Claude's Cycles

Human-AI Collaboration in Mathematical Discovery

Donald Knuth & Claude Opus 4.6 — February/March 2026

"Shock! Shock! I learned yesterday that an open problem I'd been working on for several weeks had just been solved by Claude Opus 4.6." *— Don Knuth*

The Conjecture

Consider a directed graph with **m^3 vertices** labeled (i, j, k) where $0 \le i, j, k < m$.

From each vertex, 3 arcs point to its neighbors:

i^+jk, ij^+k, ijk^+

(where $x^+ = (x+1) \bmod m$)

Goal: Decompose all arcs into exactly **3 Hamiltonian cycles of length m^3** — for all $m > 2$.

The Scale of Difficulty

Search space: **$3^{(m^3)}$** possible choices

Brute force is computationally impossible for any meaningful m.

Prior Progress

- ✓ **Knuth** solved m = 3
- ✓ **Stappers** found solutions for m = 4…16 empirically
- ✗ No general construction — problem open for years

31 Explorations: Claude as Research Collaborator

1–3

Linear & Quadratic Tests

Reformulated as group theory. Tried linear g=(ai+bj+ck) mod 3 — none worked.

4–5

Gray Code / Serpentine

Discovered the 3D serpentine (= modular m-ary Gray code). Residual graph proved rigid.

6–14

Hyperplane Strategies

Targeted the hyperplane i+j+k ≡ −1. Many approaches, no breakthrough.

15–20

Fiber Decomposition + SA

Layered digraph by (i+j+k) mod m. Simulated annealing found cases m=3, m=4.

21–25

Serpentine + Fibers

Realized odd-m structure with uniform fiber 0. SA at scale — still no general rule.

27–29

Near Miss: Coordinate Rot.

3D serpentine + rotation almost worked. 3(m−1) conflicts on one hyperplane. Dead end.

30

Key Insight from SA Output

Revisited exploration 20: choice depends on one coordinate per fiber (j for s=0; i otherwise).

31 ✦

The Construction!

Concrete Python program: valid decompositions for m=3,5,7,9,11. Tested up to m=101.

The Construction & Knuth's Proof

Claude's C Construction (simplified)

```
s = (i+j+k) % m;
if (s == 0)
  d = (j == m-1) ? "012" : "210";
else if (s == m-1)
  d = (i > 0) ? "120" : "210";
else
  d = (i == m-1) ? "201" : "102";
```

Tested for ALL ODD m from 3 to 101 — perfect decompositions every time

"All three cycles are Hamiltonian, all arcs are used, perfect decomposition!" — Claude

Knuth's Proof Strategy

1. s = (i+j+k) mod m

"Fiber" index governs routing

2. i changes only when s=0, j=m-1

m^2 vertices per i-block are consecutive

3. k steps by 2 mod m at s=0

Requires m odd — hits all residues

4. Repeats for all i-blocks

All m^3 vertices covered exactly once

Beyond Claude's Solution: The Landscape of 760

11,502

Hamiltonian cycles for m=3

Total base-case solutions

996

Generalizable to all odd m

Cycles that scale universally

760

Claude-like decompositions

Valid for all odd m > 1

Knuth's Theorem ("Claude-like" decompositions)

A Claude-like decomposition is valid for all odd m > 1 if and only if each of the three sequences it defines for m = 3 is generalizable. Exactly 760 such decompositions exist. Claude's solution was one of them — found without knowing the others existed.

Postscript — March 3, 2026: The Story Continues

⟳ Tackling the Even Case — Mar 3, 2026

After the odd-case triumph, Stappers put Claude back to work on even m for ~4 more hours.

Progress was partial: Claude built a fiber-based construction but spent most energy on search optimizations rather than finding a closed-form rule.

A turning point came when Stappers suggested Google's ORTools CP-SAT (AddCircuit constraint) — suddenly solutions for individual even m could be found within seconds.

A general construction for even m still remained elusive at this stage.

✓ Lean Formalization — Mar 4, 2026

Kevin Buzzard notified Knuth that Kim Morrison of the Lean proof assistant community had formally verified Knuth's proof of Claude's construction.

