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ON THE APPLICATION OF A VARIATIONAL METHOD TO THE STUDY OF SMALL OSCILLATIONS OF A FLUID IN MOVING TANKS

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A finite volume Ω of ideal, incompressible fluid is inside a rigid tank which is in a state of motion. We take $O_1x_1y_1z_1$ a reference system that is fixed in space and Oxyz an invariable reference system connected with the tank. S^* is the tank surface that is in contact with the fluid. Let $F(x,y,z,t) = 0$, or $F \equiv z - f(x,y,t) = 0$ be the equation of the free surface S of the fluid at the moment t ; S_o is the free surface at the initial moment $t = 0$ (the undisturbed surface). We note with $\vec{v}_a(\vec{r},t)$ and with $p(\vec{r},t)$ the absolute speed and respectively the fluid pressure at the point $P(\vec{r})$, $\vec{r} = \overrightarrow{OP}(x,y,z)$.

The nonstationary movement of the fluid is taken as a potential one, i.e. $\vec{v}_a = \nabla\Phi$ where $\Phi(\vec{r},t)$ is the velocity potential ; then $\nabla.\nabla\Phi = 0$. On the other hand, at the moment t, we have on S

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \nabla F.\frac{d\vec{r}}{dt} = \frac{\partial F}{\partial t} + (\vec{n}.\vec{v}_{rS})|\nabla F| = 0 \Rightarrow v_{Sn} = \frac{\partial f/\partial t}{\sqrt{1+f_x^2+f_y^2}}$$

where v_{Sn} is the relative velocity of the fluid from $(x,y,z) \in S$ in projection on the exterior normal at S which has the unitary vector $\vec{n}$ (S is admitted to be a fluid surface). The derivative $\partial f/\partial t$ is calculated at point $(x,y,z) \in S$. If a surface Σ , $z = f_1(x,y,t)$ - for instance the surface S^* of the tank - is connected with Oxyz then $\partial f_1/\partial t = 0$.

The nonstationary and potential movement of the fluid in the mobile domain Ω verify the equation as well as the boundary and initial conditions [1], [2], [5] :

(1) The Laplace equation : $\Delta\phi(x,y,z,t) = 0$, $(x,y,z,t)\in\Omega\times(t_0,t_1)$ $t_0 \geq 0$, $t_1 < \infty$

(2) The geometrical condition: $\frac{\partial\phi}{\partial n} = \vec{v}_t \cdot \vec{n}$ on S^*

(3) The kinematical condition: $\frac{\partial\phi}{\partial n} = \vec{v}_t \cdot \vec{n} + v_{Sn}$ on S $(z=f(x,y,t))$

(4) The dynamical condition : $\frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + U - \vec{v}_t \cdot \nabla\phi = 0 \left(= -\frac{p-p_0}{\rho}\right)$ on S

(5) $\phi(x,y,z,0) = \phi_0(x,y,z)$, $f(x,y,0) = f_0(x,y)$ — given functions

where U is the potential of exterior body forces while $\vec{v}_t = \vec{v}_0 + \vec{\omega}\times\vec{r}$ is the transport velocity of the fluid particle ($\vec{v}_0$ = translation speed ; Ω = the angular speed of the tank) and (4) represents the Cauchy integral.

The unknown functions in (1)-(4) are : $\phi(x,y,z,t)$ and $f(x,y,t)$. The study and the solving of the system (1)-(4) are difficult because: 1) the system contains nonlinear equations; 2) f is an unknown function; 3) the domain Ω is variable during the movement.

Slow motions.Linearization. Under the impact of its own weight as well as of that of the inertia forces, the fluid has slow oscillations in Ω. The free surface S_0 (the free surface of the fluid at rest) is taken to be horizontal, the plane Oxy is on S_0 $(z = 0)$ and the axis Oz is vertical. We presume that the velocity $\vec{v}$, the free surface deviation S $(z=f(x,y,t))$ from S_0 as well as the function derivatives $\vec{v}$ and f are small. Thus their powers with respect to the unit may be neglected:

$$|\nabla F| = \sqrt{1 + f_x^2 + f_y^2} \approx 1 \; ; \; v_{Sn} \approx \frac{\partial f}{\partial t} \; ; \; |\nabla \phi|^2 \approx 0 \quad \text{in (4)} \; ;$$

$$\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial z} \quad \text{on } S \; ; \quad \nabla \phi \cdot \nabla f \approx 0$$

and the conditions on S are applied on S_o , thus $\phi'_z(x,y,z,t)|_{z=f} =$ $= \phi'_z(x,y,0,t)$.

