Multi-resolution analysis, numerical methods and reverse dynamics, integrated in software instruments for performance improving in the satellite remote sensing

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Abstract-**In this paper we present the general conception and the mathematical background of a software environment aimed to provide enhanced remotely-sensed images processing capabilities. Integral and discrete transforms, multi-resolution analysis, reverse dynamics, symbolic computation principles and tools will be involved. As mathematical background, will be presented some details concerning discrete transforms, multi-resolution analysis, reverse dynamics.**

I. Introduction

Our goal in the project that is the subject of this paper consist in integrating robust components, based on advanced mathematics (integral and discrete transforms, multi-resolution analysis, reverse dynamics, symbolic computation) in the context of an ultra performing software system for satellite image and data processing, with capability of very fine details extraction, both in on-line and off-line manner, its realization being one of the major objectives of the project, actually, the final objective. The system will also integrate components for making compatible the assistance of the technology for image and satellite data transformation, with the features of the orbital movements analyzed through the reverse dynamics perspectives. At the basis of the software system development there will be, as general support, the symbolic computation, and concerning the image processing components, these will be developed starting from the complete set of fundamental operators defined and implemented according to the 120873 ISO-IEC standard, on the constructive principles of the PIKS (Programmers Imaging Kernel System) methodology.[5][6] The system will be destined both to the inter-university research and to the integration of this kind of research in the European and Euro-Atlantic excellence networks.

According to this global objective, the development of advanced satellite provided data and image processing instruments requests research in the domain of the mathematical methods, oriented on performance increase concerning the processing time, and also substantial quality increase, compared with the performance of the existing systems.

The gain will be complex, consisting in the realization of a computerized system, with advanced capabilities, and also in some high value scientific results obtainment.

II. State Of The Art

The satellite based remote sensing is, nowadays, a domain with a great impact on the majority of the socio-economic life areas, as well as a field of scientific challenges that are circumscribed to the high-level technology.

In the domain of satellite-based remote sensing involved sensors, there exist, nowadays, some experimental sensors that can perceive multiple channels. The sensors that perceive up to 10 channels are called multi-spectral sensors, those that perceive hundreds of channels are called hyper spectral sensors and the sensors that perceive more than one thousand of channels are called ultra-spectral channels. The last ones are not available for the GIS applications in the civil domain and won't be available in the near future. Nowadays, some hyper spectral sensors are being experimented on airplanes, namely AVIRIS (Advanced Visible/Infrared Imaging Spectrometer) and CASI (Compact Air bone Spectrographic Imager). These two sensors have between 224 and 288 channels with a bandwidth between 3 and 10 nm and have a narrow detection field, as well as a great radiometric resolution.

The sensors and scanners on the satellites (or airplanes) are building an image as a two dimensional matrix , as the consequence of the spatial variations of the air brightness. This method is called scanning or deflection and the result is a scanned image in raster format. Concerning the sensors that are used on satellites, the CCD sensor (Charge Coupled Device) is materialized through an electronic chip that can store a single pixel in the image. The principle is based on the magnetically charging of the chip when the EM wave intercepts the detector. The magnetically charging is proportional with the brightness of the landscape. These devices are capable to differentiate a very large brightness domain, comparing to the photographic film.

1-4244-0361-8/06/$20.00 ©2006 IEEE

The satellite operations are divided in three large groups: the operations concerning the geometric and radiometric corrections, operations concerning the registration and geo-referential activity, and operations applied on images that have as final purpose the extraction of significant information. The image provider performs the first two groups generally, after the image acquisition, so that the final user cannot interfere in changing their structure. The algorithms that were used in the geometric corrections (the panoramic distortion, the distortion due to the rotation of the Earth) and in the radiometric corrections (stripes elimination, noise elimination, the variation in time of the sun light, the atmospherically effects), are generally accepted by the international scientific community. The algorithms concerning the geo-referential activity are also considered correct up to the spatial resolution of the images that are being delivered nowadays.

The image quality improvements, for direct interpretation or for significant information extraction, are representing the main domains where contributions can be added. In general, the commercial software products (ERDAS, PCI, ER Mapper) implement two types of algorithms: those of general interest and specific algorithms. Sensors like SARF and LIDAR have specific algorithms. In the LiDAR case, the algorithms are still in development.

The image enhancement algorithms are: the contrast extension, the spatial filtering, the Principal Component Analysis, the Fourier Transform. The first two methods have more corresponding algorithms that are implemented in different ways by the commercial software. The main problem is the satellite image nature that , until the year 200 were based on Lanndsat structures. After the year of 2000, a lot of satellites with a big radiometric and spatial resolution appeared so that a great part of the algorithms should be revisited.

