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**„BABEȘ — BOLYAI" UNIVERSITY
FACULTY OF MATHEMATICS AND PHYSICS
RESEARCH SEMINARS**

...of „Babeș-Bolyai" University in Cluj-Napoca.
Besides the contributions of the active staff of our Seminar,
also publish any papers of some ancient members, as well as
some external colleagues.
Preprint No.2, 1991
The issue is dedicated to the memory of the late professor
SEMINAR ON GEOMETRY
Professor F. Rado, who died the last autumn. Professor F. Rado was the
head of the Chair of Geometry at our Faculty. He worked hardily
and incessantly for the continuous development of our depart-
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ROMANIA**

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Concerning the other papers, they deal with some preferred topics as the geometric and algebraic structures, namely with problems on the hyperbolic geometry, differential geometry and topology of differentiable manifolds.

The Editor

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FOREWORD

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The Editor

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From the academic year 1949-1950 he was appointed as assistant professor at the newly founded Pedagogical Institute in Târgu-Mureş. In January 1950 he was appointed as lecturer at

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"BABEŞ-BOLYAI" UNIVERSITY

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THE LIFE AND WORK OF THE PROFESSOR FRANCISC RADÓ (1921-1990)

by

V.Groze, M.Țarină, A.Vasiu

One of the most important teacher and research scientist in the field of mathematics, in the second half of our century at the University in Cluj-Napoca was professor Francisc Radó.

He was born in Timișoara (Roumania), on May 21, 1921 in an intellectual family. His father, Arthur Radó, a railway engineer, disclosed for his son the beauties of mathematics. So, F.Radó finished his primary and secondary education in his natal town, proving a real talent for mathematics. In 1939 he succeeded to enter the Politehnical Institute in Bucharest, but he was forbidden to continue his studies for racial reasons. During the World War II he was incorporated in a labour camp.

After the war he was accepted as a student at the Faculty of Mathematics of the Romanian University in Cluj and got his degree in January 1946. He taught mathematics at the some jewish school in Timișoara, where once he was a school boy.

From the academic year 1949-1950 he was appointed as assistant professor at the newly founded Pedagogical Institute in Timișoara. In January 1950 he was nominated as lecturer at

the hungarian "Bolyai" University in Cluj, where in the next year he was promoted as professor. He will teach there most of the basical mathematical courses.

In 1959 the two Universities in Cluj were united in the actually "Babeş-Bolyai" University, and here F. Radó was promoted as full professor in 1969, being active until his retirement in 1985. In this period he was the chief (1973-76) of the chair of geometry, still continuing to teach at the "Babeş-Bolyai" University basic and other special or optional courses. In the some time professor F. Radó was working during 1951-1961 as a scientific researcher at the Academic Institute for Computation in Cluj. Much interested in practical applications of mathematics he had an essential contribution in performing economical contracts.

With his indefatigable attendance he honoured all the mathematical meetings organized by the Academy, Mathematical Science Society and by the Ministry of Education in the country. He also attended many scientific meetings abroad. Since 1968 he took an active part in scientific lectures, and symposia in Oberwolfach (Germany), La Mendola (Italy), San Antonio (U.S.A), Waterloo (Canada), etc. During 1969-70 he acted as visiting professor at the University of Waterloo. Many of his original researches were presented at various meetings, at the University of Bochum, Giessen, Frankfurt am Main, Pavia, Hamilton (Canada), Buffalo (USA), giving a great honour to the romanian mathematical school.

His great interest for professoral mathematics turned up to be a hobby since he was a schoolboy. His wife Martha Gidály, also a well-known professor of mathematics and his daughters Eva

and Ágy were the first collucators regarding school books and programmes.

Recently retired from his work still full of creation he becomes the victim of a long mercilles illness. The family was his last refuge, where he was visited by close friends and colleagues. He died on 28-th of November 1990 and was buried in the neological jewich cemetery in Cluj-Napoca.

