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ROMANIAN ASTRONOMICAL JOURNAL

Vol. 6, No. 1, 1996

CONTENTS

G. D. CHIŞ, A. POP, V. TURCU, Primary Light Minima of X Trianguli between 1921–1995 . 1

M. S. GHIZARU, M. GOOSSENS, Numerical Model for the Study of Wave-Mean Flow Interactions in Solar Magnetohydrodynamics 21

M. BÎRLAN, CCD Photometry on the Asteroid 2419 Moldavia 31

E. GREBENICOV, On the Generators of the New Perturbation Methods 35

G. BOZIS, MIRA-CRISTIANA ANISIU, Family Boundary Curves in Rotating Systems 43

Yu. A. RYABOV, Analytic-Numerical Solutions of the Restricted Three-Body Problem 53

B. ELMABSOUT, Nonlinear Instability of some Relative Equilibrium Configurations in the $(n + 1)$-Body Problem 61

M. BĂRBOSU, La métrique de Maupertuis dans l'espace des configurations d'un système dynamique conservatif 73

CRISTINA STOICA, V. MIOC, A Qualitative Study of the Two-Body Problem in Maneff-Type Fields 81

V. MIOC, Orbits around Stars of Linearly Increasing Magnitude 89

O. VĂDUVESCU, M. BÎRLAN, Software Package for Preparing and Processing of an Astronomical Observation 97

MISCELLANEA

NATO Advanced Research Workshop "Theoretical and Observational Problems Related to Solar Eclipses" (Magdalena Stavinschi, Georgeta Mariş) 101

The Cluj-Timişoara Astronomical Observatory (1940–1945) (Eugenia Radu) 102

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FAMILY BOUNDARY CURVES IN ROTATING SYSTEMS

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Abstract. In the framework of the two-dimensional inverse problem of Dynamics with autonomous forces, we discuss the creation of the family boundary curves *(FBC)* with the aid of Dainelli's formulae. We then extend the notion of *FBC* to the case of rotating systems and we prove that these curves are loci either of points of equilibrium for the corresponding forces or points where the given family of orbits is "exterminated".

Key words: inverse problem, family boundary curves.

1. INTRODUCTION

Dainelli's formulae, as reported by Whittaker (1944, p. 96), offer the components X, Y depending on (x, y) of an autonomous force field which, acting on a material point P of unit mass moving in the inertial frame Oxy, can produce a preassigned monoparametric family of planar curves with equation

$$f(x, y) = c. \tag{1}$$

Let the function f be defined on the open set $D_f \subseteq R^2$ of the Cartesian system Oxy and $f \in C^2(D_f, I)$, the parameter $c \in I$ taking real values from an interval $I \subseteq R$.

It is known that in a specific force field $\{X, Y\}$ producing (1) neither all curves (1) in D_f for all values $c \in I$ nor the entire part of a curve for a certain $c = c_0 \in I$ are traced by the material point P. In fact, real orbits become only those curves or parts of curves (1) which lie in an allowed region $D_a \subseteq D_f$ defined by the inequality

$$\frac{f_x X + f_y Y}{W} \geq 0 \tag{2}$$

where the non-zero expression W is given by

$$W = 2f_{xy} f_x f_y - f_{xx} f_y^2 - f_{yy} f_x^2. \tag{3}$$

In general, inequality (2) defines a *genuine two-dimensional subregion* D_a of D_f and this $D_a \subset D_f$ is limited by the so-called *family boundary curves* (*FBC*) (Bozis 1994). The trivial case of having $D_a \neq \emptyset$ *one-dimensional* presents no physical interest and is not considered but several other alternatives with two-dimensional D_a may appear. It may be, for instance, that (2) is satisfied everywhere in D_f or nowhere in D_f strictly as an inequality. It may also be that (2) is satisfied everywhere in D_f and, at the same time, as an equality on one or more curves (or on certain isolated points) of the *xy* plane. In this latter case $D_a = D_f$ and these curves do not separate D_f in allowable and nonallowable regions, therefore they do not define *boundaries* for the members of (1). For the needs of the present study we propose to call them *Dainelli's curves (or points).*

On a *FBC*, as well as on a Dainelli's curve or point, the velocity of P, as it moves on an orbit (1), vanishes (in finite or infinite time). Yet *FBC* do not coincide with the well-known *zero velocity curves* unless the force field is conservative and all motions on all members of (1) are isoenergetic.

