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Dependence of the Equivalent Complex Magnetic Permeability on the Shape of the Variation in Time of the Alternating Magnetic Field and on the Shape of the Magnetisation Curve of the Ferromagnetic Materials

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With 18 Figures

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Summary

In the work a quantitative study of the $H(t)$ and $B(t)$ shape curves influence is given concerning the equivalent complex magnetic permeability of ferromagnetic materials, defined by means of the first harmonics of the functions $B(t)$ and $H(t)$ and admitting for $H(t)$ a curve close by those met in practice $H(t) = H_m \sin \omega t |\sin \omega t|^{p-1}$ with $p > 0$. The ratio between the magnetic permeability defined in this way and the static permeability obtained from the normal magnetisation curve for the peak value, for the peak value of the first harmonic and for the effective value of the magnetic field respectively is determined. The variation of these ratios is studied, for normal magnetisation curve using the expressions: a) $B = KH^{\frac{1}{n}}$; b) $B = B_0 \left(1 - \frac{H_0}{H}\right)$; c) $B = aH - b \sin cH$, in which K, n, B_0, H_0, a, b and c are constants. Several useful conclusions are drawn for the calculation of the equivalent permeability in alternating field. The calculated values agree satisfactory with the presented experimental results.

Zusammenfassung

Es wird der Einfluß der Kurvenform der Zeitverläufe von magnetischer Feldstärke und Induktion auf die äquivalente komplexe magnetische Permeabilität ferromagnetischer Materialien quantitativ untersucht. Das Verhältnis der durch die ersten Harmonischen der entsprechenden Zeitfunktionen definierten Permeabilität und der aus der Hysteresekurve für den Spitzenwert resp. für den Spitzenwert der Grundwelle und für den Effektivwert der magnetischen Feldstärke ermittelten statischen Permeabilität wird bestimmt und für verschiedene analytische Approximationen der Hysteresekurve diskutiert. Für die Berechnung der äquivalenten Permeabilität bei Wechselfeldern werden einige nützliche Folgerungen gezogen. Die berechneten Werte stimmen mit den experimentellen Ergebnissen zufriedenstellend überein.

1. Introduction

The study of the equivalent complex magnetic permeability of the ferromagnetic materials in the alternating magnetic field is required for the magnetic circuit calculation. Thus for the calculation of the electromagnetic field in ferromagnetic conductors in the hypothesis of a sinusoidal time variation of magnetic induction B, a magnetic field H and an electric field [1–4], this study enables us to obtain indications in regard to the value of the equivalent complex permeability.

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The static permeability of the ferromagnetic materials for alternating magnetic field varies in the course of a period of time. The influence of the variation in time of this quantity upon the equivalent permeability, regardless of the time, should be expressed by means of the static permeability determined from a normal magnetisation curve, which is mostly employed among the ferromagnetic material characteristics, for a certain value of the magnetic field. For this study it is necessary to take for $H(t)$ various shapes, taking into account that at present the ferromagnetic materials are required for magnetic fields which have a time variation far from the sinusoidal one.

For two given materials in the case of sinusoidal variation of the magnetic field the absolute value of the equivalent complex magnetic permeability was studied by DREYFUS [5]; by neglecting the hysteresis, the determination of the first harmonic of the magnetic induction is done by the approximation of the curve $B(t)$ with the straight segments and the decomposition in Fourier series of the trapeziums obtained in this way.

NEIMAN [1] studied the problem more generally, in the case of strong magnetic fields, admitting for the magnetisation curve a parabolical expression.

In this paper a quantitative study of the influence of the shape curves $H(t)$ and $B(t)$ on the equivalent complex permeability in some calculus hypotheses is made.

2. Definition of the equivalent complex permeability

For a non-sinusoidal variation of the magnetic field in time, the value of the equivalent complex permeability,

$$\mu^{\angle} = \frac{B^{\angle}_{\mathrm{me}}}{H^{\angle}_{\mathrm{me}}} = \mu\, \mathrm{e}^{-\mathrm{j}\chi} = \mu \cos \chi - \mathrm{j}\mu \sin \chi = \mu_1 - \mathrm{j}\mu_2 , \tag{1}$$

depends upon the manner in which the equivalent harmonics B_{me} and H_{me} of the magnetic induction and the magnetic field are defined, therefore it depends upon the shape of the curves $B(t)$ and $H(t)$, of the shape of the magnetisation curve, respectively.

For the determination of the three unknown quantities B_{me}, H_{me} and χ we have to fulfil only one condition, that is to keep unchanged the hysteresis losses during one cycle and in the unit volume:

$$w_{\mathrm{h}} = \oint H\, \mathrm{d}B = \oint H_{\mathrm{me}} \sin \omega t\, \mathrm{d}[B_{\mathrm{me}} \sin (\omega t - \chi)] = \pi H_{\mathrm{me}} B_{\mathrm{me}} \sin \chi \tag{2}$$

Taking into account that the curves $B(t)$ and $H(t)$ are in general nonsinusoidal, the determination of the other conditions would be done in several ways. DREYFUS [5] and NEIMAN [1] arrive to the conclusion that it is more suitable to consider B_{me} and H_{me} equal to the peak values of the first harmonics:

$$B_{\mathrm{me}} = B_{1\mathrm{m}}, \qquad H_{\mathrm{me}} = H_{1\mathrm{m}}. \tag{3}$$

These relations will be adopted also in this paper. In this case

$$\mu = \frac{B_{1\mathrm{m}}}{H_{1\mathrm{m}}}, \tag{4}$$

$$\sin \chi = \frac{w_{\mathrm{h}}}{\pi B_{1\mathrm{m}} H_{1\mathrm{m}}} \tag{5}$$

It should be emphasized that in order to preserve unchanged the hysteresis losses, the angle χ does not equal the phase difference between the first harmonics of the induction and the magnetic field.

The conditions (3) can be readily employed, leading to the relations which enables the $\mu^{\angle}$ calculation by means of static permeability determined from the normal magnetisation curve for a certain value of the magnetic field.

3. Calculus hypotheses

The harmonics due to the hysteresis are separated from those due to the nonlinearity of the normal magnetisation curve. In many cases the superior harmonics due to the hysteresis are negligible [6, 7].

For $H > 0$, the normal magnetisation curve is expressed by one of the relations:

$$\text{a) } B = KH^{\frac{1}{n}}, \qquad \text{b) } B = B_0\left(1 - \frac{H_0}{H}\right), \qquad \text{c) } B = aH - b \sin cH, \tag{6}$$

in which K, n, B_0, H_0, a, b, c are constants.

The intensity of the magnetic field is considered to vary in time according to a curve of the shape:

$$H(t) = H_{\mathrm{m}} \sin \omega t |\sin \omega t|^{p-1}, \tag{7}$$

in which $p > 0$; for $p = 1$ $H(t)$ varies sinusoidal, for $p < 1$ $H(t)$ has a flat shape, while for $p > 1$ – a sharp shape.