Morrison posted the machine-checked verification on GitHub on March 4, 2026 — just days after the paper appeared.

Knuth's reaction: "That's good to know, because I've been getting more errorprone lately."

The verification makes the odd-case result one of the most rigorously confirmed new mathematical results of 2026.

Post-Postscript — Mar 4–6, 2026: An Even Simpler Construction Emerges

Exocija, an anonymous contributor, found an even simpler valid decomposition for odd m — **probably the computationally simplest possible**.

The key features of Exocija's solution:

Only uses s and j — not i

One fewer variable; replaces lines 8–10 of the C program with 3 simpler cases

Identity permutation "012" at almost every step

Maximally uniform — nearly no special cases needed

It was solution #369 of the 760

Knuth: "I would have found this myself if I'd taken time to look carefully at all 760"

His proof [7] was found by passing text back and forth between GPT 5.4 (Extended Thinking) and Claude 4.6 Sonnet — a multi-AI collaborative discovery.

BREAKING NEWS

The Even Case: SOLVED

Ho Boon Suan · GPT-5.4 Pro · Keston Aquino-Michaels

Ho Boon Suan (Singapore) — Mar 4, 2026

Used GPT-5.3-codex to find a construction for all even $m \geq 8$. Tested up to $m = 2000$ (8 billion vertices!).

GPT-5.4 Pro (OpenAI) — Mar 4–5, 2026

Produced a beautifully formatted, apparently flawless 14-page proof paper [8] — entirely the machine's doing, no human editing.

Keston Aquino-Michaels — Mar 6, 2026

Brought the story to a fitting conclusion [10,12]: elegant decompositions for both odd and even m, plus a 20-page analysis of multi-agent collaboration.

A New Paradigm: Human–AI Mathematical Research

Early Feb 2026

Knuth poses problem

Human

Open conjecture from TAOCP, stuck for weeks

Feb 27, 2026

Claude explores

AI (Claude Opus 4.6)

31 iterations in ~1 hour; discovers odd-m construction

Feb 27–28, 2026

Stappers coaches + Knuth proves

Human

Stappers guides session; Knuth writes rigorous proof

Feb 28, 2026

Paper published

Knuth (Human)

"Claude's Cycles" released; even case still open

Mar 4–6, 2026

Morrison (Lean) + Exocija

AI+Community

Lean verification on GitHub; Exocija finds simpler construction via GPT 5.4 + Claude 4.6 Sonnet

Mar 4–6, 2026

Even case closed

Ho Boon Suan + GPT-5.4 Pro

GPT-5.3-codex finds even construction; GPT-5.4 Pro writes 14-page proof. All $m > 2$ solved.

What "Claude's Cycles" Means for Mathematics

> *I think Claude Shannon's spirit is probably proud to know that his name is now being associated with such advances in automatic deduction and creative problem solving.* **Hats off to Claude!**

— Donald E. Knuth, Claude's Cycles (2026)

AI as creative collaborator

Claude didn't just compute — it reformulated, strategized, and discovered structure.

Speed of iteration

Odd case (Claude): 1 hour. Even case (GPT): days. Full formal proof: days.

Humans remain essential

Proof, verification, coaching, and insight: the human role evolved but didn't disappear.

A new research paradigm

Humans pose → AI explores → humans prove → AI verifies. Fully documented in Knuth's note.

Over and Out — Knuth's Closing Words

> “ *Dear reader, I hope you have enjoyed reading this story at least half as much as I've enjoyed writing it. We are living in very interesting times indeed.*
>
> *I absolutely must get back to writing [3], which will soon contain further stories of a completely different kind, stories that I'm much more qualified to write than stories about LLMs.* **May the force be with you.** ”

— Donald E. Knuth, Claude's Cycles (revised 06 March 2026)

The problem

Open for years: decompose a 3D Cayley digraph into 3 Hamiltonian cycles for all $m > 2$

The breakthrough

Claude Opus 4.6 cracked the odd case in 1 hour, 31 explorations, then Knuth proved it

The completion

GPT-5.4 Pro proved the even case; Exocija found the simplest known odd construction

The legacy

Aquino-Michaels: a 20-page study on multi-agent AI collaboration as a new research paradigm