The main operation group that has as result the extraction of information with significance is integrated in the field of image classification. The purpose is the image simplification through the reduction of the total domain of numbers and spatial entities in more restrictive categories. IN other words, the purpose is that of building thematic images in which every pixel is assigned to a particular class of objects, based on the spectral response. The classification operations were initially implemented for Landsat images, that have a spatial multi-spectral resolution of 30 m (7 bands), a panchromatic band of 15 cm and a radiometric resolution of 8 bits/pixel. Nowadays, the spatial resolution is, in the civil domain, 0,60 m and the radiometric resolution is 12 bits/pixel. This thing imposes the creation of new classification algorithms.

Concerning the mathematical domain, the nowadays knowledge reflects an advanced level of optimisation of the mathematical algorithms, that will be involved in the software processing that is the final objective of the research. The Fourier, Gabor and Wavelet Transforms, together with their discrete variants, have known the development of some ultra rapid variants, very useful in the on-line processing.

III. Main Objectives

The problems that were proposed in the context of this paper, are especially connected to the improvement of the performances in the satellite based remote sensing, in the direction of technology improvements starting from the mathematical foundations of the signal processing, integration of methods that have been improved in complex contexts, with maximum utility and, finally, the realization of an integrated , complex, coherent and self perfectible system, through its capabilities of integration, at functional resort level, of perfected methodologies in order to test them and to directly integrate them in the system after the validation.
Concerning the image information extraction capabilities, the efforts will be concentrated on the discovery of the hidden regions, as well as on the correction of the distortions which correspond to the orbital movement of the satellite and to the curvature of the Earth.

The objectives that result from the symbolic computation involvement in the researches concerning the optimization of the results in satellite based remote sensing are:

- Using the formal computation in the study of some combinatorial problems that are connected to the subject
- Graphical representation for some complex variable functions; determination of the single valence satellite radius, convexity, results generalization
- Various methods of symbolic computation for the Fourier coefficients, asymptotic developments for the Fourier coefficients, explicit developments for various orthogonal systems, extending the previously obtained results

As a results improvement oriented objective related to the context of the project, the purpose will be to obtain some estimations of the approximation error and the improvement of the convergence rate, for various subclasses of finite energy signals.

IV. Scientific and Technical Background

The material, which will be processed by the advanced methods of multi-resolution analysis, symbolic calculus and reverse dynamics, consists of the satellite based remotely sensed images.

Generally, these systems provide essential information, even if a preliminary processing misses, but the methods and algorithms based especially on multi- resolution analysis and symbolic calculus together with powerful computer equipments are very strong tools and their performances are unlimited.

A basic concept of such a project is that of ***resolution***, defined for each equipment or phenomenon that needs a feature of this type; concerning digital images, we have ***spatial***, ***radiometric***, ***spectral*** and ***temporal resolutions***.

Spatial resolution, defined in the simplest way, is the smallest terrestrial surface whose reflectance may be measured by a sensor [1].

Concerning the ***radiometric resolution***, it is to consider that all scanners provided data have a certain intensity domain. The maximum of this intensity is corresponding to the maximum brightness of the objects, the minimum value (zero) corresponds to the objects that have not reflection property at all (so called *absolute black*). So, the ***radiometric resolution (radiometric sensitivity)*** is defined as referring to the number of digital levels used to express the data collected by the sensor. [1]

The ***spectral resolution*** refers to the width of the discrete spectral bands in which an individual image is recorded. [1]

Temporal resolution is defined as being the time interval between two consequent passages of the satellite over the same terrestrial area.

On the general way of obtaining accurate visual information, the symbolic calculus and the multi-resolution analysis are very important, especially for the so-called hidden details. The software system aimed for satellite image and data processing, will be built based on a vigorous mathematical background, some of these mathematical tools being presented in that follows:

1. The continuous time (integral) Fourier transformation associates to a continuous -time signal f(t) in $L^1(R)$ its transform in frequency domain

$$(1)\ F(\omega) = (Ff)(\omega) = \int_{-\infty}^{\infty} f(t)\exp(-i\omega t)dt;\ \omega \in R$$

also named frequency spectrum of *f(t)*. [2]

The inversion formula

$$(2)\quad f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\exp(i\omega t)d\omega;\ t \in R$$

shows that, for a wide class continuous-time signals, f(t) can be reconstituted knowing its spectrum.

So, a two sided-transfer of information time-frequency with important engineering applications is achieved.