In the presence of a *FBC*, motion takes place in the allowed region D_a either (i) on a non-crossing the *FBC* (closed or open) curve (1) or (ii) librating between two points lying on the *FBC* or (iii) asymptotically tending to a certain point of the *FBC* or (iv) escaping to infinity after being reflected on the *FBC*.

In cases (i) and (ii) the motion, if not escaping, is periodic (Lissajoux motions with irrational frequency ratio cannot be described by equation (1)). In all cases (i)–(iv), at any point of D_a not lying on a *FBC*, both directions of the velocity vector are allowed, i.e. the curve (1), as a geometrical entity, may be traced back and forth. Besides, as implied by inequality (2), the same family (1) can be traced back and forth in the region $D_f - D_a$ including its boundary under the action of the force field $\{-X, -Y\}$.

The notion of the *FBC*, as given above for the planar inverse problem formulated for a family (1) in an inertial frame and for an autonomous force field, was introduced without reference to Dainelli's (1880) formulae.

In section 2 of the present paper we study *FBC* in the light of these formulae and we show that, for inertial systems, inequality (2) is an immediate consequence of the fact that the arbitrary function *g* appearing in Dainelli's formulae is real.

In section 3 we show that, as regards the various types and properties of motion mentioned above, the picture changes if the frame is uniformly rotating. In fact, in this case: (i) at each point of D_a, there exists a prescribed orientation for motion on a member of (1) to take place in a certain force field. Consequently, librational motion can no longer appear. (ii) The field $\{-X, -Y\}$ is no longer compatible with members (or parts of members) of (1) lying in $D_f - D_a$. Motion is allowed on such pieces of (1) in $D_f - D_a$ either under the same force $\{X, Y\}$ *but in the opposite sense* (on any specific curve (1)) or in the same sense (i.e. as a continuation of the motion in D_a) *but under another force field.*

As regards the role of the *FBC* and Dainelli's curves in a rotating frame, two possibilities exist: (i) The moving point P reaches them asymptotically as $t \to \infty$ or as $t \to -\infty$, a situation also met in inertial frames. (ii) In contrast to what happens in inertial frames, after hitting them in finite time (if ever), the orbit with equation (1) is "exterminated" and the point P continues moving on an orbit not belonging to the family (1).

2. *FBC* DERIVED FROM DAINELLI'S FORMULAE IN INERTIAL SYSTEMS

In an inertial frame, Dainelli's formulae for the force components

$$X = X(x, y), \quad Y = Y(x, y) \tag{4}$$

which can produce, for adequate initial conditions, a family of orbits with equation (1) are (Whittaker, 1944, p. 96)

$$X(x,y) = g^2(f_y f_{xy} - f_x f_{yy}) + \frac{1}{2} f_y \left[f_y (g^2)_x - f_x (g^2)_y \right]$$

$$Y(x,y) = g^2(f_x f_{xy} - f_y f_{xx}) - \frac{1}{2} f_x \left[f_y (g^2)_x - f_x (g^2)_y \right],$$

where $g{:}D_f \to R$ is an arbitrary function of class C^1. We can obtain force components which are defined only on subsets of D_f, bounded by the so-called *FBC*. To this aim, let us consider: $g{:}\overline{D}_g \to R$, where D_g is an open set, g being continuous on $\overline{D}_g$ and null on its boundary, and continuously differentiable on D_g. We denote this by $g \in C_0(\overline{D}_g, R) \cap C^1(D_g)$. Obviously, $g^2 \in C_0(\overline{D}_g, R_+) \cap C^1(D_g)$ and the force field is defined in $D = D_f \cap D_g$.

Formulae (5) are easily obtained by differentiating with respect to time t the velocity components

$$\dot{x} = g(x, y) f_y, \quad \dot{y} = -g(x, y) f_x \tag{6}$$

of the moving material point P with unit mass along any orbit of the family (1).