Thus, in these hypotheses, in the case of $B(H) = K|H|^{\frac{1}{n}-1}H$, the magnetic induction varies in time according to the relation:

$$B(t) = B_{\mathrm{m}} \sin \omega t |\sin \omega t|^{\frac{p}{n}-1}, \tag{8}$$

having a sinusoidal shape for $p = n$, flat shape for $p < n$ and sharp shape for $p > n$. In Fig. 1 are represented the time variation curves of the ratios $\frac{H}{H_{\mathrm{m}}} = h$ and $\frac{B}{B_{\mathrm{m}}} = b$, for some usual values of n and p: $n = 8$ and $p = 0, 1, 8, 64$, $n = 0{\cdot}5$ and $p = 0, 0{\cdot}25, 0{\cdot}50, 1, 2$, respectively. From this figure it results that the curve shapes $B(t)$ and $H(t)$

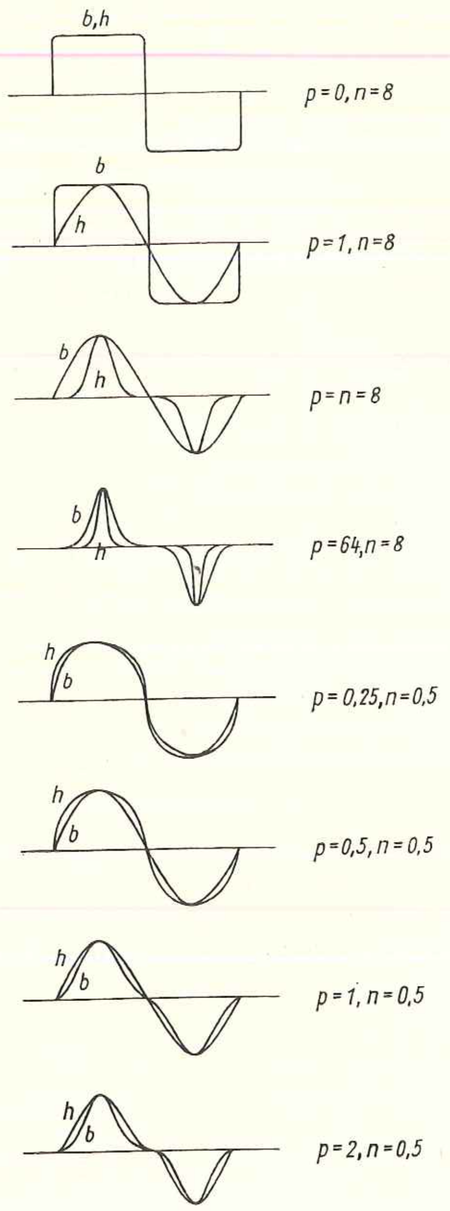

Fig. 1.

are close enough to reality: generally these curves don't move away too much from the sinusoidal variation, the parameter p varying from 1 to n.

The study of the dependence of the equivalent permeability on the shape of the time variation of the magnetic field and on the shape of the magnetisation curve will be made separately for the absolute value and the argument.

4. Absolute value of the permeability

The influence of the shape of the curves $B(t)$ and $H(t)$ on the absolute value of the equivalent complex permeability may be observed by studying the ratios of this, determined by the relation (4) to the static permeability determined from the normal magnetisation curve for the peakvalue of the magnetic field intensity μ_{stm}, for the peak value of the first harmonic of the magnetic field [therefore for the same value of the magnetic field as in the relation (4)] μ_{st1m} and for the effective value of the magnetic field (the value which can be sometimes readily measured) μ_{stef}, respectively.

a) Case $B(H) = KH^{1/n} (H > 0)$

Noting with

$$S_k = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin^k \alpha \, \mathrm{d}\alpha \tag{9}$$

and taking into consideration that the curves $B(t)$ and $H(t)$ are symmetrical in comparison with $t = \frac{T}{4}$, the peak value of the first harmonics of the magnetic induction and magnetic field will be given by the relations

$$H_{1\text{m}} = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} H(t) \sin \omega t \, \mathrm{d}(\omega t) = \frac{4}{\pi} H_{\text{m}} \int_0^{\frac{\pi}{2}} \sin^{p+1} \omega t \, \mathrm{d}(\omega t) = 2H_{\text{m}} S_{p+1}, \tag{10}$$

$$B_{1\text{m}} = \frac{4}{\pi} K H_{\text{m}}^{\frac{1}{n}} \int_0^{\frac{\pi}{2}} \sin^{\frac{p}{n}+1} \omega t \, \mathrm{d}(\omega t) = 2KH_{\text{m}}^{\frac{1}{n}} S_{\frac{p}{n}+1} \tag{11}$$

Thus, the absolute value of the equivalent complex permeability will have the expression:

$$\mu = \frac{B_{1\text{m}}}{H_{1\text{m}}} = KH_{\text{m}}^{\frac{1}{n}-1} \frac{S_{\frac{p}{n}+1}}{S_{p+1}} \tag{12}$$

For the static permeability one readily obtains the expression:

$$\mu_{\text{st}} = KH^{\frac{1}{n}-1} \tag{13}$$

From the above relations and taking into account that the effective value of the magnetic field intensity has the expression:

$$H_{\text{ef}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} H^2 \, \mathrm{d}(\omega t)} = H_{\text{m}} \sqrt{\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin^{2p} \omega t \, \mathrm{d}(\omega t)} = H_{\text{m}} S_{2p}^{\frac{1}{2}}, \tag{14}$$

for the three studied cases we obtain:

$$\frac{\mu}{\mu_{\text{stm}}} = \frac{S_{\frac{p}{n}+1}}{S_{p+1}}, \tag{15}$$

$$\frac{\mu}{\mu_{\mathrm{st1m}}} = 2^{\frac{n-1}{n}} \frac{S_{\frac{p}{n}+1}}{S_{p+1}^{\frac{1}{n}}}, \tag{16}$$

$$\frac{\mu}{\mu_{\mathrm{stef}}} = S_{2p}^{\frac{n-1}{2n}} \frac{S_{\frac{p}{n}+1}}{S_{p+1}} \tag{17}$$

Taking S_k from Fig. 2, the results of the calculuses carried out with these relations for some values of p and $n < 1$ (small magnetic fields) are represented in Fig. 3, 4 and 5. In these figures, the curves corresponding to a sinusoidal variation of the magnetic induction, for $p = n$, are represented by a interrupted line.

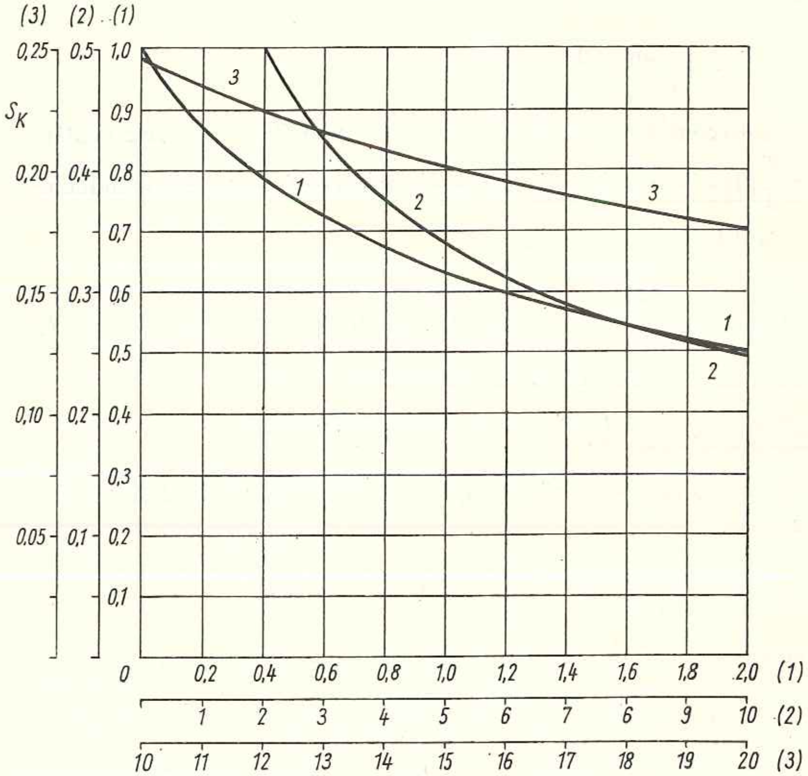

Fig. 2.