The numerical processing of the continuous-time needs a sampling in the time domain, $x_n=f(nT)$, n ϵ Z, so that we obtain a sequence of equidistant samples.

It is well-known the famous theorem WKS (Whittacker - Kotelnikov - Shannon) which shows that for each signal function of finite energy with banded frequency band (of width 2B) the sampling does not lead to the loss of the information; more exactly, the following equality

$$(3)\quad f(t) = \sum_{n=-\infty}^{\infty} f(\frac{n\pi}{B})sa(Bt - n\pi),$$

holds, where ***sa*** (or ***sinc***) is the so-called cardinal-sinus function (or sampling function).

Fourier Transform, together with Fourier series and numerical methods associated, was a good instrument concerning the study and processing of periodical or non-periodical signals belonging to some sufficiently singular classes.

Gradually, according as to the difficulties generated by the complexity of the practical applications, some drawbacks of the classical Fourier analysis have emerged. So, to calculate the spectrum of a single frequency F, there must (see (10)) be known the values of f(t) for all real values of t. Similarly, in order to make use of the inversion formula (2) we need all real values of the frequencies. Moreover, the numerical methods associated to this problems, have a slow convergence.

Practically, we need the values of f(t) only for some intervals, so that the spectral information is generated for a given frequency band. So, one introduces the notion of flexible window, time -frequency due to Nyquist, Wiener and D. Gabor. Firstly, one considered only translations of windows and the spectrum F was studied on some closed intervals of time and frequency, respectively.

Gabor Transform (named also Short-Time Fourier Transform) is defined as a function depending on two parameters (variables), associated to the time and frequency. More exactly, given a "window" h ϵ $L^1(R) \cap L^2(R)$ with th(t) ϵ $L^2(R)$, and a continuous time signal of finite energy f(t), the function

$$(4)\ F_h\ f: RxR \to C, (F_h f)(b,\omega) = \int_{-\infty}^{\infty} f(t)\overline{h}(t-b)e^{-i\omega t}dt,$$

defines the Gabor Transform associated to f and h, while the linear operator F_h: $L^2(R) \to C^{RxR}$ which associates to f ϵ $L^2(R)$ its Gabor Transform F_h f, is said to be Gabor Transformation associated to the window h.

The inversion formula

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(F_h f)(b,\omega)h(t-b)\exp(i\omega t)dbd\omega$$

is also valid.

The explosive technological development of the last twenty years needed a new and advanced mathematical methods and algorithms. In order to eliminate the main difficulty of Gabor Transformation that consists of the variation of the window only by translations, J. Morlet proposed the modification of the window by translations and dilatations, simultaneously; so, it appeared the wavelet analysis, having various applications for radar signals, geology signals or voice signal (voice recognition).

Given a function ψ: R->C and a pair (a,b) ϵ R* x R we introduce the basic set of functions $\{\psi_{a,b}\}$ generated by ψ as

$$(5)\quad \psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a}),\quad t \in R$$

where the parameter *a* refers to dilatations or contractions and the parameter *b* is associated to translations. So, varying *a* and *b*, we shall obtain a family of windows which control the time-frequency information better than in the case of Gabor Transformations.

In order to define the Wavelet Transformation, we start with a wavelet function ψ: R->C which means that $\psi \in L^1(R) \cap L^2(R)$ and the integral

$$\int_{-\infty}^{+\infty} |\omega|^{-1} |(Ff)(\omega)|^2 \, d\omega \overset{not}{=} C(\psi)$$

denoted by C(ψ) , is convergent. If $f \in L^2(R)$ is a given continuous-time signal, then using (5), the function

$$(6)\ W_\psi f : R^* x R -> C,\ (W_\psi f)(a,b) = \int_{-\infty}^{\infty} f(t) \bar{\psi}_{a,b}(t) dt$$

is said to be the continuous-time (integral) wavelet transform of f, with respect to ψ. The operator

$$W_\psi :\ L^2(R) -> C^{RxR},\ f -> W_\psi f$$

is named continuous-time (integral) Wavelet Transformation associated to the given function .[2]

This transformation is linear and has also properties of translation and scaling.

The Inversion Formula

$$(7)\quad f(t) = \frac{1}{C(\psi)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{a^2} (W_\psi f)(a,b) \psi_{a,b}(t) da\, db, \quad \forall t \in R,$$

holds too, if $f \in C(R) \cap L^1(R) \cap L^2(R)$ and $F_f \in L^1(R)$.

The formulas (6) and (7) are used also in modern geological explorations, together with the information obtained by means of the artificial satellites and making use of powerful mathematical algorithms related to approximation methods and high-performance computer equipments.