The didactical work of professor Radó took up all his experience gathered during his activity with his pupils, later with his students about lessons and courses at the University. All who had known him, had the possibility to follow his very impresive and original lectures, his subtle spirit, the clearsi-ghted way of tackling the mathematical problems.

He taught a wide-range of disciplines as geometry, algebra, theory of numbers and theory of complex functions. His lessons always proved his distinctive professoral qualities. The transparency and accuracy of his speeches succeeded in attracting the students interest. Professor F. Radó proved to be an unrivalled master, both in research methods and professor speech.

In the last 40 years of professoral activity he obtained good results and managed to train a pleiad of students, most of them being active at different universities high-schools and research institutes in our country. All his achievement that makes him the most exquisite educator in the field of mathematics are reflected in his didactical publications and papers (see the list B).

During his activity he published with P. Szilágyi the course of Analytic Geometry (B.1) as well as a collection of problems (B.4). The book entitled "Lessons on Nomography" [B.2], written with the coauthor Lasca Bal, appeared in the "Editura Tehnică" Publishing Company, includes the lectures from 1965 held in Bucharest within the General Association of Engineers in Romania (AGIR). The course of Geometry [B.5] from 1972 sets forth an original outlook of the author upon the teaching methods of geometry for the first year of study, as well as a collection of problems [B.8] written with the members of his group together. The Course of Linear Programming [B.6] was reproduced on his lectures held for didactical staff of the faculty. The course of Geometry [B.7] written by the request of the Ministry of Education with contributor professor G. Galbură is a basic reference of the geometry. An outstanding hand book was published in 1981 by "Dacia" Publishing Company in Cluj-Napoca with contributor B. Orbán upon the modern viewpoints on the geometry [B.9]. Beside he had a remarkable contribution in drawing up methodical books for teachers [B.10, B.12], or publishing a mathematical lexicon (pocket-book) [B.11] edited in hungarian by "Kriterion" Editor in Bucharest.

The scientific activity of professor F. Radó included many mathematical domains concerning both fundamental and applicative problems. There is a natural link which explains the passing from a theme to another. His first researches referred to some problems which were intensively studied in the period between the two World Wars, by the mathematicians from Cluj,

i.e. Functional Equations. Paper [5] concerning the linear dependence of 3 continuous functions is related to T. Popoviciu's researches. In [11] he studied the functional equation of associativity and generalized bisymmetry, together with their particular cases. These problems lead him straightforward to the applications in the domain of nomography, geometrical webs and quasigroups [12, 13]. This time he begins the collaboration with foreign mathematicians like J. Aczel, M. Hosszú, G. Pickert. This Ph.D. Thesis held in 1959 at the University of Cluj, treated functional equations in connection with the nomography [11]. In fact, the methods used in the study of functional equations will play an important role in his algebraic and geometrical researches and their applications. This kind of considerations were enrolled also in the last years [43, 45], in connection both with the Perider's equation [66] and by some linear programming applications.

The theory of webs was a major subject of his research concerning both the regularity conditions [17] and special generalizations [16] or the associate algebraic structures [21]. In [28], F. Radó gave necessary and sufficient conditions for embedding of a semi-web into a regular-web and of a semi-group into a group.

Most of the latest papers are included in what F. Radó used to call Geometrical structures, a research domain related to those mentioned before. We shall present here some of his main contributions in the domain he was concerned with.

The Incidence structures were considered from the point of view of embedding them into a projective plane with

some closure conditions.

The Projective planes constitute a main studied object of professor F.Radó and of his collaborators. He studied non injective collineations defined on some sets of a Desarguesian projective plane, generalizing a Klingenberg's theorem referring to the extension of these collineations [35,36]. Instead of coordinates method, he adjusted Artin's method which can be applied in the study of geometry on rings also.

The Translation planes are involved in the congruences preserving the isomorphisms of the associated translation group [38] as well as in the problem of the extension of a collineation defined on some given subsets of the plane [40].

