data assimilation and model calibration in high-dimensional systems.

This is joint work with Alexander Lobbe and Dan Crisan (Imperial College London).



# Inverse source estimates for parabolic problems with boundary observations

Cătălin Lefter

*Al.I. Cuza University and Octav Mayer Institute, Iași*

## Abstract

This presentation is devoted to studying the null internal controllability of a time-dependent Kirchoff-Love thin plate with a middle surface having a comb-like shaped structure described by a small parameter  $\varepsilon$ . It is often impossible to directly approach such a problem numerically, due to the large number of thin fingers. So an asymptotic analysis is needed. In this paper, we first prove that the problem is null controllable at each level  $\varepsilon$ . We then prove that the sequence of the respective controls with minimal  $L^2$  norm converges, as  $\varepsilon$  vanishes, to a limit control function ensuring the optimal null controllability of a degenerate limit problem set in a domain without oscillating boundary.



# Spherical interpolation of scattered data by means of combined Shepard operators

Andra Malina

*ICTP, Romanian Academy and Babeş-Bolyai University, Cluj-Napoca*

## Abstract

We consider the problem of interpolating large sets of scattered data on the sphere using combined Shepard operators. The first approach involves a local Shepard-type method and two spherical radial basis functions. The second method combines Bernoulli operators with two different Shepard-type basis functions. We study the interpolation properties and approximation errors of the proposed methods and evaluate their efficiency using several test functions and datasets. The practical applicability is demonstrated through two real data problems.





# Central limit theorems for smooth optimal transport maps

Tudor Manole  
*MIT, USA*

## Abstract

One of the central objects in the theory of optimal transport is the Brenier map: the unique monotone transformation which pushes forward an absolutely continuous probability law onto any other given law. A large body of recent work has studied the question of estimating Brenier maps between unknown measures, on the basis of i.i.d. observations. In this work, we derive an estimator for Brenier maps which satisfies a pointwise central limit theorem. This provides a first step toward the question of performing uncertainty quantification for these maps. Our proofs hinge upon a quantitative linearization of the Monge-Ampere equation governing the optimal transport problem, which may be of independent interest.



# Tangent space reconstruction of statistical manifolds arising in deep learning

Sorin Mitran

*University of North Carolina at Chapel Hill, USA*

## Abstract

An algorithm to accelerate deep neural network (DNN) training is presented based upon computational reconstruction of the statistical manifold describing DNN weights. DNN weights encountered during the training process (e.g., by gradient descent) are interpreted as data points of an undefined statistical manifold. Simplicial complexes are constructed to represent the manifold, and the essential structure is extracted through persistent homology. The resulting reduced-dimensional representation of the statistical manifold is then used to choose subsequent gradient descent directions.



# An inverse geometric problem in 2D steady-state anisotropic heat conduction

Liviu Marin

*University of Bucharest and ISMMA, Romanian Academy, Bucharest*

## Abstract

The inverse problem investigated is concerned with the numerical reconstruction of the unknown inner boundary, in a 2D anisotropic solid from homogeneous Dirichlet data available on the entire boundary of and an additional measurement of the Neumann data on the outer boundary assumed to be known. This so-called inverse geometric problem is tackled by defining an operator that maps an admissible inner boundary belonging to the space of  $2\pi$ -periodic and twice continuously differentiable functions into the Neumann data on the outer boundary assumed to be continuous. It is proved that this operator is differentiable and hence a gradient based algorithm that uses the anisotropic single layer representation of the solution to a mixed problem for the 2D anisotropic heat conduction is developed for approximating the unknown inner boundary. The numerical implementation is realised for both exact and perturbed Neumann data on the outer boundary and employs a quadratic finite element method (FEM) and a suitable quadrature for the anisotropic single layer representation of the solution, and shows the convergence and stability of the proposed method.

This is a joint work with Andrei Tiberiu Pantea (École Polytechnique, Institut Polytechnique de Paris).



# Remarks on a continuum model for hierarchical fibril assembly

Adrian Muntean

*Karlstads Universitet, Sweden*

## Abstract

Biological polymers that make up our cells and tissues are usually hierarchically structured. Such structural hierarchy must be encoded in the self-assembly process, from the earliest stages onward, in order to be able to produce the appropriate sub-structures. In this talk, we explore specific interaction features linked to the kinetics of multistage self-assembly processes via a multiscale discrete population dynamics model of Becker-Döring type. After performing (formally) a discrete-to continuum limit followed by a suitable two-scale homogenization asymptotics, we derive a set of continuum equations that can interpret turbidimetry data on the self-assembly of collagen fibrils. This presentation revisits old results reported in B. S. van Lith, A. Muntean, C. Storm, EPL 106 (2014) 68004 and points out new possible research questions.