The studied ratios vary much with p, the variation limits with p being larger while n is smaller. For the usual values of n and the common shapes of $B(t)$ and $H(t)$ curves, these values are relatively small, especially for the two ratios. Thus for $p = 0{\cdot}5 \ldots 1{\cdot}0$ and $n = 0{\cdot}5$ $\frac{\mu}{\mu_{\mathrm{stm}}} = 0{\cdot}85 \cdots 0{\cdot}90$; $\frac{\mu}{\mu_{\mathrm{st1m}}} = 0{\cdot}80 \cdots 0{\cdot}85$ and $\frac{\mu}{\mu_{\mathrm{stef}}} = 1{\cdot}10 \cdots 1{\cdot}20$.

In the case of strong magnetic fields, the first ratio varies a great deal with n and p. For the usual ferromagnetic materials ($4 < n < 16$), the last two ratios vary by a narrow

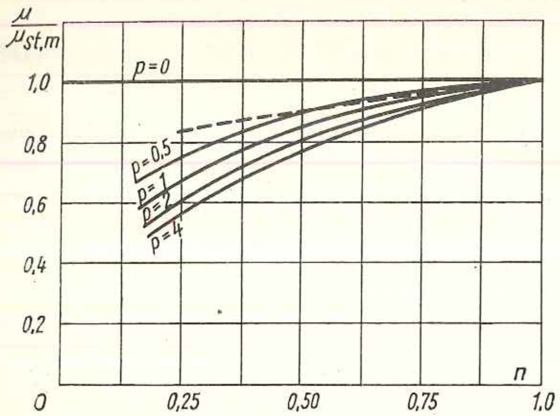

Fig. 3.

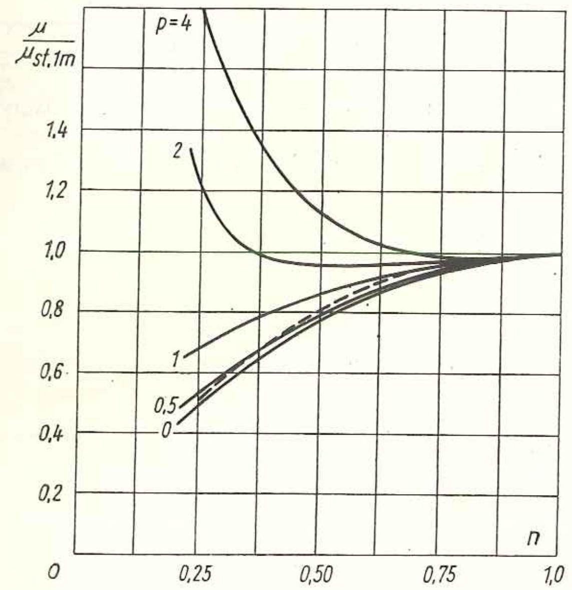

Fig. 4.

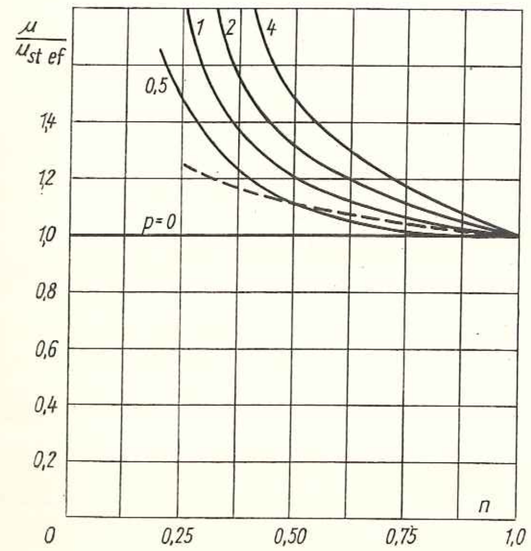

Fig. 5.

limit: $\dfrac{\mu}{\mu_{\mathrm{st\,1m}}} = 1{\cdot}20 \cdots 1{\cdot}25$ for $p = 1$ (a sinusoidal variation of the magnetic field) and $1{\cdot}00 \cdots 1{\cdot}06$ for $p = n$ (a sinusoidal variation of the magnetic induction); $\dfrac{\mu}{\mu_{\mathrm{stef}}} = 0{\cdot}90 \cdots 1{\cdot}05$ for $p = 1 \cdots n$.

b) Case $B = B_0\left(1 - \dfrac{H_0}{H}\right)$

From this expression it results that $B = 0$ for $H = H_0$ and $B \to B_0$ for $H \to \infty$; the static permeability $\mu_{\mathrm{st}} = \dfrac{B_0}{H}\left(1 - \dfrac{H_0}{H}\right)$ is maximum for $H_{\mathrm{M}} = 2H_0$. Will be considered $B = 0$ for $H \leqq H_0$, the expression $B = B_0\left(1 - \dfrac{H_0}{H}\right) = B_0\left(1 - \dfrac{\eta}{2}\right)$, with $\eta = \dfrac{2H_0}{H}$, being good for strong magnetic fields $H > 4H_0$, $\eta < 0{\cdot}5$ respectively.

In this case the peak value of the first harmonic of the magnetic induction will have the expression:

$$B_{1\mathrm{m}} = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} B_0\left(1 - \frac{H_0}{H_{\mathrm{m}}\sin^p \omega t}\right)\sin \omega t \, \mathrm{d}(\omega t) = B_0\left[\frac{4}{\pi}\cos \omega t_0 - \frac{H_0}{H_{\mathrm{m}}} F_1(\omega t_0, p)\right], \tag{18}$$

in which ωt_0 is determined from the condition $H_0 = H_{\mathrm{m}} \sin^p \omega t_0$,

$$\omega t_0 = \arcsin\left(\frac{H_0}{H_{\mathrm{m}}}\right)^{\frac{1}{p}}, \tag{19}$$

$$\cos \omega t_0 = \sqrt{1 - \left(\frac{H_0}{H_{\mathrm{m}}}\right)^{\frac{2}{p}}} \text{ respectively, and}$$

$$F_1(\omega t_0, p) = \frac{4}{\pi}\int_{\omega t_0}^{\frac{\pi}{2}} \sin^{1-p} \omega t \, \mathrm{d}(\omega t) \tag{20}$$

being non-integrable analyticly only for some values of p, it was done with a digital electronic computer (see Appendix).

