

# Claude's Cycles

## Human-AI Collaboration in Mathematical Discovery

Donald Knuth & Claude Opus 4.6 — February/March 2026

**"Shock! Shock!** I learned yesterday that an open problem I'd been working on for several weeks had just been solved by Claude Opus 4.6." — *Don Knuth*

## The Conjecture

Consider a directed graph with  $m^3$  vertices labeled  $(i, j, k)$  where  $0 \leq i, j, k < m$ .

From each vertex, 3 arcs point to its neighbors:

$i^+jk, ij^+k, ijk^+$

(where  $x^+ = (x+1) \bmod m$ )

**Goal:** Decompose all arcs into exactly **3 Hamiltonian cycles of length  $m^3$**  — for all  $m > 2$ .

## The Scale of Difficulty

**Search space:**  $3^{(m^3)}$  possible choices

Brute force is computationally impossible for any meaningful  $m$ .

## Prior Progress

- ✓ **Knuth** solved  $m = 3$
- ✓ **Stappers** found solutions for  $m = 4 \dots 16$  empirically
- ✗ No general construction — problem open for years

# 31 Explorations: Claude as Research Collaborator

1–3

## Linear & Quadratic Tests

Reformulated as group theory. Tried linear  $g=(ai+bj+ck) \bmod 3$  — none worked.

4–5

## Gray Code / Serpentine

Discovered the 3D serpentine (= modular  $m$ -ary Gray code). Residual graph proved rigid.

6–14

## Hyperplane Strategies

Targeted the hyperplane  $i+j+k \equiv -1$ . Many approaches, no breakthrough.

15–20

## Fiber Decomposition + SA

Layered digraph by  $(i+j+k) \bmod m$ . Simulated annealing found cases  $m=3$ ,  $m=4$ .

21–25

## Serpentine + Fibers

Realized odd- $m$  structure with uniform fiber 0. SA at scale — still no general rule.

27–29

## Near Miss: Coordinate Rot.

3D serpentine + rotation almost worked.  $3(m-1)$  conflicts on one hyperplane. Dead end.

30

## Key Insight from SA Output

Revisited exploration 20: choice depends on one coordinate per fiber ( $j$  for  $s=0$ ;  $i$  otherwise).

31 ✨

## The Construction!

Concrete Python program: valid decompositions for  $m=3,5,7,9,11$ . Tested up to  $m=101$ .

# The Construction & Knuth's Proof

## Claude's C Construction (simplified)

```
s = (i+j+k) % m;  
if (s == 0)  
    d = (j == m-1) ? "012" : "210";  
else if (s == m-1)  
    d = (i > 0) ? "120" : "210";  
else  
    d = (i == m-1) ? "201" : "102";
```

**Tested for ALL ODD m from 3 to 101 — perfect decompositions every time**

*"All three cycles are Hamiltonian, all arcs are used, perfect decomposition!" — Claude*

## Knuth's Proof Strategy

1.  $s = (i+j+k) \bmod m$

"Fiber" index governs routing

2.  $i$  changes only when  $s=0$ ,  $j=m-1$

$m^2$  vertices per  $i$ -block are consecutive

3.  $k$  steps by  $2 \bmod m$  at  $s=0$

Requires  $m$  odd — hits all residues

4. Repeats for all  $i$ -blocks

All  $m^3$  vertices covered exactly once

# Beyond Claude's Solution: The Landscape of 760

11,502

**Hamiltonian cycles for  $m=3$**

Total base-case solutions

996

**Generalizable to all odd  $m$**

Cycles that scale universally

760

**Claude-like decompositions**

Valid for all odd  $m > 1$

## Knuth's Theorem ("Claude-like" decompositions)

A Claude-like decomposition is valid for all odd  $m > 1$  if and only if each of the three sequences it defines for  $m = 3$  is generalizable. Exactly 760 such decompositions exist. Claude's solution was one of them — found without knowing the others existed.

# Postscript — March 3, 2026: The Story Continues

## 🕒 Tackling the Even Case — Mar 3, 2026

After the odd-case triumph, Stappers put Claude back to work on even  $m$  for  $\sim 4$  more hours.

Progress was partial: Claude built a fiber-based construction but spent most energy on search optimizations rather than finding a closed-form rule.

A turning point came when Stappers suggested Google's ORTools CP-SAT (AddCircuit constraint) — suddenly solutions for individual even  $m$  could be found within seconds.

A general construction for even  $m$  still remained elusive at this stage.