# Updated Lagrangian for hyperelastic material with contact and impact absorption

Cornel Murea

*Université de Haute Alsace, France*

## Abstract

We study numerically the dynamic impact/contact of an elastic body on a moving foundation using the mid-point algorithm. Stability results are presented when foundation is decreasing. Numerical simulations on two-dimensional problems are included and we show that the energy is absorbed in the case of decreasing foundation compared to the fixed one.

Ann. Acad. Rom. Sci. Ser. Math. Appl. Vol. 15, No. 1-2 (2023), 353-366 [1].



## Stabilized state estimators for time-dependent PDEs

Mihai Nechita

*ICTP, Romanian Academy and Babeş-Bolyai University, Cluj-Napoca*

### Abstract

In many relevant applications modeled by partial differential equations (PDEs), one does not have access to full data in order to deal with well-posed problems. For example, a part (or the whole) of the boundary could be inaccessible for measurements or the initial condition could be unknown. In this talk, we will consider such ill-posed problems with partial interior data for time-dependent PDEs.

We will discuss a regularization approach that combines a stabilized finite element method (FEM) – to assimilate the partial data – with a Luenberger observer containing a nudging term.

The stabilized FEM is based on PDE-constrained optimization with discrete regularization and will provide completed data (boundary conditions) for the Luenberger observer.

Apart from the heat and the Stokes' equations, we will consider a fluid-structure interaction (FSI) model in which noisy velocity measurements are given in a subset of the computational domain. The goal would be to estimate the whole fluid state, in particular the pressure. Such problems are related to blood flow and medical imaging data, e.g. 4d-flow MRI data measuring the 3d velocity field of a tissue.

This is joint work with Miguel Fernandez, Muriel Boulakia, Damiano Lombardi.



# Stochastic approximations and delays

Silviu Niculescu  
*CNRS, France*

## Abstract

tba



# Feature translation for exemplar-free class-incremental learning with hill-climbing

Darian Onchiş

*West University of Timișoara*

## Abstract

Exemplar-free class-incremental learning (EFCIL) poses significant challenges, primarily due to catastrophic forgetting, necessitating a delicate balance between stability and plasticity to accurately recognize both new and previous classes. Traditional EFCIL approaches typically skew towards either model plasticity through successive fine-tuning or stability by employing a fixed feature extractor beyond the initial incremental state. Building upon the foundational FeTrIL framework, our research extends into novel experimental domains to examine the efficacy of various oversampling techniques and dynamic optimization strategies across multiple challenging datasets and incremental settings. We specifically explore how oversampling impacts accuracy relative to feature availability and how different optimization methodologies, including dynamic recalibration and feature pool diversification, influence incremental learning outcomes. Notably, our extensions reveal the nuanced impacts of oversampling and optimization on EFCIL, contributing to a more refined understanding of feature-space manipulation for class incremental learning.





# A probabilistic Takens embedding theorem

Maria Oprea

*Cornell University, USA*

## Abstract

In the theory of dynamical systems, Takens embedding theorem allows the reconstruction of the true underlying attractor from time series data with missing states. In particular, it proves that an embedding map exists between the two, which comes from the time delay of an observable. However, data measurements are often polluted by noise. Hence, we shift the paradigm from Lagrangian to Eulerian by looking at probability densities in the Wasserstein spaces and gradient flows. In this work, we generalize the Takens embedding theorem to the Eulerian framework by considering an embedding between Wasserstein spaces. We show that the classic delay embedding map as a push-forward map provides an embedding between Wasserstein spaces. We present theoretical guarantees for reconstructing the attractor from noisy data and when the dynamics are inherently stochastic. Moreover, the weaker condition we impose when learning the delay embedding map can help improve the algorithm stability.



# Fixed point results for non-self operators on $\mathbb{R}_+^m$ -metric spaces

Diana Otrocol

*ICTP, Romanian Academy and Technical University of Cluj-Napoca*

## Abstract

The purpose of this paper is to discuss some problems of the fixed point theory for non-self operators on  $\mathbb{R}_+^m$ -metric spaces. The results complement and extend some known results given in the paper:

[1] A. Chis-Novac, R. Precup, I.A. Rus, Data dependence of fixed points for non-self generalized contractions, *Fixed Point Theory*, 10(2009), No. 1, 73–87.

[2] D. Popa, I. Rasa, Hyers-Ulam stability of the Laplace operator. *Fixed Point Theory*, 2018, 19(1), 379-382.



# Stability of hydroelastic waves

Emilian Părău

*University of East Anglia, UK*

## Abstract

Two-dimensional periodic travelling hydroelastic waves on water of infinite depth are investigated numerically. The stability of these periodic waves is examined using a surface-variable formulation in which a linearised eigenproblem is stated on the basis of Floquet theory and solved numerically. The eigenspectrum is discussed encompassing both superharmonic and subharmonic perturbations.