Thus taking into account the relation (9) and marking with the three ratios that concern us, will have the expressions:

$$\frac{\mu}{\mu_{\mathrm{stm}}} = \frac{\dfrac{4}{\pi}\cos \omega t_0 - \dfrac{\eta_{\mathrm{m}}}{2} F_1(\omega t_0, p)}{2S_{p+1}\left(1 - \dfrac{\eta_m}{2}\right)}, \tag{21}$$

$$\frac{\mu}{\mu_{\mathrm{st\,1m}}} = \frac{\dfrac{4}{\pi}\cos \omega t_0 - \dfrac{\eta_{\mathrm{m}}}{2} F_1(\omega t_0, p)}{1 - \dfrac{\eta_{\mathrm{m}}}{4S_{p+1}}}, \tag{22}$$

$$\frac{\mu}{\mu_{\mathrm{stef}}} = \frac{\dfrac{4}{\pi}\cos \omega t_0 - \dfrac{\eta_{\mathrm{m}}}{2} F_1(\omega t_0, p)}{\dfrac{2S_{p+1}}{S_{2p}^{1/2}}\left(1 - \dfrac{\eta_{\mathrm{m}}}{2S_{2p}^{1/2}}\right)} \tag{23}$$

The limit case $\eta_{\mathrm{m}} = 0$ ($H_0 = 0$), in other words for a rectangular magnetisation curve $B = B_0 \operatorname{sign} H$, these ratios can be also determined in an another way. For any value of p, $B(t)$ will have a rectangular variation shape; for this curve $B(t)$ is known that $B_{1\mathrm{m}} = \dfrac{4}{\pi} B_0$. Thus

$$\left(\frac{\mu}{\mu_{\mathrm{stm}}}\right)_0 = \frac{2}{\pi}\frac{1}{S_{p+1}} = \frac{0{\cdot}6366}{S_{p+1}}, \tag{21'}$$

$$\left(\frac{\mu}{\mu_{\mathrm{st\,1m}}}\right)_0 = \frac{4}{\pi} = 1{\cdot}273, \tag{22'}$$

$$\left(\frac{\mu}{\mu_{\mathrm{stef}}}\right)_0 = \frac{2}{\pi}\frac{S_{2p}^{\frac{1}{2}}}{S_{p+1}} = 0{\cdot}6366\frac{S_{2p}^{\frac{1}{2}}}{S_{p+1}} \tag{23'}$$

The variation with p of the first and the last of these ratios is represented in Fig. 6 and 7, the second being constant. The first ratio monotonous increases with p from 1 ($p = 0$) to 5·73 for $p = 50$; the last ratio decreases initially from 1 ($p = 0$) to 0·9 ($p = 1$) then increasing with p to 1·62 for $p = 50$.

The variation with $\eta_{\mathrm{m}} \in [0,1]$ for different values of p of the studied ratios is given in Fig. 8 to 10; in Fig. 9 bis and 10 bis the variation of the ratios are represented in function of $\eta_{1\mathrm{m}} = \dfrac{2H_0}{H_{1\mathrm{m}}} = \dfrac{\eta_{\mathrm{m}}}{2S_{p+1}}$, $\eta_{\mathrm{ef}} = \dfrac{2H_0}{H_{\mathrm{ef}}} = \dfrac{\eta_{\mathrm{m}}}{S_{2p}^{1/2}}$ respectively.

The ratio $\dfrac{\mu}{\mu_{\mathrm{stm}}}$ for $p \leqq 2$ or $\eta_{\mathrm{m}} > 0{\cdot}3$ decreases almost linearly with η_{m}.

The ratio $\dfrac{\mu}{\mu_{\mathrm{st\,1m}}}$ decreases monotonous from $\dfrac{4}{\pi}$ ($\eta_{\mathrm{m}} = 0$) for $p < 3$, having a minimum over this value of p. The variation of this ratio with η_{m} is the more important, the greater is p.

The ratio $\dfrac{\mu}{\mu_{\mathrm{stef}}}$ increases monotonous with η_{m} for $p < 1$, being smaller than 1 for $\eta_{\mathrm{ef}} < 1{\cdot}25$, and having a minimum over this value of p; its variations are the more

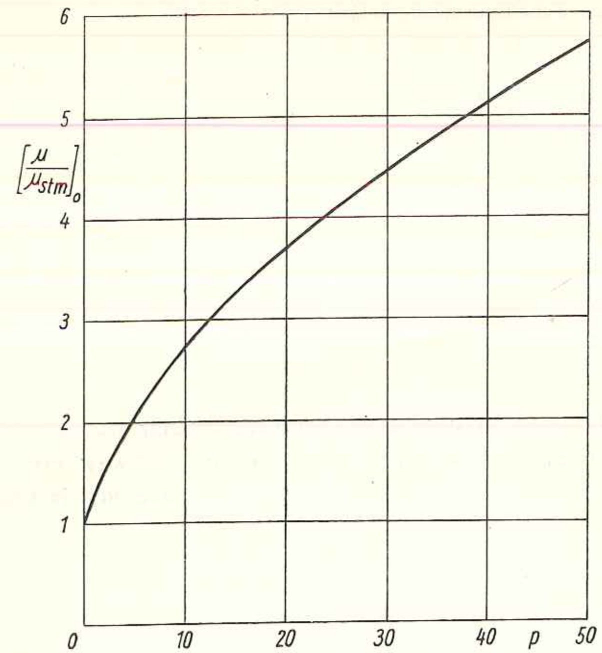

Fig. 6.

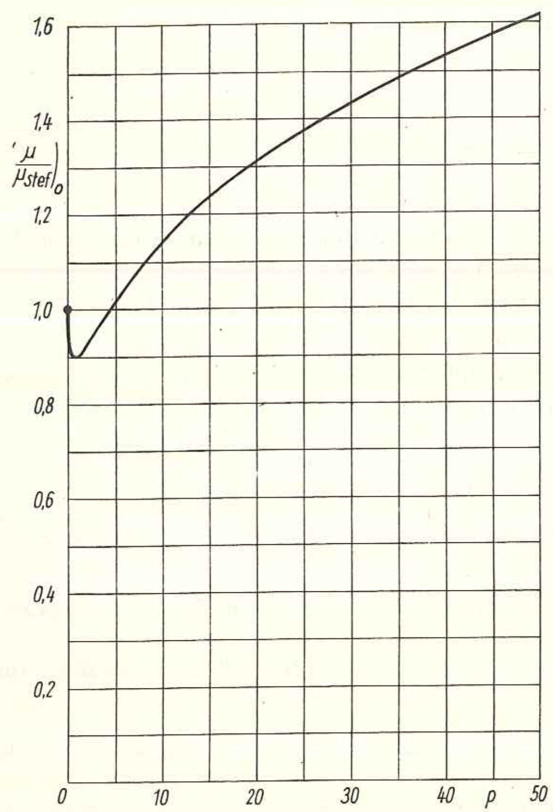

Fig. 7.