If the pair of functions (f, g) leads to a certain force field (5), then so does the pair $(f, -g)$, i.e., in this case, both direct and retrograde motion on the curves (1) can be effected by the same force field.

Let us consider the locus L of points of the plane, where $g(x, y) = 0$, i.e. $L = \{(x, y) \in \overline{D}_g \mid g(x, y) = 0\}$. According to equations (6), the velocity of the point P moving on an orbit (1), as it reaches the curve $g(x, y) = 0$, becomes zero. So the locus L includes both the *FBC* and the Dainelli's curves and points which possibly exist for a certain pair of family (1) and force field (5).

Points of L may or may not lie in the domain D_g. Thus, for $g(x, y) = y - x^2$, $D_g = R^2$ and $L \subseteq D_g$ whereas for $g(x, y) = \sqrt{y - x^2}$, $D_g = \{(x, y) \in R^2 \mid y > x^2\}$ and $L \cap D_g = \varnothing$. For $g(x, y) = x^{1/3}$, $D_g = R^2 - \{(0, y) \mid y \in R\}$ and $L \cap D_g = \varnothing$.

The *FBC* is the *boundary of the region* $\overline{D}_g$. The other points in L will be Dainelli's curves or points. Multiplying by f_x and f_y the two equations (5) respectively and adding we obtain

$$\frac{f_x X + f_y Y}{W} = g^2. \tag{7}$$

Since $g^2 \geq 0$, the requirement that g is real is equivalent to inequality (2), so $D_a = \overline{D}_g$.

The fact that the function g^2 (rather than g) appears in formulae (5) makes librational motion possible. In its way back and forth, the moving point P changes the sign of g in accordance with formulae (6). The force components acting on P when P is on on the locus L are found from (5) equal to

$$X_g = \frac{1}{2} f_y\left[f_y(g^2)_x - f_x(g^2)_y\right], \quad Y_g = -\frac{1}{2} f_x\left[f_y(g^2)_x - f_x(g^2)_y\right]. \tag{8}$$

Consider, as an example, the family of circles

$$f(x, y) = x^2 + y^2 = c, \tag{9}$$

for which $D_f = R^2$ and $I = R_+$.

Dealing with (9) and having the possibility of selecting the arbitrary function g, we can *program* various types of circular motion. Thus:

(a) For $g(x, y) = \sqrt{x^2 + 1}$, $D_g = R^2$ and no *FBC* exists. From (5) we obtain the force

$$X = 4x(y^2 - x^2 - 1), \qquad Y = -4y(2x^2 + 1).$$

The motion (direct for $-g$ or retrograde for g) is periodic on full circles (Fig. 1*a*) everywhere in $D_a = D_f = R^2$.

(b) For $g(x, y) = \sqrt{y - x^2}$ one has $D_g = \{(x, y) \mid y > x^2\}$, so $D_a = \{(x, y) \mid y \geq x^2\}$ and P is librating on circular arcs inside the nonshaded region D_a (Fig. 1*b*) driven by the force

$$X = 4x(x^2 - y^2) - 6xy, \qquad Y = 2x^2(1 + 4y) - 4y^2.$$

On the *FBC* $y = x^2$ the force is $X_g = -2x^3(1 + 2x^2)$, $Y_g = 2x^2(1 + 2x^2)$.

(c) For $g(x, y) = x^{1/3}$, $D_g = R^2 - \{(0, y) \mid y \in R\}$ and $D_a = R^2$, so allowed is the entire plane, as in (a) above. The $x = 0$ axis is then a Dainelli's curve. The force

$$X = -4x^{\frac{5}{3}} + \frac{4}{3}x^{-\frac{1}{3}}y^2, \qquad Y = -\frac{16}{3}x^{\frac{2}{3}}y$$

is not defined for $x = 0$ and regularization is needed for the numerical integration of the motion. Since for $y \neq 0$ $\lim_{x\to\pm 0} X = \pm\infty$, $\lim_{x\to\pm 0} Y = 0$ one expects semicircular librational motion as shown in Fig. 1*c*.