# Scalable approaches for contact mechanics using homotopy methods and algebraic multigrid

Cosmin Petra

*Lawrence Livermore National Laboratory, USA*

## Abstract

We present path-following continuation numerical methods for nonlinear complementarity problems with focus on solving contact mechanics using scalable algebraic multigrid methods (AMG). We introduce a general non-interior regularized homotopy proposed for  $P_0$  complementarity problems that uses a filter trust-region globalization with Newton-like centering steps and Newton targeting predictor steps to efficiently follow and stay within a controlled neighborhood of the homotopy path towards a solution of complementarity problem. A salient feature of the proposed method is that it does not break when used for complementarity problems that do not possess the regularity (such as monotonicity,  $P$ ,  $P^*$ , or  $P_0$  properties) needed by the well-posedness of the homotopy. Second, we present the performance of a more mainstream homotopy approach, namely a nonlinear interior-point method, with focus on using AMG methods for solving the contact equilibrium equations governed by elastic deformations. Numerical results obtained with the two approaches are discussed at the end, including high-performance simulations of challenging contact problems.



# An information geometric viewpoint on the Bregman proximal gradient method

Ștefania Petra

*Heidelberg University, Germany*

## Abstract

We explore the dual interpretations of the exponentiated gradient method as both a Bregman proximal gradient method and a Riemannian gradient descent on the parameter manifold of a distribution within the exponential family. This duality enables the integration of constraints while facilitating accelerated iterates. Our focus is on minimizing the Kullback-Leibler divergence between a linear model  $Ax$  and a positive vector  $b$  across various convex domains, including the positive orthant,  $n$ -dimensional box, and probability simplex. In this context, the exponentiated gradient method yields efficient multiplicative updates. To demonstrate the effectiveness of the proposed acceleration schemes, we present results from extensive numerical experiments on large-scale datasets.



# On soft, hard and relaxed symmetry priors for Equivariant Neural Networks

Mircea Petrache

*Pontificia Universidad Católica de Chile*

## Abstract

We will present recent work in collaboration with Shubhendu Trivedi (NeurIPS 2023) on theoretical estimation on the trade-off between generalization and approximation error gains when applying relaxed versions of equivariance, in neural network learning of partially and approximately equivariant data. This requires a rethinking of classical risk bounds in PAC-learning, keeping in mind the equivariance properties of the data and of the model. Our framework allows to justify the previously heuristically described principle, according to which there exists a “correct quantity of symmetries” to be imposed to a learning model, for optimal test error performance. I will also pinpoint related works and open questions, if time allows.



# On solving a Stefan problem for nonlinear degenerate equations

Sorin Pop

*Hasselt University, Belgium*

## Abstract

This work is motivated by unsaturated flow, or two-phase flow in a porous medium. Commonly used mathematical models assume that quantities like saturation, phase pressure differences, or relative permeability are related by monotone, algebraic relationships. These are determined experimentally, under equilibrium conditions. The mathematical models derived under such conditions involve nonlinear, possibly degenerate parabolic equations. Their (weak) solutions do satisfy a maximum principle, which rules out several phenomena that have been observed in experiments, like saturation overshoot, or the formation of finger profile.

These shortcomings are caused by the modelling paradigm, relying on the assumption that the relationships mentioned above are algebraic and disregarding the dynamics of the flow. In the past decades, non-equilibrium models have been developed, where dynamic or hysteretic effects are included in the above-mentioned relationships. This leads to nonlinear pseudo-parabolic and degenerate equations, possibly involving differential inclusions.

In this presentation we briefly address the existence and uniqueness of weak solutions to such models. We then discuss different numerical schemes, including aspects like the rigorous convergence of the discretization, domain decomposition, and solving the emerging nonlinear time-discrete or fully discrete problems.

This is a joint work with K. Mitra (Eindhoven), S. Lunowa (Munich), S. Karpinski (Esprit Engineering), F. Radu (Bergen), X. Cao (Toronto), C. Rohde (Stuttgart)



# Robust and efficient solvers for different poromechanics models

Florin Radu

*University of Bergen, Norway*

## Abstract

We will present robust and efficient fixed-stress type splitting solvers for poromechanics, i.e. for fully coupled flow and deformation in porous media. Stabilization and optimization will be discussed [3]. A new family of splitting schemes based on approximate Schur complement [2] and a nonlinear poromechanics model [1] will be presented as well.

[1] J. Kraus, K. Kumar, M. Lymbery, F.A. Radu, A fixed-stress splitting method for nonlinear poroelasticity, *Engineering with Computers*, 2024.