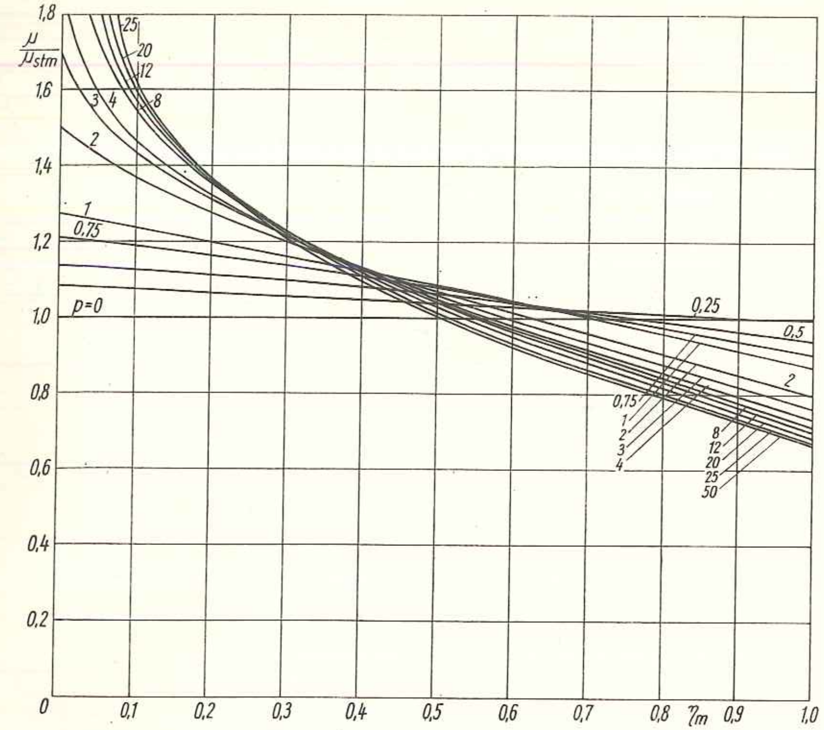

Fig. 8.

important, the greater is p. For $p = 1$ and $\eta_{ef} < 1$ it has a value approximately constant and equal to 0·9.

The variation of the ratios $\frac{\mu}{\mu_{st1m}}$, $\frac{\mu}{\mu_{stef}}$ respectively are slowlier as function of η_{1m}, η_{ef} respectively.

These results agree with those obtained approximating the normal magnetisation curve with a parabola.

c) Case $B = aH - b \sin cH$

As results from [8], the static permeability $\mu_{st} = \frac{B}{H} = a - b\frac{\sin cH}{H}$ has a minimum, the initial permeability $\mu_i = a - bc$, for $cH = 0$; it has a maximum $\mu_M = a + 0{\cdot}2172bc$ for $cH_M = 4{\cdot}4934$ and an another minimum $\mu_{min} = a - 0{\cdot}1284bc$ for $cH_{min} = 7{\cdot}725$. From here it results that this expression approximates the magnetisation curve in the domain of the medium magnetic fields, $cH < 8$, in which the static permeability has the maximum.

From the above relations, knowing the initial permeability μ_i, the maximum static permeability μ_M and the corresponding magnetic field intensity H_M, can be determined

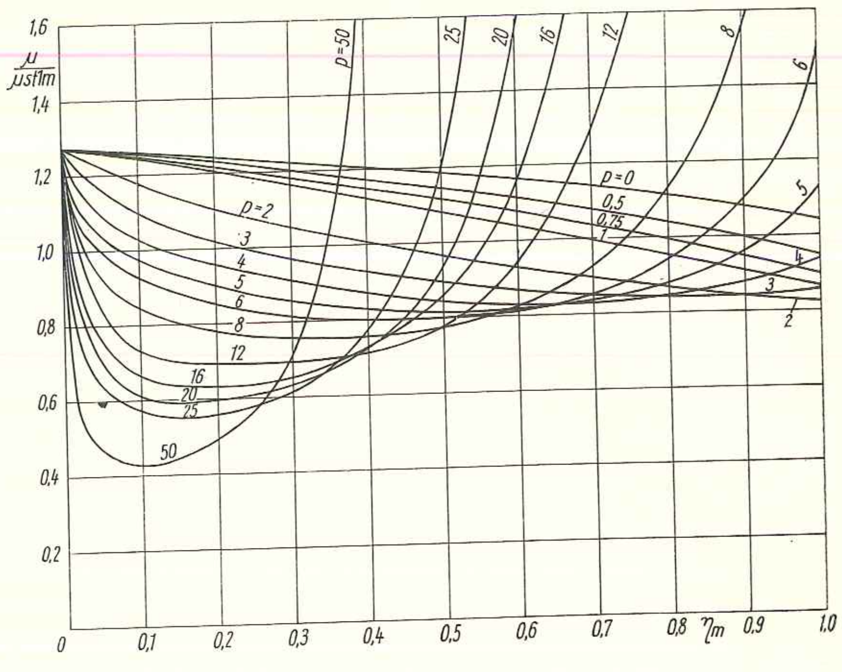

Fig. 9a.

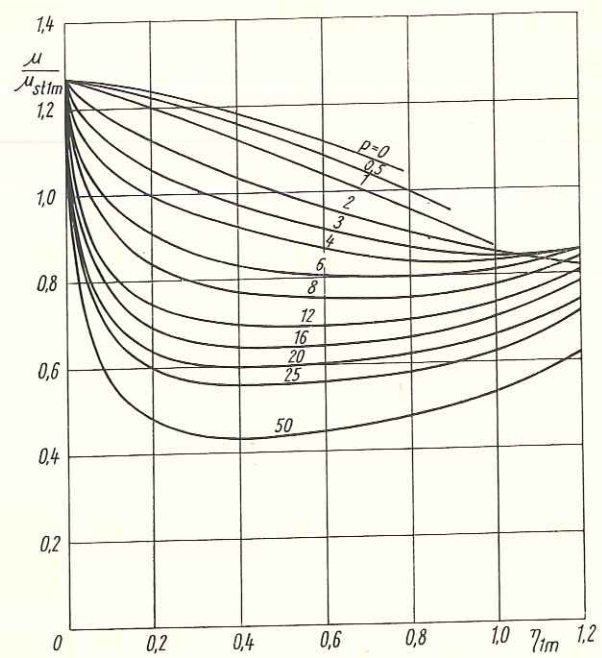

Fig. 9b.

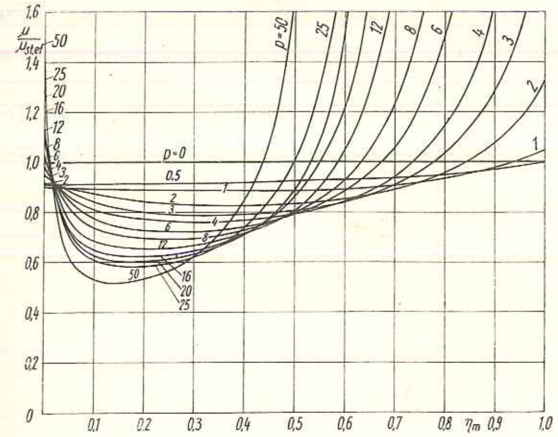

Fig. 10a.