The Geometries on Rings represents an important mathematical domain in which remarkable results and researches had been performed. The works of J.Hjelmslew (1929) and D.Barbilian (1940-41) had been continued by W.Klingenberg (1954-56) and under the affine point of view by W.Benz, H.J.Arnold and W.Leissner. Starting from a geometrical characterization of an affine Barbilian plane over a $\mathbb{Z}$ -ring, given by W.Leissner (i.e. over a ring with unity having the property $ab=1 \Rightarrow ba=1$). F.Radó [49,53] gave an extension of this result in the case of planes over arbitrary rings, by a suitable modification of the axioms system which defines the geometrical structure, requiring only the existence of a minimal line which is incident with two neighbouring points. This represents a weaker form than the axiom given by H.J.Arnold.

The characterization of isometries and semi-isometries, in some metrical affine space is another important research domain of the last decades. Starting with the results of A.D.Alexandrov (1950) and F.S.Bachmann-D.A.Quarles (1953) concerning the Minkowski spaces other researches had been made by W.Benz (1977),

M.J.Borcher-C.C.Hegerfeldt (1972). J.A.Lester (1981), H.Schröder (1980). The isometries of the Euclidean and Minkowskian spaces over the real field are characterized by the invariance of the unit distances. More generally for an arbitrary field, F.Radó [51] proved that the previous condition implies only the semi linearity of the mapping. Some extensions of Borcher's-Hegerfeldt's result are obtained, concerning both the theorem conditions and the basic field. He extends a similar result of W.Benz for the case of fields K for which $\text{char } K \neq 2, 3$ or 5 , and for the case of Galois field $K = \text{GF}(5^m)$ respectively [56]. Using the existence condition of a regular simplex in a Galois field, F.Radó [62] extends the mentioned result at the case of metric vector spaces V over $\text{GF}(p^m)$, $p > 2$, which satisfy the condition $3 \leq \dim V = n < \infty$, $n \neq 0, -1, -2 \pmod{p}$. Similar results are obtained in other papers [55], [58], [59], [60] separately or in collaboration [57,63].

Associated algebraic structures. The problems concerning geometrical structures are naturally related to the considering associated algebraic structures. So, in a joint paper with J.Aczel, J.A.Baker, D.Z.Djocović, P.L.Kannapan, professor F.Radó gives the extension of some semigroups homomorphisms to groups homomorphisms and analogously he considers different problems connected with quasigroups [42] or with Abelian groups [44]. In collaboration with professor R.Miron [46], he also treats problems concerning the independence of the axioms ^{for} modules or vector spaces respectively. Regarding with the geometries on rings, he gives also some generalizations of valuations in vector spaces [44,48] and in skew-fields [50].

Another direction in which professor F.Radó has a great contribution, from the very beginning is the Numerical Analysis. In one of his first papers, written in Hungarian [8], F.Radó gives a method of approximative calculus for simple roots of an algebraic equation $P(x) = 0$ by successive divisions of the polynomial P ,

with $x-x_1$, where x_1 is an approximation of the root x_1 of P . He elaborated a method for determining the extremal values of a real valued function of one variable without using derivatives [18]. He also gave an algorithm for solving some mathematical programming problems [24]. In 1954, in a joint paper with G.Călugăreanu, P.Radó [2] gives an algorithm for numerical solving of a partial differential equation with boundary conditions arising by a heat transfer problem.

An important field in P.Radó's work is that of Operations Researches. In this chapter can be included P.Radó's first researches in the domain of nomography, wearing a theoretical character, related to the possibility of separation of the variables in different equations (The anamorphosis problem) [4,9]. In collaboration with L. Bal, E.Gergely and G.Ionescu he studies some problems on remarkable classes of nomograms (as romboidal nomograms, nomograms with transparency a.o) at the Institute for Computation [6]. Together with V.Groze, B.Orbán, A.Vasiu he studied problems on the optimisation of some alignment nomograms by projective transformations or by more general transformations [27, 29, 33] studying also the error evaluation [47].

