APPROXIMATION PROCEDURES FOR THE TEMPERATURE FIELD IN THE VISCOUS INCOMPRESSIBLE FLOW THROUGH CYLINDRICAL TUBES

by

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In this paper one determines the unsteady field of temperature in the incompressible viscous fluid flow through cylindrical tubes by using the weighted integral relations method (moments). The results are compared to those provided by the finite differences method (Crank-Nicolson).

1. Basic equations. We consider a semi-infinite circular cylindrical tube of radius R in which flows in steady state an incompressible viscous fluid subjected to a pressure gradient $(p_o - p_L)/L$, where p_o is the pressure in the cross section $z=o$ and p_L is the fluid pressure in the circular section $z=L$. Oz is the tube axis and L is the tube length.

The motion of the incompressible viscous fluid in the tube is governed by the equation

$$(1) \qquad \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} = -\frac{dp}{dz} \quad , \quad (x,y,z) \in D \quad , \quad z > o$$

$$v_z|_S = o \quad , \quad S = \partial D \quad ,$$

where D is the domain of the motion, $\vec{v}(o,o,v_z)$ is the velocity and p is the pressure in the fluid.

Poisson's equation (1) has the solution

$$(2) \qquad v_z = \frac{R^2 \Delta p}{4 \mu L} \left(1 - \frac{r^2}{R^2} \right)$$

$$(r^2 = x^2 + y^2 \quad , \quad \Delta p = p_o - p_L > o)$$

In what follows we shall deal with the unsteady heat transfer in an incompressible viscous fluid in steady motion. It is assumed that at the initial moment the temperature of the fluid is a constant $\tilde{T}_o$ and the temperature of the tube wall is a constant value $\tilde{T}_w$. We suppose also that the motion of the fluid is dissipative.

The unsteady heat transfer equation under the above-mentioned conditions is

$$(3) \qquad \frac{\partial T}{\partial t} = m\left(\frac{dU}{dy}\right)^2 + \frac{1}{\sigma}\,\frac{1}{y}\,\frac{\partial}{\partial y}\left(y\,\frac{\partial T}{\partial y}\right), \quad (y,t)\in\Omega$$

$$(4) \qquad T(y,o) = o\,, \quad T(o,t) = \text{finite}\left(\frac{\partial T}{\partial y}(o,t)=o\right), \quad T(1,t) = 1$$

$$\Omega = \{(y,t) \mid o<y<1\,,\ o<t<t^*\}$$

where

$$y = \frac{r}{R}\,, \quad t = \frac{\mu\tau}{\rho R^2}\,, \quad U = \frac{v_z}{\left(-\frac{dp}{dz}\right)\frac{R^2}{4\mu}} = 1 - y^2$$

$$m = \frac{R^4 (dp/dz)^2}{16\mu^2 c_p(\tilde{T}_w-\tilde{T}_o)}\,, \quad \sigma = \frac{\mu c_p}{\lambda}\,, \quad T = \frac{\tilde{T}-\tilde{T}_o}{\tilde{T}_w-\tilde{T}_o}$$

In these formulas r is the radial coordinate in a circular section of the tube, τ is time, $\tilde{T}$ is temperature, ρ - density, μ - the viscosity dynamic coefficient, c_p - the specific heat, λ - the thermal conductivity coefficient, σ - the Prandtl number, and m is a constant. The unknown function in this initial-boundary value problem is the dimensionless temperature $T(y,t)$.