## ✓ Lean Formalization — Mar 4, 2026

Kevin Buzzard notified Knuth that Kim Morrison of the Lean proof assistant community had formally verified Knuth's proof of Claude's construction.

Morrison posted the machine-checked verification on GitHub on March 4, 2026 — just days after the paper appeared.

Knuth's reaction: "That's good to know, because I've been getting more errorprone lately."

The verification makes the odd-case result one of the most rigorously confirmed new mathematical results of 2026.

# Post-Postscript — Mar 4–6, 2026: An Even Simpler Construction Emerges

**Maximilian Reitbauer**, aka "Exocija", found an even simpler valid decomposition for odd  $m$  — **probably the computationally simplest possible**.

**The key features of Reitbauer's solution:**

**Only uses  $s$  and  $j$  — not  $i$**

One fewer variable; replaces lines 8–10 of the C program with 3 simpler cases

**Identity permutation "012" at almost every step**

Maximally uniform — nearly no special cases needed

**It was solution #369 of the 760**

Knuth: "I would have found this myself if I'd taken time to look carefully at all 760"

*His proof [7] was found by passing text back and forth between GPT 5.4 (Extended Thinking) and Claude 4.6 Sonnet — a multi-AI collaborative discovery.*



## BREAKING NEWS



# The Even Case: SOLVED

Ho Boon Suan · GPT-5.4 Pro · Keston Aquino-Michaels

**Ho Boon Suan (Singapore) — Mar 4, 2026**

Used GPT-5.3-codex to find a construction for all even  $m \geq 8$ . Tested up to  $m = 2000$  (8 billion vertices!).

**GPT-5.4 Pro (OpenAI) — Mar 4–5, 2026**

Produced a beautifully formatted, apparently flawless 14-page proof paper [8] — entirely the machine's doing, no human editing.

**Keston Aquino-Michaels — Mar 6, 2026**

Brought the story to a fitting conclusion [10,12]: elegant decompositions for both odd and even  $m$ , plus a 20-page analysis of multi-agent collaboration.

# A New Paradigm: Human–AI Mathematical Research

 Early Feb 2026

## Knuth poses problem

*Human*

Open conjecture from TAOCP, stuck for weeks

 Feb 27, 2026

## Claude explores

*AI (Claude Opus 4.6)*

31 iterations in ~1 hour; discovers odd- $m$  construction

 Feb 27–28, 2026

## Stappers coaches + Knuth proves

*Human*

Stappers guides session; Knuth writes rigorous proof

 Feb 28, 2026

## Paper published

*Knuth (Human)*

"Claude's Cycles" released; even case still open

 Mar 4–6, 2026

## Morrison (Lean) + Reitbauer

*AI+Community*

Lean verification on GitHub; Reitbauer finds simpler construction via GPT 5.4 + Claude 4.6 Sonnet

 Mar 4–6, 2026

## Even case closed

*Ho Boon Suan + GPT-5.4 Pro*

GPT-5.3-codex finds even construction; GPT-5.4 Pro writes 14-page proof. All  $m > 2$  solved.

# What "Claude's Cycles" Means for Mathematics

“ *I think Claude Shannon's spirit is probably proud to know that his name is now being associated with such advances in automatic deduction and creative problem solving. **Hats off to Claude!*** ”

— Donald E. Knuth, *Claude's Cycles* (2026)

## AI as creative collaborator

Claude didn't just compute — it reformulated, strategized, and discovered structure.

## Speed of iteration

Odd case (Claude): 1 hour. Even case (GPT): days. Full formal proof: days.

## Humans remain essential

Proof, verification, coaching, and insight: the human role evolved but didn't disappear.

## A new research paradigm

Humans pose → AI explores → humans prove → AI verifies. Fully documented in Knuth's note.

# Over and Out — Knuth's Closing Words

“ Dear reader, I hope you have enjoyed reading this story at least half as much as I've enjoyed writing it. We are living in very interesting times indeed.

I absolutely must get back to writing [3], which will soon contain further stories of a completely different kind, stories that I'm much more qualified to write than stories about LLMs. **May the force be with you.** ”

— Donald E. Knuth, *Claude's Cycles* (revised 06 March 2026)

## The problem

Open for years: decompose a 3D Cayley digraph into 3 Hamiltonian cycles for all  $m > 2$

## The completion

GPT-5.4 Pro proved the even case; Reitbauer found the simplest known odd construction

## The breakthrough

Claude Opus 4.6 cracked the odd case in 1 hour, 31 explorations, then Knuth proved it

## The legacy

Aquino-Michaels: a 20-page study on multi-agent AI collaboration as a new research paradigm