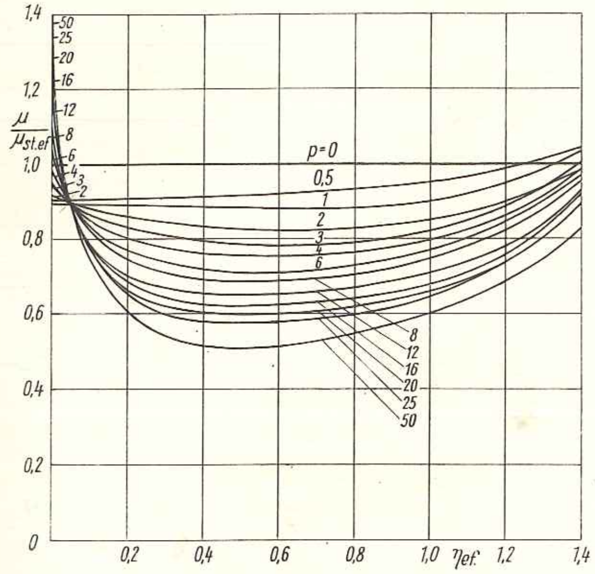

Fig. 10b.

the quantities a, b and c:

$$a = 0{\cdot}8215\mu_i + 0{\cdot}1785\mu_M,$$

$$b = 0{\cdot}1825 H_M(\mu_M - \mu_i), \tag{24}$$

$$c = \frac{4{\cdot}4934}{H_M} = \frac{1}{0{\cdot}2225 H_M}$$

In this case the peak value of the first harmonic of the magnetic induction will have the expression

$$B_{1m} = 2aH_m \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin^{p+1}\omega t\, d(\omega t) - b\frac{4}{\pi}\int_0^{\frac{\pi}{2}} \sin(cH_m \sin^p \omega t)\sin\omega t\, d(\omega t) \quad (25)$$

The last integral, being not possible effectuated by analytical means, was numerical calculated by a digital electronic computer (see Appendix); noting with

$$F_2(cH_m, p) = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} \sin(cH_m \sin^p \omega t)\sin\omega t\, d(\omega t), \quad (26)$$

the expression of B_{1m} obtains the form

$$B_{1m} = 2aH_m S_{p+1} - bF_2(cH_m, p) \quad (25')$$

Taking into account the relations (9), (14) and (24), the three studied ratios will be given by the expressions:

$$\frac{\mu}{\mu_{stm}} = \frac{a - \dfrac{b}{2}\dfrac{F_2(cH_m, p)}{S_{p+1}H_m}}{a - b\dfrac{\sin cH_m}{H_m}} = \frac{1 - \dfrac{1 - \dfrac{\mu_i}{\mu_M}}{1 + 0{\cdot}2172\dfrac{\mu_i}{\mu_M}}\,\dfrac{F_2(cH_m, p)}{2S_{p+1}cH_m}}{1 - \dfrac{1 - \dfrac{\mu_i}{\mu_M}}{1 + 0{\cdot}2172\dfrac{\mu_i}{\mu_M}}\,\dfrac{\sin cH_m}{cH_m}}, \quad (27)$$

$$\frac{\mu}{\mu_{st1m}} = \frac{a - \dfrac{b}{2}\dfrac{F_2(cH_m, p)}{S_{p+1}H_m}}{a - b\dfrac{\sin cH_{1m}}{H_{1m}}} = \frac{1 - \dfrac{1 - \dfrac{\mu_i}{\mu_M}}{1 + 0{\cdot}2172\dfrac{\mu_i}{\mu_M}}\,\dfrac{F_2(cH_m, p)}{2S_{p+1}cH_m}}{1 - \dfrac{1 - \dfrac{\mu_i}{\mu_M}}{1 + 0{\cdot}2172\dfrac{\mu_i}{\mu_M}}\,\dfrac{\sin 2S_{p+1}cH_m}{2S_{p+1}cH_m}}, \quad (28)$$

$$\frac{\mu}{\mu_{stef}} = \frac{a - \dfrac{b}{2}\dfrac{F_2(cH_m, p)}{S_{p+1}H_m}}{a - b\dfrac{\sin cH_{ef}}{H_{ef}}} = \frac{1 - \dfrac{1 - \dfrac{\mu_i}{\mu_M}}{1 + 0{\cdot}2172\dfrac{\mu_i}{\mu_M}}\,\dfrac{F_2(cH_m, p)}{2S_{p+1}cH_m}}{1 - \dfrac{1 - \dfrac{\mu_i}{\mu_M}}{1 + 0{\cdot}2172\dfrac{\mu_i}{\mu_M}}\,\dfrac{\sin S_{2p}^{\frac{1}{2}}cH_m}{S_{2p}^{\frac{1}{2}}cH_m}} \quad (29)$$

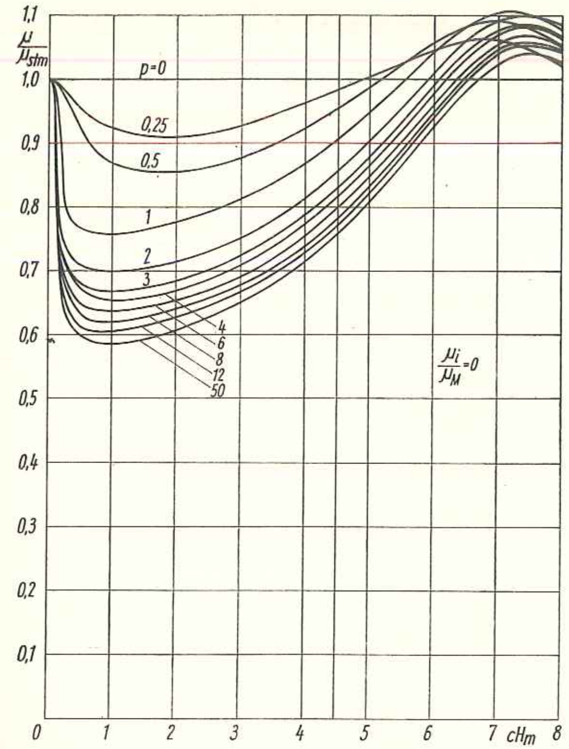

Fig. 11.

The limit values for $cH_m \to 0$ can be readily determined, observing that

$$\lim_{cH_m \to 0} F_2(cH_m, p) = \frac{4}{\pi} cH_m \int_0^{\frac{\pi}{2}} \sin^{p+1}\omega t\, d(\omega t) = 2cH_m S_{p+1} \quad (26')$$

Thus the three ratios will have the values

$$\lim_{cH_m \to 0}\frac{\mu}{\mu_{stm}} = 1; \quad \lim_{cH_m \to 0}\frac{\mu}{\mu_{st1m}} = 1; \quad \lim\frac{\mu}{\mu_{stef}} = 1 \quad (27'–29')$$

The studied ratios depend upon p, cH_m and $\frac{\mu_i}{\mu_M}$. Figures 11–16 represent the variations of these ratios in function of cH_m for different values of p and values of the ratio $\frac{\mu_i}{\mu_M} = 0$ and 0·25 (value more frequently met). In these figures are ploted straight lines at $cH_m = 4{\cdot}4934$, the value for which the static permeability is maximum.

The ratio $\frac{\mu}{\mu_{stm}}$ decreases from 1, goes through a minimum the smaller the bigger is p, and then attains a maximum of 1·025 ⋯ 1·10 for $cH_m \approx 7$; thus the maximum of μ will be for $H_m > H_M$.