2. The choice of the approximation solution. We look for an approximate solution of the form

$$(5) \qquad T_n(y,t) = 1 + \frac{\sigma m}{4}(1-y^4)G_o(t) + \sum_{k=1}^{n} G_{2k}(t)\Psi_{2k}(y)$$

In this problem the unknown functions are G_{2k}, $k=o,\dots,n$. The functions $\Psi_{2k}(y)$, $y\in[o,1]$ are some bounded functions, linearly independent, chosen out of a complete system of functions and verify the symmetry condition $\partial T/\partial y = o$ for $y=o$. Using the boundary condition $T(1,t) = 1$ one can eliminate in (5) the function G_4 and denoting

G_o with G_4, the approximation solution becomes

(6) $$T_n(y,t) = 1 + \sum_{k=1}^{n} G_{2k}(t)\varphi_{2k}(y)$$

The coordinate functions φ_{2k} have the form

(7) $$\varphi_{2k}(y) = \begin{cases} \psi_{2k}(y) - \psi_{2k}(1)\dfrac{\psi_4(y)}{\psi_4(1)} & , k \neq 2 \\ \dfrac{6m}{4}(1-y^4) & , k = 2 \end{cases} \quad , k = \overline{1,n}$$

These functions fulfil the conditions required by the residual method.

3. The application of the weighted integral relations method. The method consists in imposing the orthogonality condition which requires the residual to be orthogonal to each member of a set of weighted functions y^{i-1} , $i = \overline{1,n}$, that is [3]

(8) $$\int_0^1 [A(T_n(y,t)) - \sigma g(y)]\, y^{i-1}\, dy = 0 \; , \; i = \overline{1,n}$$

where

$$A(T_n) = \left[\sigma \frac{\partial}{\partial t} - \frac{1}{y}\frac{\partial}{\partial y}\left(y \frac{\partial}{\partial y}\right)\right] T_n$$

$$g(y) = m\left(\frac{dU}{dy}\right)^2$$

The integral relations (8) lead to a system of n differential equations of first order with constant coefficients for determining the functions G_{2k} :

(9) $$\sum_{k=1}^{n} A_{i,k}\frac{dG_{2k}(t)}{dt} - \frac{1}{\sigma}\sum_{k=1}^{n}(B_{i,k} + C_{i,k})G_{2k}(t) = b_i \; , \; i = \overline{1,n}$$

The numbers $A_{i,k}, B_{i,k}, C_{i,k}$ and b_i have the expressions

$$A_{i,k} = (y^{i-1}, \varphi_{2k}) = \int_0^1 y^{i-1}\varphi_{2k}(y)\, dy$$

$$B_{i,k} = (y^{i-2}, \varphi'_{2k}) = \int_0^1 y^{i-2}\varphi'_{2k}(y)\,dy$$

(10) $$C_{i,k} = (y^{i-1}, \varphi''_{2k}) = \int_0^1 y^{i-1}\varphi''_{2k}(y)\,dy \quad ;\ i,k = \overline{1,n}$$

$$b_i = (y^{i-1}, g) = \int_0^1 y^{i-1} g(y)\,dy$$

4. <u>The initial condition</u>. One applies the orthogonality condition of the weighted integral relations method at $t=o$. Thus, it is required that

(11) $$\int_0^1 T_n(y,o)y^{i-1}\,dy = o \quad , \quad i = \overline{1,n}$$

By replacing $T_n(y,o)$, we find the algebraic system of equations

(12) $$\sum_{k=1}^{n} A_{i,k}G_{2k}(o) = -\frac{1}{i} \quad , \quad i = \overline{1,n}$$

The Gram determinant of this system,

$$D_n = \det\left[A_{i,k}\right]_{i,k=1}^{n} = \det\left[(y^{i-1}, \varphi_{2k})\right]_{i,k=1}^{n}$$

is non-zero because the functions φ_{2k}, $k=\overline{1,n}$ are linearly independ ent. Consequently, the system (12) is a Cramer system with the solution

(13) $$G_{2k}(o) = \frac{D_{(2k)}}{D_n} \quad , \quad k = \overline{1,n}$$

where $D_{(2k)}$ is the determinant obtained out of D_n by substituting the column k with rightside column of the system.