[2] R. Nuca, E. Storvik, F.A. Radu, M. Icardi, Splitting schemes for coupled differential equations: block Schur-based approaches and partial Jacobi approximation, *Computers and Mathematics with Applications* 161, 2024.

[3] E. Storvik, J. W. Both, K. Kumar, J.M. Nordbotten, On the optimization of the fixed-stress splitting for Biot's equations, *IJNME* 120, 2019.





# Uniform boundary controllability property of the semi-discrete 1D wave equation

Ionel Roventă

*University of Craiova, Romania*

## Abstract

We consider a finite-difference semi-discrete scheme for the approximation of boundary controls for the one-dimensional wave equation. It is well-known that high frequency numerical spurious oscillations lead to a loss of the uniform (with respect to the mesh size) controllability property of the semi-discrete model in the natural setting. By filtering high frequencies of the initial data in an optimal range, we restore the uniform controllability property. Moreover, we obtain a relation between the range of filtration and the minimal time of control needed to ensure the uniform controllability.



# A mutual control problem for semilinear systems via fixed point approach

Andrei Stan, Radu Precup

*ICTP, Romanian Academy and Babeş-Bolyai University, Cluj-Napoca*

## Abstract

The concept of a mutual control problem is introduced and discussed using a vector fixed-point approach, drawing on the theorems of Perov, Schauder, and Avramescu. Additionally, our analysis incorporates a novel technique involving Bielecki equivalent norms.

This is a joint work with R. Precup.



# Optimal contraction orders of tree tensor networks

Mihail Stoian

*University of Technology Nuremberg, Germany*

## Abstract

We show, through a surprising connection to database systems, that tree tensor networks admit optimal linear contraction orders. Tree tensor networks are of increasing interest today in quantum physics and quantum chemistry due to their expressibility. Our work thus enables fast processing times for these downstream applications. The algorithm is publicly available at <https://github.com/stoianmihail/netzwerk>.

This is a joint work with the Quantum Computing group at TUM.



# Global random walk for Stefan-like moving-boundary problems

Nicolae Suciu

*ICTP, Romanian Academy, Cluj-Napoca*

## Abstract

This talk presents global random walk approximations of solutions to one-dimensional Stefan-type moving-boundary problems. We are particularly interested in the case when the moving boundary is driven by an explicit representation of its speed. This situation is usually referred to in the literature as moving-boundary problem with kinetic condition. As a direct application, we propose a numerical scheme to forecast the penetration of small diffusants into a rubber-based material. To check the quality of our results, we compare the numerical results obtained by global random walks either using the analytical solution to selected benchmark cases or relying on finite element approximations with a priori known convergence rates. It turns out that the global random walk concept can be used to produce good quality approximations of the weak solutions to the target class of problems.

Work in collaboration with: Surendra Nepal, Yosief Wondmagegne, Magnus Ögren, and Adrian Muntean.



# On a non-local inversion problem in planar vector tomography

Alexandru Tămășan  
*University of Central Florida, USA*

## Abstract

I will present some recent results on the reconstruction of a vector field from its momenta ray-transform. Of particular interest is a specific Region-of-Interest problem from partial data.

This is joint work with H. Fujiwara (Kyoto University, Japan) and K. Sadiq (Radon Institute for Computational and Applied Mathematics, Austria).



# Computational models of bioconvective flows

Cătălin Trenchea

*University of Pittsburgh, USA*

## Abstract

Bioconvection describes the mixing of fluid and micro-organisms exhibiting negative geotaxis movement under the force of gravity.

The collective population moves towards the surface of the fluid, generating a Rayleigh–Taylor instability, where initial fingers of organisms plummet to the bottom.

The inherent drive to swim vertically generates large collective flow patterns that persist in time.

The mathematical model consists of the Navier-Stokes equations for an incompressible, viscous fluid, coupled with the transport equation describing the concentration of the micro-organisms.

We propose a numerical method based on a partitioned, second-order, symplectic, time adaptive algorithm and the finite element method for spatial discretization.

The method is long time energy-stable, and for small time steps, the iterative procedure in the partitioned algorithm is linearly convergent.



# Time density in physics-informed neural networks (PINNs)

Gabriel Turinici

*Université Paris Dauphine, France*

## Abstract

In order to develop fast optimization methods, we analyze the asymptotic behavior of a large class of autonomous dissipative inertial continuous dynamics. The function to be minimized (not necessarily convex) enters the dynamic via its gradient, which is assumed to be Lipschitz continuous on bounded subsets. This results in autonomous dynamical systems with nonlinear damping and nonlinear driving force. Stabilization of the trajectories and convergence rates are discussed.