(d) For $g(x, y) = y - x^2$, one has $D_g = R^2$, $D_a = D_f = R^2$. The force is

$$X = 4x(y - x^2)(x^2 - 2y^2 - 2y), \qquad Y = 4(y - x^2)(3x^2y + x^2 - y^2)$$

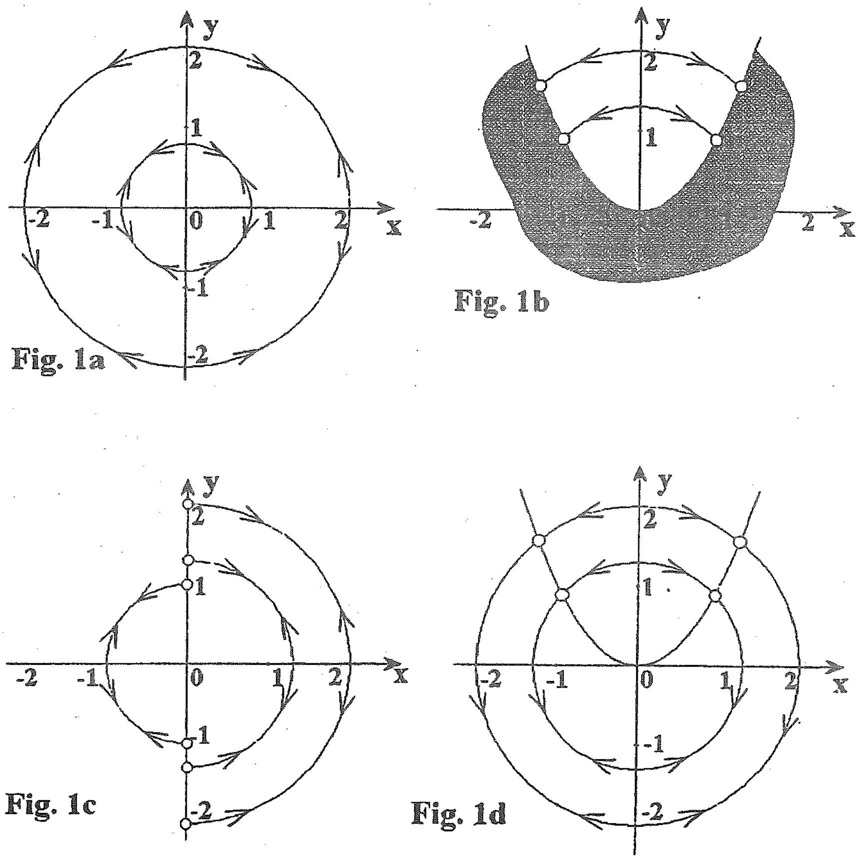

Fig. 1

and, as can be checked from (2), $y = x^2$ is a Dainelli's curve. In accordance with (8) one obtains $X_g = Y_g = 0$, so the curve $y = x^2$ is locus of points of equilibrium. Starting with the appropriate velocity (in the direct or retrograde sense) at any point of the circle of Fig. 1*d*, the point P tends asymptotically to the first crossing of the circle with the Dainelli's curve. It can be checked however that the above equilibrium points are unstable. As a result and as numerical integration reveals, P is moving on the entire circle. Its motion becomes very slow in the vicinity of the equilibrium points where the direction changes in this or the other sense "unexpectedly", as it happens with a pendulum at the higher unstable point of equilibrium.

3. *FBC* IN ROTATING FRAMES

In a system Oxy, rotating uniformly with constant angular velocity ω (usually taken equal to 1), the equations of motion are

$$\ddot{x} - 2\omega\dot{y} = \tilde{X}(x, y), \quad \ddot{y} + 2\omega\dot{x} = \tilde{Y}(x, y). \tag{10}$$

For a family of orbits (1) traced by a unit mass point P in this frame, the velocity components are again taken from formulae (6) and the force components acting on $P(x, y)$ are (Pal and Anisiu, 1995)

$$\tilde{X} = X + 2\omega g f_x, \quad \tilde{Y} = Y + 2\omega g f_y, \tag{11}$$

where X, Y are given by (5).