The equations (1)-(4) for small oscillations get reduced to their linear form:

$$\Delta \phi(x,y,z,t) = 0 \quad \text{in } \Omega, \; t \in (t_o, t_1], t_1 < \infty$$

$$L_1 \equiv \frac{\partial \phi}{\partial n} - \vec{v}_t \cdot \vec{n} = 0 \quad \text{on } S$$

(6)

$$L_2 \equiv \frac{\partial \phi}{\partial z} - \vec{v}_t \cdot \vec{n} - \frac{\partial f}{\partial t} = 0 \quad \text{on } S_o \; ; \; \left(\frac{\partial f}{\partial t} = v_{Sn} \right)$$

(7) $$L_3 \equiv \frac{\partial \phi}{\partial t} + U - \vec{v}_t \cdot \nabla \phi = 0 \quad \text{on } S_o$$

where the unitary normal on S is replaced with the unitary vector $\vec{k}$ on the axis Oz (i.e. $\vec{n} = \vec{k}$ on S_o).

<u>The variational equation of small oscillations</u>. Either the variational Galerkin method [3] or the Bateman principle [4] can be used in order to infer this equation. If the former is used, the following formulation is added [4]:

(8) $$\int_{t_o}^{t_1} \left\{ \int_{\Omega} (-\Delta \phi) \delta \phi \, d\Omega + \int_{S^*} L_1 \delta \phi \, dS^* + \int_{S_o} L_2 \delta \lambda_1 dS_o + \int_{S_o} L_3 \delta \lambda_2 dS_o = 0 \right.$$

$$(\delta \phi(x,y,z,t_o) = 0, \; \delta \phi(x,y,z,t_1) = 0)$$

where the variations (multipliers) are chosen as follows.

If we use Green's formula

$$-\int_{\Omega} \Delta\phi \delta\phi d\Omega = -\int_{S_o \cup S^*} \frac{\partial \phi}{\partial n} \delta \phi \, dS + \frac{1}{2} \int_{\Omega} \delta |\nabla \phi|^2 d\Omega$$

and if we choose $\delta \lambda_1 = \delta \phi$, the Galerkin equation (8) is written under the form ($\partial f / \partial t = v_{Sn}$)

$$\int_{t_o}^{t_1}\Big\{-\int_{S^*}\vec{v}_t\cdot\vec{n}\,\delta\phi\,dS-\int_{S_o}(\vec{v}_t\cdot\vec{n}+v_{Sn})\delta\phi\,dS_o+$$

(9)

$$+\int_{S_o}\Big(\frac{\partial\phi}{\partial t}+U-\vec{v}_t\cdot\nabla\phi\Big)\delta\lambda_2\,dS_o\Big\}dt=0$$

As we have $\delta\dot{\phi}=\partial(\delta\phi)/\partial t$ and $v_{Sn}=0$ on S^* (S^* is fixed in Oxyz) we infer

$$\int_{t_o}^{t_1}\Big(\int_{s}v_{Sn}\delta\phi\,dS\Big)dt=\int_{t_o}^{t_1}\Big(\int_{S\cup S^*}v_{Sn}\delta\phi\,dS\Big)dt=\int_{t_o}^{t_1}\Big(\frac{d}{dt}\int_{\Omega(t)}\delta\phi\,d\Omega-$$

$$-\int_{\Omega}\frac{\partial}{\partial t}\delta\phi\,d\Omega\Big)dt=-\int_{t_o}^{t_1}\int_{\Omega}\delta\Big(\frac{\partial\phi}{\partial t}\Big)d\Omega\,dt\ ;\ (S\rightarrow S_o)\ ;$$

$$\int_{S_o\cup S^*}\vec{v}_t\cdot\vec{n}\,\delta\phi\,dS=\int_{\Omega}\nabla\cdot(\vec{v}_t\delta\phi)d\Omega=\int_{\Omega}\delta(\vec{v}_t\cdot\nabla\phi)d$$

$$\big(\nabla\cdot(\vec{v}_t\delta\phi)=(\nabla\cdot\vec{v}_t)\delta\phi+\vec{v}_t\cdot\nabla\delta\phi\ ,\ \nabla\cdot\vec{v}_t=\nabla\cdot\vec{v}_o+\nabla\cdot(\vec{\omega}\times\vec{r})=0\big)$$