5. <u>The choice of the coordinate functions</u>. If we take the case of the $\psi_{2k} : [o,1] \to [o,1]$ of the form $\psi_{2k}(y) = y^{2k}$, we have

(13) $$\varphi_{2k}(y) = \begin{cases} y^{2k} - y^4 & , k \neq 2 \\ \frac{G \cdot m}{4}(1-y^4) & , k = 2 \end{cases} \quad , \quad k = \overline{1,n}$$

The formulas (1o) lead to the following expressions for coefficients

$$A_{i,k} = \begin{cases} 2\,\dfrac{2-k}{(i+4)(i+2k)} & , k \neq 2 \\ \dfrac{m\sigma}{i(i+4)} & , k = 2 \end{cases}$$

(14)
$$B_{i,k} = \begin{cases} \dfrac{2(i-2)(k-2)}{(i+2)(i+2k-2)} & , k \neq 2 \\ -\dfrac{m\sigma}{i+2} & , k = 2 \end{cases}$$

$$C_{i,k} = \begin{cases} \dfrac{2(2ki+3i+4k-6)(k-2)}{(i+2k-2)(i+2)} & , k \neq 2 \\ -\dfrac{3\sigma m}{i+2} & , k = 2 \end{cases}$$

$$b_i = \frac{4m}{i+2}$$

6. <u>First order approximation</u> ($n=1$; $i=1$; $k=2$). The equation and the boundary condition of the problem in first order approximation is obtained from (9) and (12), in the form

(15) $$3\sigma G_4'(t) + 2o\, G_4(t) = 2o \quad , \quad G_4(o) = -\frac{5}{\sigma m} \quad .$$

The solution of this problem is

$$G_4(t) = 1 - \left(1 + \frac{5}{\sigma m}\right) e^{-\frac{2o}{3\sigma}t}$$

Consequently, the solution of the thermal problem (3)-(4) is

(16) $$T_1(y,t) = 1 + \frac{\sigma m}{4}\left[1 - \left(1 + \frac{5}{\sigma m}\right) e^{-\frac{2o}{3\sigma}t}\right](1 - y^4) \quad .$$

7. <u>Second order approximation</u> ($n=2$; $k=1,2$; $i=1,2$). The approximation solution in this case has the expression

(17) $$T_2(y,t) = 1 + G_2(t)\,\varphi_2(y) + G_4(t)\,\varphi_4(y) =$$

$$= 1 + (y^2 - y^4)\, G_2(t) + \frac{\sigma m}{4}(1 - y^4)\, G_4(t)$$ -

and the differential equations system (9), in order to determine the functions G_2 and G_4, is reduced to

$$\begin{aligned} &2\sigma G_2' + 3\sigma^2 m\, G_4' + 20\,(G_2 + \sigma m\, G_4) = 20\, \sigma m \\ &\sigma G_2' + \sigma^2 m\, G_4' + 12\,(2G_2 + \sigma m\, G_4) = 12\, \sigma m \end{aligned} \tag{18}$$

In this approximation, the system (12) has the solution

$$G_2(0) = -3 \quad , \quad G_4(0) = -\frac{3}{\sigma m} \quad . \tag{19}$$

The following transformation is made

$$G_2(t) = V(t) \;,\quad G_4(t) = -W(t) + 1$$

and the system (18) with the conditions (19), after the solving in terms of the derivatives, is written in the homogeneous form

$$\begin{aligned} \frac{dV}{dt} &= -\frac{52}{\sigma} V + 16\, m\, W \\ \frac{dW}{dt} &= -\frac{28}{\sigma^2 m} V + \frac{4}{\sigma} W \end{aligned} \tag{20}$$

$$V(0) = -3 \;,\quad W(0) = 1 + \frac{3}{\sigma m} \quad . \tag{21}$$

The solution of this problem is

$$\begin{aligned} V(t) &= 16m\,(C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}) \\ W(t) &= \left(\frac{52}{\sigma} + \lambda_1 \right) C_1 e^{\lambda_1 t} + \left(\frac{52}{\sigma} + \lambda_2 \right) C_2 e^{\lambda_2 t} \end{aligned} \tag{22}$$

where

$$\lambda_1 = \frac{4}{\sigma}(6 - \sqrt{21}) \quad , \quad \lambda_2 = \frac{4}{\sigma}(6 + \sqrt{21})$$

$$\begin{aligned} C_1 &= \frac{1}{\lambda_1 - \lambda_2}\left[1 + \frac{3}{\sigma m} + \frac{3}{16m}\left(\frac{52}{\sigma} + \lambda_2 \right)\right] \\ C_2 &= -\frac{1}{\lambda_1 - \lambda_2}\left[1 + \frac{3}{\sigma m} + \frac{3}{16m}\left(\frac{52}{\sigma} + \lambda_1 \right)\right] \end{aligned} \tag{23}$$