The equation which is analogous to (7) (and is obtained from (11) in the same manner as (7) from (5)) is

$$Wg^2 + 2\omega g(f_x^2 + f_y^2) - (f_x\tilde{X} + f_y\tilde{Y}) = 0 \tag{12}$$

and for g to be real it is necessary and sufficient that

$$\omega^2(f_x^2 + f_y^2)^2 + W(f_x\tilde{X} + f_y\tilde{Y}) \geq 0. \tag{13}$$

As expected, for $\omega = 0$ the above inequality is equivalent to (2). Of course, for a force field $\{\tilde{X}, \tilde{Y}\}$ derived from (11) by specifying a function $g \in C_0(\overline{D}_g) \cap C^1(D_g)$, inequality (13) is identically satisfied in D_g. But suppose that, in some other way, we happen to know a compatible triplet $(f, \tilde{X}, \tilde{Y})$ in a frame rotating with known angular velocity ω. Then inequality (13) (analogously to inequality (2) for inertial frames) can serve to provide the *FBC* or, equivalently, equation (12) can serve to offer the arbitrary function g used in formulae (11).

A first important consequence of formulae (11) is that, in the rotating frame, *librational motion is not possible.* Indeed, consider a regular point $P_0(x_0, y_0)$ of the family (1) (i.e. with $(f_x^2 + f_y^2)P_0 \neq 0$) not lying on the locus L (i.e. with $g(x_0, y_0) \neq 0$). If at P_0 both directions of the velocity were allowed, according to formulae (6), the arbitrary function g would have to change sign. Denoting e.g. by $X(g)$ the X component of the force corresponding to the arbitrary function g, we see from (11) that

$$\tilde{X}(g) \neq \tilde{X}(-g), \quad \tilde{Y}(g) \neq \tilde{Y}(-g), \tag{14}$$

meaning that, in this case, *the force field would have to change* as P travels on (1) and this, of course, cannot happen.

It is also seen from formulae (11) that the force components on P at the time when P is on the locus L(*FBC* or Dainelli's curve or point) are the same as in the case of inertial frames, i.e.

$$\tilde{X}_g = X_g, \quad \tilde{Y}_g = Y_g, \tag{15}$$

where X_g, Y_g are given by formulae (8).

If $\{\tilde{X}(g), \tilde{Y}(g)\}$ gives rise to the family (1) traced in a certain sense (direct or retrograde) in the neighbourhood of a point, then the field $\{\tilde{X}(-g), \tilde{Y}(-g)\}$, which, for the same ω, is entirely different, creates the same family traced in the opposite sense.

It follows from the above that, in the rotating frame, the locus L (where $g(x, y) = 0$) consists either of libration points (if $\tilde{X}_g = \tilde{Y}_g = 0$) or of points where the family (1) terminates abruptly (it $\tilde{X}_g^2 + \tilde{Y}_g^2 \neq 0$) in the sense that (1) is no longer the equation of the orbits.

As an example consider the family of parabolas

$$f(x, y) = y - x^2 = c \tag{16}$$

and take constantly $\omega = 1$.

(a) Working with $g(x, y) = \varepsilon(y - x)$, $D_g = R^2$, for $\varepsilon = \pm 1$, we find the two force fields

$$\tilde{X} = (y - x)((2 - 4\varepsilon)x - 1), \quad \tilde{Y} = 2(y - x)(2x^2 - 2x + y + \varepsilon) \tag{17}$$

It is seen that $D_a = D_g = D_f = R^2$, thus no FBC exists. The straight line $y = x$ stands for a Dainelli's curve and, as seen from (6) and (15), not only the velocity of P but also the force on P vanish there. Both, for $\varepsilon = 1$ and $\varepsilon = -1$, on either side of $y = x$, there exist parabolic arcs (or full parabolas (16)). If created by the field (17) with $\varepsilon = 1$ they are traced as shown by the arrows in Fig. $2a_1$.

The very same orbits are traced, at corresponding points in Fig. $2a_2$ of the xy rotating frame, in the opposite sense under the (entirely different) force field obtained from (17) for $\varepsilon = -1$.