If we employ these estimations, the equation (9) becomes

$$(10)\quad \int_{t_o}^{t_1}\Big\{\int_{\Omega}\delta\Big(\frac{\partial\phi}{\partial t}-\vec{v}_t\cdot\nabla\phi\Big)d\Omega+\int_{S_o}\Big(\frac{\partial\phi}{\partial t}+U-\vec{v}_t\cdot\nabla\phi\Big)\delta\lambda_2\,dS_o\Big\}dt=0$$

We take $z=h^*(x,y)$ the S^* surface equation (the tank surface in contact with the fluid at the moment t). Then, for a function G we have

$$\delta\Big(\int_{\Omega}G(\phi(x,y,z))d\Omega=\delta\int_{S_o}\Big(\int_{h^*}^{f}G(\phi)dz\Big)dS=\int_{\Omega}\delta G\,d\Omega+\int_{S_o}G\,\delta f\,dS_o$$

If we take $\delta\lambda_2=\delta f$, since U, t_o, t and t_1 are not subject to variations, (10) becomes

$$(11)\quad \delta\int_{t_o}^{t_1}\int_{\Omega}\Big(\frac{\partial\phi}{\partial t}+U-\vec{v}_t\cdot\nabla\phi\Big)d\Omega\,dt=0\Rightarrow\delta\int_{t_o}^{t_1}\int_{\Omega}p\,d\Omega\,dt=0$$

The formulation (11) represents Bateman's principle : for the slow nonstationary real movement that takes the volume of ideal incompressible fluid Ω from one position (P_o) at the fixed moment t_o to another position (P_1) at the fixed moment t_1, the integral functional

$$J(\phi,f) = \int_{t_o}^{t_1}\int_{\Omega} p(\phi,f)\, d\Omega\, dt$$

is stationary. [p - the pressure in the ideal incompressible fluid, $\phi \in C^{2,1}(\Omega \times (t_o,t_1))$, $f \in C^{1,1}(S_o \times (t_o,t_1))$].

The variational equation of slow motions for the heavy ideal incompressible fluid in moving tanks is obtained from (8), with $\delta\lambda_2 = \delta f$ and $U = \vec{g}.\vec{r}$, and has the form [1]

(12)
$$\int_{t_o}^{t_1}\Big\{\int_{\Omega}(-\Delta\phi)\delta\phi\, d\Omega + \int_{S^*}\Big(\frac{\partial\phi}{\partial n} - \vec{v}_t.\vec{n}\Big)\delta\phi\, dS + \int_{S_o}\Big[\Big(\frac{\partial\phi}{\partial z} - \vec{v}_t.\vec{k} - \frac{\partial f}{\partial t}\Big)\Big|_{z=0} + \Big(\frac{\partial\phi}{\partial t} - \vec{g}.\vec{r} - \vec{v}_t.\nabla\phi\Big)\Big|_{z=0}\delta f\Big]dS\Big\}dt = 0$$

where $\vec{v}_t.\nabla\phi$ may be considered a small term of second order (negligible) if $\vec{v}_t$ is small.

<u>The tank has only a translation movement</u> ($\vec{\omega} = 0$, $\vec{v}_t = \vec{v}_o(t)$). Solutions in the form of series are chosen for the differential problem (6)-(7), [1],

(13)
$$f(x,y,t) = \sum_{i=1}^{\infty} a_i(t)\, f_i(x,y)$$

(14)
$$\phi(x,y,z,t) = \sum_{m=1}^{\infty} c_m(t)\, \varphi_m(x,y,z) + \vec{v}_o.\vec{r}$$

$$\Big(a_i(t) = \int_{S_o} f f_i\, dS_o \Big)$$

where the coefficients $a_i(t)$ and $c_m(t)$ are the unknown functions and f_i and φ_m are known functions (similar to the function systems in a generalized Fourier expansion, or in a direct approximation method (Ritz, Galerkin)) which should verify the properties necessary for securing the validity of the series (13) and (14) as well as for making possible the effective determination of the functions a_i and c_m with respect to the time t.