In the same field of applied mathematics are included his researches concerning the Optimization Theory and Mathematical Programming with important practical applications ([15] or [22]). One of the last publications [67] refers to a location problems generalizing a classical problem of Fermat to the Euclidean space, which extends an algorithm given by W.Weiszfeld and establishes, in more general hypothesis the convergence and the optimality of this method. Others problems with applied character solved by P.Radó concern some algebraic structures related to the ordering problems [31].

Besides his books, lecture notes and scientific papers, P.Radó has redacted also some survey papers with methodical or informative character. Some of them [42,61] refer to the fundamentals of geometry, others [C2,C3] (from the list C) to the algebraic structures. His final lesson [50] at the festive course at the close of the academic year 1987-88 was an exceptional presentation of the great ideas developed in the history of the geometry.

So, by all his life and work, as well as by his human qualities, professor Francisc Radó remains a remarkable figure in the gallery of the professors of mathematics at the University of Cluj-Napoca.

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PROF. F. RADO'S PUBLICATIONS

A. Research papers

1. Observații asupra unui sistem liniar infinit (Remarks on an infinite linear system; romanian); Bull.st.sect.st.mat.fiz., 5(1953), 285-292.
2. (With G.Călugăreanu) Asupra unei probleme de propagare a căldurii (On a problem of heat propagation; romanian). Bul.st.sect.st.mat.fiz., 6(1954), 17-30.
3. (With L.Bal). Două teoreme referitoare la separarea variabilelor în nomografie. Comunic.Acad.R.P.R., 5(1955), 285-290 (Two theorems concerning the separation of variables in nomography).
4. (With L.Bal) Separarea variabilelor în nomografie (The separation of variables in nomography). Comunic.Acad.R.P.R., 5(1955), 303-305.
5. Condiții de dependență liniară pentru trei funcții continue (Linear dependence conditions for three continuous functions). Studii și cercet. Cluj, ser.I, 6(1955), 51-63.
6. (With E.Gergely, L.Bal and G.Ionescu) Despre nomogramele romboïdale. (On rhomboidal nomograms). Lucrările conf. de geometrie diferențială, Timișoara, 361-366 (1955).
7. Certaines propriétés des intégrales des équations différentielles linéaires. Lucrările celui de al IV-lea Congres al matematicienilor români, 1956, 162-163.
8. Az algebrai egyenletek numerikus megoldásáról (On the numerical solution of the algebraic equations; hungarian). Bul.Univ. "Babeș-Bolyai", ediția I; maghiara, 2(1957), 13-24.
9. Cea mai bună transformare proiectivă a scărilor la nomograme cu puncte aliniate (The best projective transformation of the scales of alignment nomograms). Studii și cercet. de matematică, Cluj, 8(1957), 161-168.
10. Despre contactele nomogramelor cu transparent (On the contacts of the nomograms with transparents). Studii și cercet. de matem. Cluj, 8(1957), 319-329.

11. Ecuații funcționale în legătură cu nomografia (Functional equations in connection with nomography). Studii și cercet.de matematica Cluj, 9(1958), 249-320.
12. Equations fonctionnelles caractérisant les nomogrammes avec trois échelles rectilignes. Mathematica, 1(24), fasc.1(1959), 143-166.
13. Sur quelques équations fonctionnelles avec plusieurs fonctions a deux variables, ibid., 1(24), fasc.2(1959), 321-339.
14. (With J.Acsél and G.Pickert) Nomogramme, Gewebe und Quasigruppen. Mathematica, 2(25), (1960), 5-24.
15. Despre problema programării liniare (On the problem of linear programming). Studii și cercet.de mat., Cluj, Fasc.anexa, 11(1960), 167-177.
16. (With E.Munteanu) Calculul sarcinilor celor mai economice la cup-toarele de topit fontă (The calculation of the most economic charges of the furnace to melt cast iron). Studii și cercet.de mat., Cluj, Fasc.anexa, 11(1960), 149-158.
17. Generalizarea țesuturilor spațiale pentru structuri algebrice (Generalization of spatial webs related to algebraic structures). Studia Univ."Babeș-Bolyai", 1960, ser.I, fasc.1, 41-55.
18. Eine Bedingung für die Regularität der Gewebe. Mathematica, 2(25), (1960), 325-334.
19. Le calcul approximatif des extrêmes des fonctions. Mathematica, 3(26), (1961), 171-177.
20. Caractérisation de l'ensemble des intégrales des équations différentielles linéaires homogènes à coefficients constantes d'ordre donné. Mathematica 4(27), (1962), 131-143.
21. Charakteristika spojnicových nomogramu rodu nula funkcni rovnicemi (The characterization of alignment nomograms of genus zero by functional equations) (czech language). Nomografické Metody, Praha, 1962, 86-91.
22. Nomograme, țesuturi și cuasigrupuri (Nomograms, webs and quasi-groups). Studii și cercet.de mat., fasc.anexa 13(1962), 305-319.
23. (With L.Németi) Ein Wartezeitproblem in der Produktion. Mathematica 5(28), (1963), 65-95.
24. Programare liniară cu condiții logice (Linear programming with logical conditions). Comunicările Acad.R.P.R., 13, nr.12(1963), 1039-1042.