The velocity at any point $P_0(x_0, y_0)$ of both Figures $2a_1$ and $2a_2$, found from (6) and (16), is

$$\dot{x}_0 = \varepsilon(y_0 - x_0), \qquad \dot{y}_0 = 2\varepsilon x_0(y_0 - x_0).$$

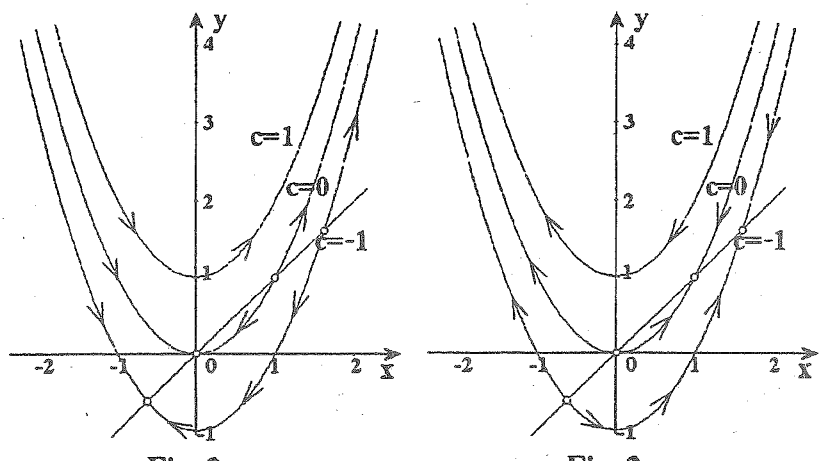

Fig. $2a_1$ Fig. $2a_2$

Fig. 2a

(b) If, for the same family (16), we select $g(x, y) = \varepsilon\sqrt{y}$, $\varepsilon = \pm 1$, we have $D_g = \{(x, y) \mid y > 0\}$ and $D_a = \{(x, y) \mid y \geq 0\}$. So, we make the x-axis a *FBC*. The corresponding force fields, for $\varepsilon = \pm 1$, are

$$\tilde{X} = x(1 - 4\varepsilon\sqrt{y}), \quad \tilde{Y} = 2(x^2 + y + \varepsilon\sqrt{y}) \tag{18}$$

and the force $\tilde{X}_g = x$, $\tilde{Y}_g = 2x^2$ on the *FBC* is not zero (except at the point (0, 0)). Members of (16) are drawn in Figs. $2b_1$, $2b_2$.

The *FBC* acts in this case as a "destructor" of the members of the family (16). Parabolic arcs which reach the x-axis with zero velocity are reflected to the positive y's, following orbits (shown in figures $2b_1$, $2b_2$ by dashed arrows) whose equation is no longer (16). Such is e.g. the orbit with initial conditions $x_0 = -2$, $y_0 = 3$, $\dot{x}_0 = \sqrt{3}$, $\dot{y}_0 = -4\sqrt{3}$ in Fig. $2b_1$ under the force (18) for $\varepsilon = 1$ and also the orbit of Fig. $2b_2$ passing through the point $P_0(2, 3)$ with velocity $(-\sqrt{3}, -4\sqrt{3})$ and traced in the force field (18) with $\varepsilon = -1$. The above "loss of its path" which the moving point undergoes is a feature met only in rotating frames. In fact it is a property of the function g (for the case at hand: $\varepsilon\sqrt{y}$) and not of the specific family. Indeed, instead of (16), consider any family (1) with the provision that the x – *axis* belongs to its D_f. According to formulae (15) it would be $\tilde{X}_g = -1/2 f_x f_y \neq 0$, $\tilde{Y}_g = 1/2 f_x^2 > 0$ at all points of the x – *axis* except at certain points where possibly $f(x, 0) = 0$.