The functions a_i and c_m are so determined that (13)-(14) should verify the equation (12) for any variations δa_i and δc_m. By using (13)-(14), the equation (12) turns into the identity (with respect to δa_i, δc_m)

$$\sum_{i=1}^{\infty}\Big[\sum_{m=1}^{\infty}\frac{dc_m}{dt}\int_{S_0} f_i(\varphi_m)\big|_{z=0}dS_0 + (\vec{v}_0 - \vec{g})\cdot\int_{S_0}\vec{r}\, f_i\, dS_0 -$$

$$-\sum_{m=1}^{\infty} c_m\vec{v}_0\cdot\int_{S_0}(\nabla\varphi_m)\big|_{z=0} f_i\, dS_0 - v_0^2\int_{S_0} f_i\, dS_0\Big]\delta a_i +$$

$$+\sum_{m=1}^{\infty}\Big\{\sum_{k=1}^{\infty} c_k\int_{\Omega}(-\Delta\varphi_k)\varphi_m\, d\Omega + \sum_{k=1}^{\infty} c_k\int_{S_0\cup S}\frac{\partial\varphi_k}{\partial n}\varphi_m\, dS -$$

$$-\sum_{i=1}^{\infty}\dot{a}_i\int_{S_0} f_i(\varphi_m)\big|_{z=0}dS_0 + \int_{S^*}\Big[\frac{\partial}{\partial n}(\vec{v}_0\cdot\vec{r}) - \vec{v}_0\cdot\vec{n}\Big]\varphi_m\, dS^*\Big\}\delta c_m \equiv 0 \qquad (15)$$

Now, we notice that we have on S^*,

$$\frac{\partial}{\partial n}(\vec{v}_0\cdot\vec{r}) = \vec{n}\cdot\nabla(\vec{v}_0\cdot\vec{r}) = \vec{n}\cdot\vec{v}_0$$

$$\Big(\int_{S^*}\varphi_m dS \text{ - finite}\Big)$$

Then, as the variations δa_i and δc_m are independent and arbitrary on their definition domain, from the identity (15) we get the infinite system of first order differential equations with respect to the functions $a_i(t)$ and $c_m(t)$:

$$(16)\qquad \sum_{k=1}^{\infty} E_{mk}\, c_k - \sum_{i=1}^{\infty} A_{mi} \frac{da_i}{dt} + \sum_{k=1}^{\infty} B_{mk}\, c_k = 0 , \quad m=1,2,\ldots$$

$$(17)\qquad \sum_{m=1}^{\infty} C_{im} \frac{dc_m}{dt} + (\dot{\vec{v}}_o - \vec{g})\cdot \vec{b}_i - \dot{\vec{v}}_o \cdot \sum_{m=1}^{\infty} \vec{T}_{im} c_m - v_o^2 T_i = 0, \quad i=1,2,\ldots$$

if we introduce the notations

$$E_{mk} = - \int_{\Omega} \varphi_m \Delta \varphi_k \, d\Omega$$

$$A_{mi} = \int_{S_o} (\varphi_m)|_{z=0} \, f_i \, dS_o$$

$$B_{mk} = \int_{S_o \cup S^*} \varphi_m \frac{\partial \varphi_k}{\partial n} \, dS$$

$$C_{im} = \int_{S_o} f_i \, (\varphi_m)|_{z=0} \, dS_o$$

$$\vec{T}_{im} = \int_{S_o} f_i \, (\nabla \varphi_m)|_{z=0} \, dS_o$$

$$\vec{b}_i = \int_{S_o} \vec{r} \, f_i \, dS_o , \qquad T_i = \int_{S_o} f_i \, dS_o$$