8. <u>Third order approximation</u> ($n=3$; $k=1,2,3$; $i=1,2,3$). In this case the approximating solution is of the form

$$T_3(y,t) = 1 + G_2(t)\,\varphi_2(y) + G_4(t)\,\varphi_4(y) + G_6(t)\,\varphi_6(y) = \tag{24}$$

$$= 1 + (y^2 - y^4)G_2(t) + \frac{\sigma m}{4}(1 - y^4)G_4(t) + (y^6 - y^4)G_6(t) \quad .$$

The unknown functions G_2 , G_4 and G_6 are the solutions of the system

$$(25) \quad \begin{aligned} &14\sigma G_2' + 21\sigma^2 m G_4' - 6\sigma G_6' + 28(5G_2 + 5\sigma m G_4 - 7G_6) = 140\sigma m \\ &2\sigma G_2' + 2\sigma^2 m G_4' - \sigma G_6' + 24(2G_2 + \sigma m G_4 - 2G_6) = 24\sigma m \\ &18\sigma G_2' + 15\sigma^2 m G_4' - 10\sigma G_6' + 12(49G_2 + 21\sigma m G_4 - 51G_6) = 252\sigma m \end{aligned}$$

under the initial conditions obtained from (12)

$$(25') \quad \begin{aligned} &14G_2(0) + 21\sigma m G_4(0) - 6G_6(0) = -105 \\ &2G_2(0) + 2\sigma m G_4(0) - G_6(0) = -12 \\ &18G_2(0) + 15\sigma m G_4(0) - 10G_6(0) = -105 \end{aligned}$$

By solving (25') it follows the values

$$G_2(0) = \frac{15}{4} \quad , \quad G_4(0) = -\frac{9}{2\sigma m} \quad , \quad G_6(0) = \frac{21}{2} \quad .$$

We make the functions change

$$G_2(t) = V_1(t) \quad , \quad G_4(t) = 1 - V_2(t) \quad , \quad G_6(t) = V_3(t)$$

and we obtain from (25) the following linear and homogoneous system

$$(26) \quad \begin{aligned} \frac{dV_1}{dt} &= \frac{29}{\sigma}V_1 - 11m\,V_2 - \frac{91}{\sigma}V_3 \\ \frac{dV_2}{dt} &= -\frac{10}{\sigma^2 m}V_1 - \frac{2}{\sigma}V_2 - \frac{10}{\sigma^2 m}V_3 \\ \frac{dV_3}{dt} &= \frac{126}{\sigma}V_1 - 42m V_2 - \frac{210}{\sigma}V_3 \end{aligned}$$

where the functions V_1 , V_2 ; V_3 fulfil the initial conditions

$$(27) \quad V_1(0) = \frac{15}{4} \quad , \quad V_2(0) = 1 + \frac{9}{2\sigma m} \quad , \quad V_3(0) = \frac{21}{2} \quad .$$

The solution of the system (26) is sought in the form

$$V_i(t) = a_i\, e^{\lambda t} \quad , \quad a_i \in \mathbb{R} \ ; \quad i = 1,2,3 \ .$$