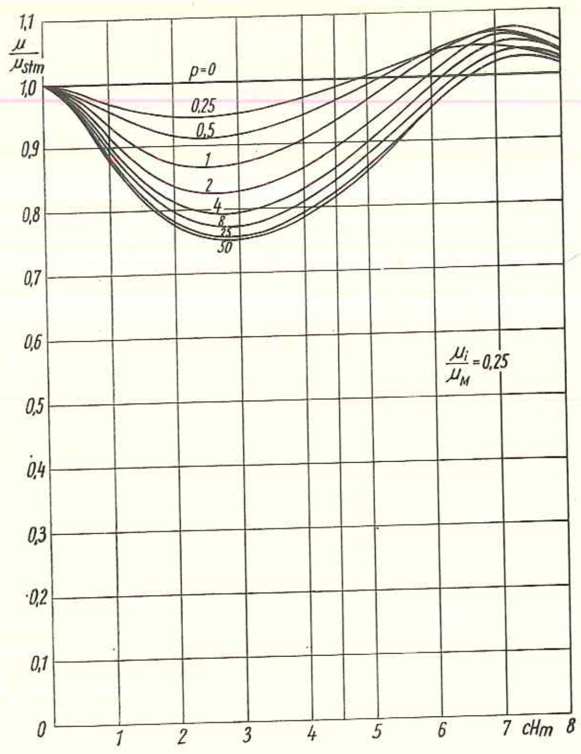

Fig. 12.

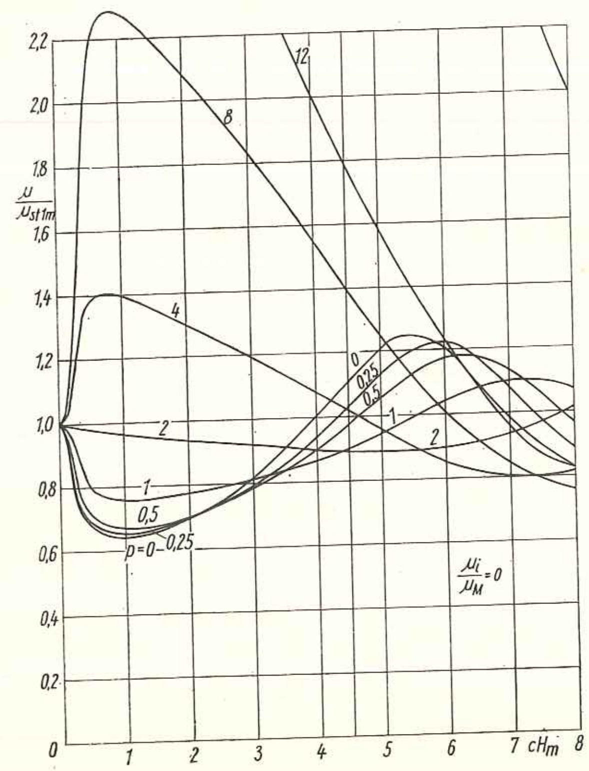

Fig. 13.

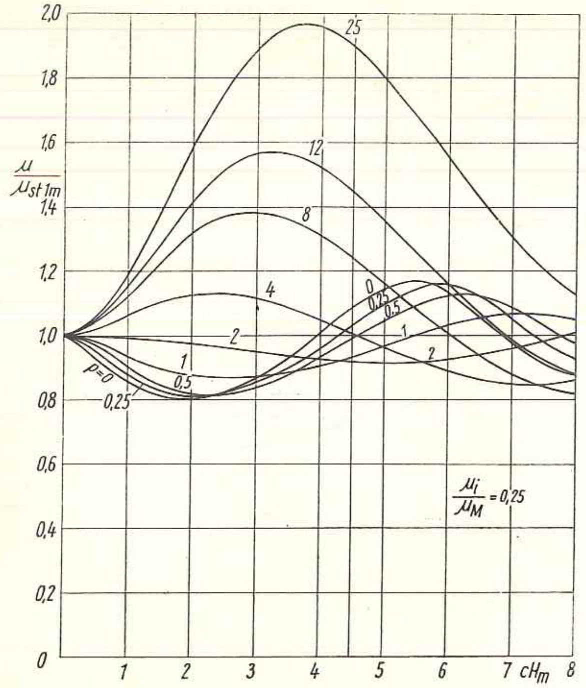

Fig. 14.

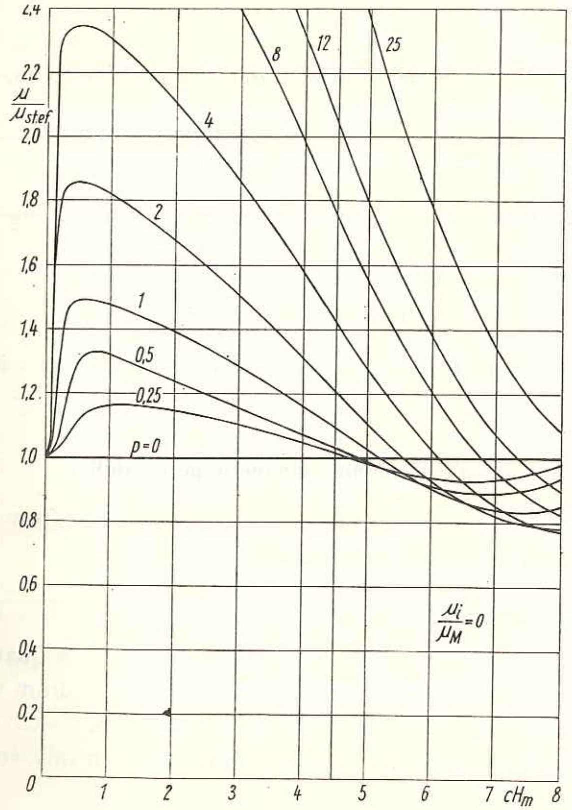

Fig. 15.

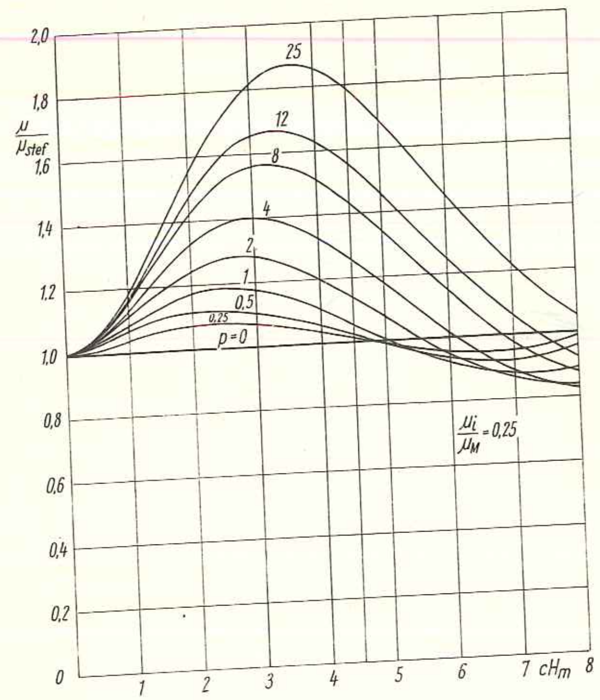

Fig. 16.

The ratio $\frac{\mu}{\mu_{\text{st1m}}}$ varies a great deal with cH_{m} and with p, being possible to have values both smaller and bigger than one. For $p < 2$ it passes through a minimum, but for $p > 2$ the ratio passes through a maximum.

The ratio $\frac{\mu}{\mu_{\text{stef}}}$ passes through a maximum, after which it decreases, being possible to become smaller than one for $cH_{\text{m}} > 4{\cdot}75$.