<u>The choise of functions φ_m and f_m</u>. From (14) we find that the function

$$\varphi(x,y,z,t) = \sum_{m=1}^{\infty} c_m(t)\, \varphi_m(x,y,z)$$

is the velocity potential of the ideal fluid for the irrotational movement in tanks which are fixed in space. In the case of the fixed tank ($\vec{v}_o = 0$), if we put $\varphi(x,y,z,t) = \tilde{\varphi}(x,y,z)\cos(\sigma t + \varepsilon)$ then $\tilde{\varphi}$ satisfies the equation as well as the linear conditions (an eigenvalue problem)

$$\Delta\tilde{\varphi}(x,y,z,t) = 0 \quad in\ \Omega \tag{18}$$

$$\frac{\partial\tilde{\varphi}}{\partial z} = \lambda\tilde{\varphi} \quad \text{on } S_o \ , \quad \frac{\partial\tilde{\varphi}}{\partial n} = 0 \quad \text{on } S^*, \ (\lambda = \sigma^2/g)$$

which can be inferred from (6)-(7) by putting $\phi = \tilde{\varphi}$, $\vec{v}_t = 0$ and combining the kinematic and dynamic conditions on S_o.

The functions φ_m are compared to $\tilde{\varphi}$ and we may choose

$$\Delta\varphi_m(x,y,z) = 0 \ , \ (x,y,z)\in\Omega \quad ; \quad \frac{\partial\varphi_m}{\partial n} = 0 \quad \text{on } S^* \tag{19}$$

The functions f_i are required to form an orthogonal (and complete) harmonic function system

$$\Delta f_i(x,y) = 0 \quad \text{on } S_o$$

$$(f_i, f_j) = \int_{S_o} f_i f_j \, dS_o = \begin{cases} I_i^2 \ , & \text{if } j = i \\ 0 \ , & \text{if } j \neq i \end{cases} \tag{20}$$

and, according to (18) let us put

$$f_m(x,y) = \lambda_m \varphi_m(x,y,0)$$

$$\left(\frac{\partial\varphi_m}{\partial z} = f_m(x,y) \ \text{on } S_o\right)$$

The functions φ_m and f_m employed to apply the variational equation (12) may also be picked up from harmonic function subspaces of the Sobolev space $H^1(\Omega)$ and $H^1(S_o)$ respectively. The systems $\{\varphi_m\}$ and $\{f_m\}$ will be complete in $H^1(\Omega)$, $H^1(S_o)$.

The fluid motion equations (the equations of the coefficients $a_i(t)$ and $c_m(t)$). The following expressions are obtained if we use the formulas (19)-(20)

$$I_m^2 = \int_{S_o} f_m^2(x,y)\, dS_o \ ;$$

$$E_{mk} = 0$$

(21)

$$A_{mi} = \frac{1}{\lambda_m} \int_{S_o} f_m f_i \, dS_o = \begin{cases} \frac{-1}{\lambda_m} I_m^2 & , i = m \\ 0 & , i \neq m \end{cases} ;$$

$$B_{mk} = \int_{S_o} \left(\varphi_m \frac{\partial \varphi_k}{\partial z} \right)\Big|_{z=0} dS_o = \frac{1}{\lambda_m} \int_{S_o} f_m f_k dS_o = \begin{cases} \frac{1}{\lambda_m} I_m^2 & , k=m \\ 0 & , k \neq m \end{cases}$$

$$C_{im} = \begin{cases} \frac{1}{\lambda_i} I_i^2 & , m = i \\ 0 & , m \neq i \end{cases}$$

Considering $\vec{b_i} = (b_{i1}, b_{i2}, b_{i3})$, for the components b_{ik}, $k = 1,2,3$, we have the expressions

$$b_{i1} = \int_{S_o} x f_i \, dS_o \ ;$$

(22)

$$b_{i2} = \int_{S_o} y f_i \, dS_o \ ;$$

$$b_{i3} = a_i \int_{S_o} f_i^2 \, dS_o = a_i I_i^2$$

$$\left(b_{i3} = \int_{S_o} z f_i \, dS_o = \int_{S_o} f f_i \, dS_o = \int_{S_o} \left(\sum_{k=1}^{\infty} a_k f_k \right) f_i dS_o = a_i \int_{S_o} f_i^2 dS_o \right)$$

According to the linear theory, the parameters that characterize the movement of the mechanic system (tank and fluid) are small values and subsequently their products may be neglected. Therefore, the nonlinear terms with respect to c_m, a_i and v_o from (16) and (17) may be removed.