By verifying in (26), we find the linear and homogeneous algebraic system for the determination of the coefficients a_i

$$(28)\qquad \begin{aligned} &\left(\frac{29}{\sigma}-\lambda\right) a_1 + 11\, m\, a_2 - \frac{91}{\sigma} a_3 = 0 \\ &\frac{10}{m\sigma^2} a_1 + \left(\frac{2}{\sigma}+\lambda\right) a_2 + \frac{10}{m\sigma^2} a_3 = 0 \\ &\frac{126}{\sigma} a_1 - 42\, m\, a_2 - \left(\frac{210}{\sigma}+\lambda\right) a_3 = 0 \end{aligned}$$

If we denote $\lambda\sigma = \mu$, the characteristic equation for (28) is

$$(29)\qquad \mu^3 + 183\mu^2 + 5208\mu + 24192 = 0$$

with the roots $\mu_1=-5,783257$, $\mu_2=-28,041603$, $\mu_3=-149,175140$, determined by a residual procedure in [2].

For the eigenvalue $\lambda_i = \frac{1}{\sigma}\mu_i$ we choose the eigenvector $v_i(a_1^{(i)}, a_2^{(i)}, a_3^{(i)})$, where

$$(30)\qquad \begin{aligned} a_1^{(i)} &= -\frac{m\sigma(72 + 91\lambda_i\sigma)}{10(120-\lambda_i\sigma)} \\ a_2^{(i)} &= 1 \qquad\qquad i=1,2,3 \\ a_3^{(i)} &= -\frac{m\sigma(168 + 27\lambda_i\sigma - \lambda_i^2\sigma^2)}{10(120-\sigma\lambda_i)} \end{aligned}$$

The general integrals of the equations (26) have the form

$$(31)\qquad V_1(t)=\sum_{i=1}^{3} C_i a_1^{(i)} e^{\lambda_i t},\quad V_2(t)=\sum_{i=1}^{3} C_i e^{\lambda_i t},\quad V_3(t)=\sum_{i=3}^{3} C_i a_3^{(i)} e^{\lambda_i t}$$

The constant values $C_i \in \mathbb{R}$ are determined by verifying the initial conditions (27) and we get

$$C_1 = \frac{1}{4\Delta}\left[42(a_1^{(2)}-a_1^{(3)})-15(a_3^{(2)}-a_3^{(3)})-2\left(1+\frac{9}{26m}\right)(a_1^{(2)}a_3^{(3)}-a_1^{(3)}a_3^{(2)})\right]$$

$$C_2 = \frac{1}{4\Delta}\left[42(a_1^{(3)}-a_1^{(1)})-15(a_3^{(3)}-a_3^{(1)})-2\left(1+\frac{9}{26m}\right)(a_1^{(3)}a_3^{(1)}-a_1^{(1)}a_3^{(3)})\right]$$

$$C_3 = \frac{1}{4\Delta}\left[42(a_1^{(1)}-a_1^{(2)})-15(a_3^{(1)}-a_3^{(2)})-2\left(1+\frac{9}{26m}\right)(a_1^{(1)}a_3^{(2)}-a_1^{(2)}a_3^{(1)})\right]$$

where

$$\Delta = (a_1^{(2)} - a_3^{(1)})(a_3^{(1)} - a_3^{(3)}) - (a_1^{(1)} - a_1^{(3)})(a_3^{(2)} - a_3^{(3)}) \quad .$$

The solution $T_3(y,t)$ represents an approximation for the temperature field in the fluid flow.

In what follows we shall also give a numerical solving by finite differences of the above problem.

9. <u>The finite differences scheme of Crank-Nicolson type</u>. In the plane Oyt we consider the point grid $(y_j, t_n) \equiv (j,n)$ with mesh size Δy and Δt, where $y_j = j\,\Delta y$ and $t_n = n\,\Delta t$, $j=\overline{o,J}$, $n=\overline{o,N}$, $J\Delta y = 1$.

We apply the heat transfer equation (3) in the point $(y_j, t_{n-\frac{1}{2}})$ and we obtain

$$(32) \qquad A_\Delta \equiv \left[\frac{\partial T}{\partial t} - \frac{1}{\sigma} \left(\frac{1}{y}\frac{\partial T}{\partial y} + \frac{\partial^2 T}{\partial y^2} \right) \right]_{j,n-\frac{1}{2}} = g_j \quad .$$

The derivatives at the node $(j,n-\frac{1}{2})$ are replaced by means of the formulas of numerical derivation at the integer nodes, [4].