The variations of these three ratios are the smaller the bigger is $\frac{\mu_{\text{i}}}{\mu_{\text{M}}}$.

In the case of the small magnetic fields, these results are in agreement with those obtained in case a.

5. The argument of the equivalent complex magnetic permeability

The angle χ is determined by the relation (5), in which w_{h} is obtained from the area under the hysteresis cycles or with the empirical relation

$$w_{\text{h}} = \varkappa B_{\text{m}}^{\nu}; \tag{30}$$

in the last expression B_{m} is the peak value of the magnetic induction, $\varkappa$ a quantity empirical determined, $\nu = 2$ for large or small values and $\nu = 1{\cdot}6$ for medium values of the magnetic induction (approximately $0{\cdot}1\ \text{Wb/m}^2 < B_{\text{m}} < 1\ \text{Wb/m}^2$).

The values B_{m}, $B_{1\text{m}}$ or $H_{1\text{m}}$ are expressed with the relations given in this work.

Thus in the case $B = KH^{1/n}$, for $\sin\chi$ is obtained

$$\sin\chi = \frac{\varkappa B_{\text{m}}^{\nu}}{\pi B_{1\text{m}} H_{1\text{m}}} = \frac{1}{\pi}\,\mu\,\frac{\varkappa}{\left(2S_{\frac{p}{n}+1}\right)^{\nu}}\,B_{1\text{m}}^{\nu-2} \tag{31}$$

This expression is simplified for $\nu = 2$ or $p = n$.

6. Experimental verifications

In Fig. 17 are represented the curves $\frac{\mu_{\text{st}}(H_{\text{m}})}{\mu_0}$ and $\frac{\mu(H_{\text{m}})}{\mu_0}$ obtained for a material in the case when $H = H_{\text{m}} \sin \omega t$. $B(t)$ is obtained point by point from the hysteresis cycle, obtaining $B_{1\text{m}}$ from the harmonical analysis. In the case of the strong magnetic fields, on the same figure, the points obtained with the relations proposed in this work are represented with small crosses.

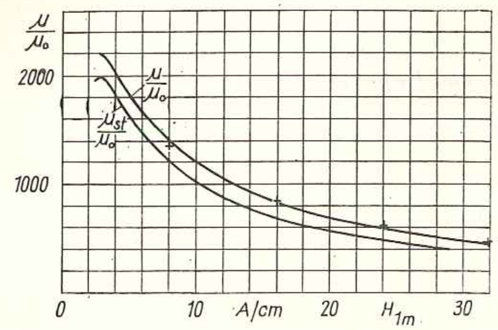

Fig. 17.

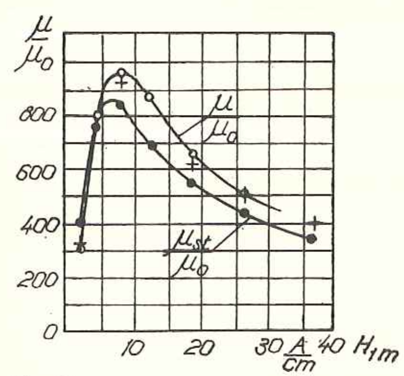

Fig. 18.

In the Fig. 18 are represented the same curves for an another material. The points obtained with the relations proposed in this work are also marked with small crosses; in case c the ratio $\frac{\mu_{\text{i}}}{\mu_{\text{M}}}$ is taken approximately equal to $0{\cdot}25$.

From these figures results that the calculated values, according to the indications given in this work, are in good agreement with the experimental values.

7. Conclusions

In this work is given a quantitative study of the $H(t)$ and $B(t)$ shape curves influence concerning the equivalent complex magnetic permeability of the ferromagnetic materials. The ratio between the absolute value of the equivalent complex permeability and the static permeability obtained from the normal magnetisation curve for the peak value, for the peak value of the first harmonic and for the effective value of the magnetic field respectively are determined. The variation of these ratio is studied for the normal magnetisation curve using the expressions: a) $B = KH^{1/n}$; b) $B = B_0\left(1 - \frac{H_0}{H}\right)$; c) $B = aH - b\sin cH$, in which K, n, B_0, H_0, a, b, c are constants. By means of the given curves, with a given p, not being necessary a harmonical analysis of the $B(t)$ curve, μ_{st} being easily obtained, the equivalent permeability may be readily determined. The presented experimental results are in satisfactory agreement with those calculated.

Appendix: Calculation of F_1 and F_2 integrals

For the numerical approximation of the integrals

$$F_1(p, x_0) = \frac{4}{\pi}\int_{x_0}^{\frac{\pi}{2}} \sin^{1-p} x \, dx,$$

$$F_2(p, A) = \frac{4}{\pi}\int_{0}^{\frac{\pi}{2}} \sin(A \sin^p x) \sin x \, dx,$$

in which

$$x_0 = 5°, 10°, \ldots, 85°,$$

$$p = 0;\ 0{\cdot}25;\ 0{\cdot}50;\ 0{\cdot}75;\ 1;\ 2;\ \ldots;\ 10;\ 12;\ 14;\ \ldots;\ 20;\ 25;\ 50,$$

$$A = 0{\cdot}5;\ 1{\cdot}0;\ 1{\cdot}5;\ \ldots;\ 7{\cdot}5;\ 8{\cdot}0,$$

was used the Gauss' quadratic formula

$$\int_{-1}^{+1} f(x)\, dx = \sum_{k=1}^{n} A_k^{(n)} f(x_k^{(n)}) + R(f),$$

where the nods $x_k^{(n)}$ $(k = 1, 2, \ldots, n)$ are the Legendre's polynomial roots of n degree

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n(x^2 - 1)^n}{dx^n},$$

and the coefficients $A_k^{(n)}$ $(k = 1, 2, \ldots, n)$ are determined by the relation

$$A_k^{(n)} = \frac{2}{[1 - (x_k^{(n)})^2]\,[P_n'(x_k^{(n)})]^2},$$

the rest being

$$R(f) = \frac{2^{2n+1}}{(2n+1)(2n)!}\left[\frac{(n!)^2}{(2n)!}\right]^2 f^{(2n)}(\xi), \quad \xi \in [-1, +1];\ f \in C^{2n}$$

In our concrete case $n = 16$, the nods $x_k^{(16)}$ and the coefficients $A_k^{(16)}$ $(k = 1, 2, \ldots, 16)$ are found in Krilov's book [9].

To approximate the function $f(x) = \sin\frac{\pi}{2}x$ $(|x| \leqq 1)$ was used the polynom

$$\sin\frac{\pi}{2}x = c_1x + c_3x^3 + \cdots + c_9x^9,$$

where

$$c_1 = 0{\cdot}78539815923 \,.\, 2^1,$$

$$c_3 = -0{\cdot}64596371106 \,.\, 2^0,$$

$$c_5 = 0{\cdot}6375174344 \,.\, 2^{-3},$$

$$c_7 = -0{\cdot}59824199296 \,.\, 2^{-7},$$

$$c_9 = 0{\cdot}62047924224 \,.\, 2^{-12},$$

of which error does not overpass $0{\cdot}5 \,.\, 10^{-8}$.

The calculations were made at the Computation Institute of the R.S.R. Academy, Cluj city Branch, using the DACICC-1 computer.

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