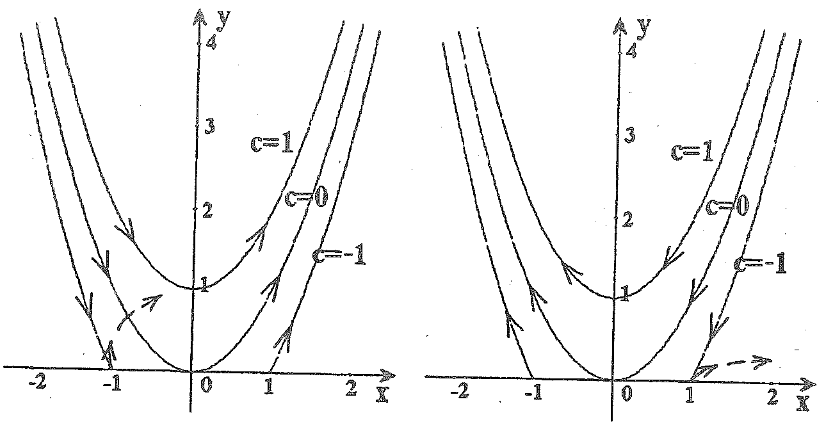

Fig. $2b_1$ **Fig. $2b_2$**

Fig. 2b

(c) Having to deal with a definite family (1) (as e.g. (16)) we can, in a way, *program motion* on it (Bozis, 1995). For the case at hand, for instance, we can make the x-axis a Dainelli's curve, instead of a *FBC* as in (b) above, so that motion is allowed in the entire rotating plane. Actually, we can further choose between making this Dainelli's curve a locus of points of equilibrium (as in (a) above) or a *family – destroying curve* (as in (b) above).

For the former choice we can select e.g. $g(x, y) = y$, leading to

$$\tilde{X} = -2xy, \qquad \tilde{Y} = 2y(2x^2 + y + 1)$$

with $\tilde{X}_g = \tilde{Y}_g = 0$ (Fig. $2c_1$). At any point (x_0, y_0) the velocity is $\dot{x}_0 = y_0$, $\dot{y}_0 = 2x_0y_0$ and the arrows are shown in Fig. $2c_1$.

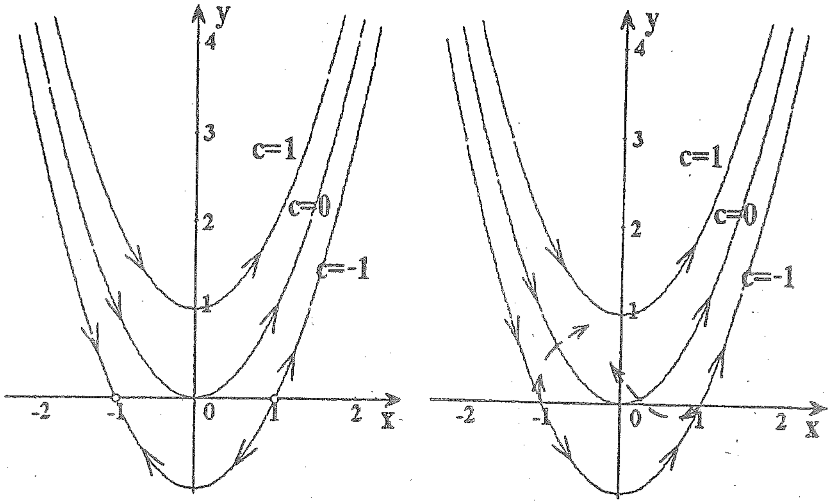

Fig. $2c_1$ Fig. $2c_2$

Fig. 2c

For the latter choice, take e.g. $g(x, y) = \sqrt{|y|}$. The force field defined on $D_g = R^2 - \{(x, 0) \mid x \in R\}$ is

$$\tilde{X} = x(\theta - 4\sqrt{|y|}), \qquad \tilde{Y} = 2(\theta x^2 + \theta y + \sqrt{|y|})$$

with $\theta = \pm 1$ for $y > 0$ and $y < 0$, correspondingly, the velocity at (x_0, y_0) is $\dot{x}_0 = \sqrt{|y_0|}$, $\dot{y}_0 = 2x_0\sqrt{|y_0|}$ and the arrows are shown in Fig. $2c_2$.

Comment: The transformation $\omega \to -\omega$, $g \to -g$ leaves the force field unaltered. So, as naturally expected, all the oriented orbits of Figs. 2 (drawn with $\omega = 1$) would be traced by the same pertinent force fields in the opposite sense if we had taken $\omega = -1$.

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