The finite differences scheme, associated to the equation (32) is represented in the form (Crank-Nicolson) :

$$-\left(1-\frac{1}{2j}\right)T_{j-1,n} + 2\left(1+\frac{\sigma}{r}\right)T_{j,n} - \left(1+\frac{1}{2j}\right)T_{j+1,n} =$$

$$(33) \qquad = \left(1-\frac{1}{2j}\right)T_{j-1,n-1} - 2\left(1-\frac{\sigma}{r}\right)T_{j,n-1} + \left(1+\frac{1}{2j}\right)T_{j+1,n-1} +$$

$$+ \frac{2\sigma\Delta t}{r}\, g_j \quad .$$

$$\left(n = \overline{1,N} \quad ; \quad j = \overline{1,J-1} \quad ; \quad r = \frac{\Delta t}{(\Delta y)^2} \right)$$

under the boundary conditions

$$(34) \qquad T_{j,o} = o \quad , \quad j=\overline{1,J-1} \quad , \quad T_{J,n} = 1 \quad , \quad n=\overline{1,N} \quad .$$

In these equations, $T_{j,n}$, $j=o,1,2,\ldots,J-1$ are unknown values. But at $y = o$ the term $(1/y)(\partial T/\partial y)$ introduces a singularity and the calculation in (33) cannot begin with $j = o$. Therefore, we shall take into account that in the vicinity of the point (o,t) the heat transfer equation (3) is reduced to

$$(35) \qquad \frac{\partial T}{\partial t} - \frac{2}{\sigma}\frac{\partial^2 T}{\partial y^2} = g(y)$$

with

$$(36) \qquad \frac{\partial T}{\partial y} = o \qquad \text{for} \quad y = o .$$

Thus, the singularity in $y = o$ is excluded.

For this case, the implicit finite differences equation of Crank-Nicolson is

$$T_{j,n} - T_{j,n-1} - \frac{r}{\sigma}(T_{j-1,n} - 2 T_{j,n} + T_{j+1,n} +$$

$$+ T_{j-1,n-1} - 2 T_{j,n-1} + T_{j+1,n-1}) = \Delta t . g_j$$

$$T_{1,n} - T_{-1,n} = o ,$$

which is applied for $j = o$.

Consequently, the finite differences scheme for the determination of the dimensionless temperature $T(y,t)$ has the form

$$(1+ \frac{\sigma}{2r}) T_{o,n} - T_{1,n} = b_{o,n-1}$$

$$(37) \quad - (1- \frac{1}{2j}) T_{j-1,n} + 2 (1+ \frac{\sigma}{r}) T_{j,n} - (1+ \frac{1}{2j}) T_{j+1,n} =$$

$$= b_{j,n-1} \qquad , \qquad j = \overline{1,J-2}$$

$$- (1- \frac{1}{2J-2}) T_{J-2,n} + 2 (1+ \frac{\sigma}{r}) T_{J-1,n} = b_{J-1,n-1}$$

where

$$b_{o,n-1} = - (1- \frac{\sigma}{2r}) T_{o,n-1} + T_{1,n-1}$$

$$b_{j,n-1} = (1-\frac{1}{2j})\,T_{j-1,n-1} - 2\,(1-\frac{\sigma}{r})\,T_{j,n-1} +$$

$$+ (1+\frac{1}{2j})\,T_{j+1,n-1} + 2\,\frac{\sigma}{r}\,\Delta t.g_j \qquad , \; j=\overline{1,J-2}$$

(38)

$$b_{J-1,n-1} = (1-\frac{1}{2J-2})\,T_{J-2,n-1} - 2(1-\frac{\sigma}{r})\,T_{J-1,n-1} +$$

$$+ 2\,(1+\frac{1}{2J-2}) + 2\,\frac{\sigma}{r}\,\Delta t.g_{J-1}$$

An advantage in calculations is obtained if we consider the case $r = \sigma$, for which some coefficients are null.