If we admit the linearization approximations and we use the formulas (21)-(22), the equations (16)-(17) are reduced to the form

$$(23) \qquad \frac{da_m(t)}{dt} - c_m(t) = 0 \qquad m = 1,2,\ldots$$

$$(24) \qquad \frac{I_i^2}{\lambda_i} \frac{dc_i(t)}{dt} + (\dot{\vec{v}}_o - \vec{g}).[\vec{k}\, I_i^2\, a_i(t) + \vec{b}_i^{\,*}] = 0 \quad , \; i = 1,2,..$$

$$(25) \qquad \vec{b}_i^{\,*} = (b_{i1}, b_{i2}, 0) \quad , \quad |\vec{k}| = 1$$

The functions $a_i(t)$ are solutions for the infinite system of second order equations

$$(26) \qquad \frac{d^2 a_i(t)}{dt^2} + \sigma_i^2 a_i(t) = -\frac{1}{\mu_i}(\dot{\vec{v}}_o - \vec{g}).\vec{b}_i^{\,*} \quad , \; i = 1,2,\ldots$$

where the vector $\vec{b}_i^{\,*}$ is given in (25) and

$$(27) \qquad \mu_i = \frac{I_i^2}{\lambda_i} \quad , \quad \sigma_i^2 = \lambda_i(\dot{\vec{v}}_o - \vec{g}).\vec{k} = \lambda_i(\dot{\vec{v}}_o - \vec{g})_{Oz}$$

Initial conditions or periodicity conditions are associated to the system (26). The coefficients of these equations are calculated by means of the above formulas, by using the solutions (harmonic oscillations) of the fluid motion in a fixed tank. For $\vec{v}_o = 0$, the previous equations correspond to the fixed tank.

<u>A particular case</u>. <u>A uniformly accelerated translation on the vertical axis Oz</u> ($\parallel \vec{g}$). In this case we have (a = tank acceleration)

$$\dot{\vec{v}}_0(t) = \dot{v}_{Oz}(t)\,\vec{k} = a\,\vec{k}\ , \quad v_{Oz}(t) = at\ , \quad a = \text{const}\,(>0)$$

$$(28)\qquad \frac{d^2 a_i}{dt} + \sigma_i^2\, a_i(t) = 0\ , \quad \sigma_i^2 = (a+g)\,\lambda_i\ , \quad i = 1,2,\ldots$$

The general solution to this equation is

$$a_i(t) = \bar{A}_i \cos \sigma_i t + \bar{B}_i \sin \sigma_i t\ , \qquad i = 1,2,\ldots$$

After (13)-(14) the solutions of the problem are represented by the series (simple harmonic oscillations)

$$(29)\qquad f(x,y,t) = \sum_{i=1}^{\infty} \lambda_i A_i \sin(\sigma_i t + \varepsilon_i)\, \varphi_i(x,y,0)$$

$$(30)\qquad \varphi(x,y,z,t) = \sum_{m=1}^{\infty} \sigma_m A_m \sin(\sigma_m t + \varepsilon_m)\, \varphi_m(x,y,z)$$

The series (29) defines the free surface S. Each term of the series represents a stationary wave. A point on S has a harmonic oscillatory movement of $A_i \varphi_i(x,y,0)$ amplitude and respectively σ_i frequency.

R E F E R E N C E S

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Summary

By using a Galerkin method, Bateman's principle is demonstrated and a variational equation is inferred for the linearized motion of an ideal, incompressible and potential liquid in a mobile tank. In the case of tank translation, the variational equation is solved by choosing, for the velocity potential and for the free surface function by time dependent generalized series. We have obtained differential equations of the coefficients as well as the types of solutions in the case of an uniformly accelerated motion of the tank.

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This paper is in final form and no version of it is or will be submitted for publication elsewhere.