An essential concept of the wavelet analysis is that of multi-resolution analysis.

By definition, a multi-resolution in $L^2(R)$ is a sequence of closed subspaces $(V_j)_{j \in Z}$ which satisfies the following conditions:[3][4]

(i) Ascending:

$$V_j \subset V_{j+1},\ \forall\ j \in Z$$

(ii) Scaling:

$$\forall j \in Z, \quad v(t) \in V_j \Leftrightarrow v(2t) \in V_{j+1}$$

(iii) Invariance of translations for V

$$\forall\, v(t) \in V_0 \Rightarrow v(t-k) \in V_0, \forall\, k \in Z$$

(iv) Density: The reunion

$$\bigcup_{j \in Z} V_j \ \text{este densa in}\ L^2(R)$$

(v) Separation:

$$\bigcap_{j \in Z} V_j = \{0\}$$

(vi) Scaling function: it exists $g \in V_0$ so that the sequence $\{ g(t-k) \}_{k \in Z}$ is a Riesz basis of V_0.

$$f = \sum_{j \in Z} f_j$$

Next, we shall introduce the notion of "zoom of order n" as the projection of $f \in L^2(R)$ on the space V_n. Given an orthonormal basis $\{\psi_{j,k}: j,k \in Z\}$ of $L^2(R)$ we obtain the decomposition of f as a sum of its "voices" f_n, namely

where

$$f_j = \sum_{k \in Z} < f, \psi_{j,k} > \psi_{j,k}, \quad j \in Z$$

The expression of the "zoom of order n" of f is

$$F_n = \sum_{j=-\infty}^{n-1} f_j$$

and the limit of the sequence (F_n) is f (in $L^2(R)$). [3][4]

The equality $F_{n+1}=F_n+f_n$ shows that the voice f_n is the detail which must be added to the "zoom of order n" in order to obtain the " zoom of order n+1".

The above results refer to continuous-time signals 1D and most of them can be extended to the case of images 2D and 3D.

2. Generally speaking, the inverse problem of dynamics aims to determine the potential under whose action a particle of unit mass can describe a given family of curves. More precisely, given a planar family of curves

$$f(x, y) = C$$

one looks for a potential V for which this family is a family of trajectories of the particle, the motion in the inertial frame of coordinates Oxy being modelled by the equations

$$x'' = - V_x$$

$$y'' = - V_y$$

This system admits the energy integral

$$(x')^2 + (y')^2 = 2(E - V)$$

E having a constant value on each trajectory of the system. For a given family of curves, in the case that it will be a family of trajectories of the considered system, the energy E will be a function denoted by $E(f)$, in order to emphasize that it is constant on each member of the family.

Szebehely (1974) obtained a partial differential equation of the first order satisfied by the potential, this equation bearing his name. Using the notation

$$\gamma = f_y / f_x , \Gamma = \gamma \gamma_x - \gamma_y$$

Szebehely's equation was written in a simpler form (Bozis, 1983)

$$V_x + \gamma V_y + (2\Gamma(E(f) - V)) / (1+ \gamma^2)$$

From the expression of the energy integral, one has

$$E(f) - V \geq 0$$

this inequality meaning that the kinetic energy cannot be negative. It follows that, for $\Gamma \neq 0$, the potential V satisfies the inequality (Bozis and Ichtiaroglou, 1994)

$$(V_x + \gamma V_y) / \Gamma \leq 0$$

By eliminating the energy from Szebehely's equation, (Bozis, 1984) has obtained, for $\Gamma \neq 0$, with the notation

$$\chi = 1/ \gamma - \gamma , \quad \lambda = (\Gamma_y - \gamma \Gamma_x) / \gamma \Gamma , \ \mu = \lambda\gamma + 3\Gamma/ \gamma$$

the partial differential equation of the second order verified by the potential

$$-V_{xx} + \chi V_{xy} = \lambda V_x + \mu V_y$$

Recently have been obtained the basic equations of the inverse problem of dynamics, in a unifying way, by Mira Cristiana Anisiu (Romanian Academy Branch of Cluj-Napoca).

V. Conclusions

Using the possibilities of the multi-resolution analysis especially in the extraction of hidden details, and involving also reverse dynamics to rigorously trait the distortions issued by both satellite movement and terrestrial curvature, the software system will provide a powerful computerized environment, aimed to enhance the actual satellite remote sensing provided image processing methods.

This chances is given especially by our access to the BCUM-COLLABORATOR endowment - a multiple users research base endowed with a high performance, multiprocessor computing system, being the most performant one in the south-eastern european area.

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