1o. Numerical results.A comparison between the analytical and numerical solutions.In order to compare the results obtained we consider the case $r = \sigma = 1$ and $a = 1$. The distribution of the non-dimensional temperature $T_3(y,t)$ for $0{,}05 \leq t \leq 0.1$, with the mesh size $\Delta y = 0.1$ and $\Delta t = 0.01$ is given in Table 1. For the same values of t , in Table 2 are presented the values of the non-dimensional temperature $T(y,t)$ which we obtained by the finite differences method.In order to solve the algebraic systems (37)-(38) , Gauss's elimination method was used.

Table 1

y	$T_3(y,0.05)$	$T_3(y,0.06)$	$T_3(y,0.07)$	$T_3(y,0.08)$	$T_3(y,0.09)$	$T_3(y,0.1)$
0	0.0215	0.0478	0.0830	0.1241	0.1689	0.2157
0.1	0.0274	0.0562	0.0931	0.1353	0.1806	0.2277
0.2	0.0466	0.0822	0.1236	0.1686	0.2156	0.2632
0.3	0.0826	0.1278	0.1755	0.2243	0.2731	0.3213
0.4	0.1407	0.1960	0.2498	0.3019	0.3520	0.3999
0.5	0.2265	0.2891	0.3469	0.4003	0.4498	0.4959
0.6	0.3437	0.4084	0.4657	0.5185	0.5630	0.6049
0.7	0.4924	0.5513	0.6021	0.6463	0.6854	0.7202
0.8	0.6653	0.7100	0.7478	0.7802	0.8083	0.8331
0.9	0.8451	0.8686	0.8881	0.9047	0.9190	0.9316
1.0	1	1	1	1	1	1

Table 2

y	T(y,0.05)	T(y,0.06)	T(y,0.07)	T(y,0.08)	T(y,0.09)	T(y,0.1)
0	0.0365	0.0613	0.0933	0.1313	0.1734	0.2181
0.1	0.0417	0.0684	0.1022	0.1414	0.1843	0.2295
0.2	0.0584	0.0910	0.1295	0.1721	0.2173	0.2636
0.3	0.0904	0.1320	0.1774	0.2247	0.2726	0.3202
0.4	0.1438	0.1960	0.2486	0.3002	0.3501	0.3981
0.5	0.2255	0.2871	0.3448	0.3985	0.4483	0.4946
0.6	0.3408	0.4069	0.4650	0.5167	0.5631	0.6050
0.7	0.4901	0.5518	0.6039	0.6486	0.6875	0.7220
0.8	0.6637	0.7117	0.7506	0.7832	0.8111	0.8354
0.9	0.8432	0.8687	0.8893	0.9061	0.9204	0.9327
1.0	1	1	1	1	1	1

References

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Table 2

y	T(y,0.05)	T(y,0.06)	T(y,0.07)	T(y,0.08)	T(y,0.09)	T(y,0.1)
0	0.0365	0.0613	0.0933	0.1313	0.1734	0.2181
0.1	0.0417	0.0684	0.1022	0.1414	0.1843	0.2295
0.2	0.0584	0.0910	0.1295	0.1721	0.2173	0.2636
0.3	0.0904	0.1320	0.1774	0.2247	0.2726	0.3202
0.4	0.1438	0.1960	0.2486	0.3002	0.3501	0.3981
0.5	0.2255	0.2871	0.3448	0.3985	0.4483	0.4946
0.6	0.3408	0.4069	0.4650	0.5167	0.5631	0.6050
0.7	0.4901	0.5518	0.6039	0.6486	0.6875	0.7220
0.8	0.6637	0.7117	0.7506	0.7832	0.8111	0.8354
0.9	0.8432	0.8687	0.8893	0.9061	0.9204	0.9327
1.0	1	1	1	1	1	1

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