25. Un algorithme pour résoudre certains problèmes de programmation mathématique. *Mathematica* 6(29), (1964), 105-116.
26. Tesuturi ternare regulate (Regular ternary webs). *Studia Univ. "Babeş-Bolyai"*, ser.1, fasc.1, 1964, 39-60.
27. (With M.Hosszu) Über die Klasse von ternären Quasigruppen. *Acta math.Acad.sci.Hung.*, 15(1964), 29-36.
28. (With V.Groze and B.Orbán) Propriétés extrémales dans une classe de fonctions et application à la transformation des nomogrammes. *Mathematica* 6(29), (1964), 307-326.
29. Einbettung eines Halbgewebe in ein reguläres Gewebe und eines Halbgruppoids in eine Gruppe. *Mathem.Zeitschrift*, 89(1965), 395-410.
30. Über die beste projektive Transformation der genadlinigen Leiter. *Zeitschr.f.angewandte Mathematik und Mechanik*, 45(1965), 356-359.
31. (With L.Németi) Sur la programmation temporelle de la fabrication. *Mathematica*, 8(31), (1966), 109-115.
32. (With V.Peteanu) Structures algébriques rattachées aux problèmes d'ordonnement. *Lucrările colocv.de teoria aproximativă a funcțiilor*, Cluj, 1967, 1-35.
33. Sur le problème de flût à cout minimum. *Studia Univ.Babeş-Bolyai*, ser.Math.Phys., fasc.1, 1968, 67-79.
34. (With V.Groze) Determinarea transformărilor optime ale nomograme-lor cu puncte aliniate (The determination of the optimal transformations of alignment nomograms). *Studia Univ.Babeş-Bolyai*, ser.Math.Phys., fasc.2, 1969, 9-14.
35. Einbettung von Inzidenzstrukturen in projektive Ebenen mit vorgegebenen Schliessungssätzen. *Archiv der Mathematik*, 20(1969), 311-319.
36. Darstellung nicht bijektiver Kollineationen eines projektiven Raumes durch verallgemeinerte semilineare Abbildungen. *Math. Zeitschr.*, 110(1969), 153-170.
37. Non-Injective Collineations on Some Sets in Desarguesian Projective Planes and Extension of Non-Commutative Valuations. *Aequationes Math.* 4(1970), 307-321.
38. Congruence Preserving Isomorphisms of the Translation Group Associated with a Translation Plane. *Canad.Journal of Math.*, 23(1971), 214-221.

39. A Branch and Cut Method for Solving Mixed One-Zero Programs. *Publ.de L'Univ.de Montréal, Fac.de Sciences*, 22(1970), 1-18.
40. (With J.Acztél, J.A.Baker, D.Z.Djocović, P.L.Kannappan) Extensions of Certain Homomorphisms of Semigroups to Homomorphisms of Groups. *Aequat.Math.*, 6(1971), 263-271.
41. Extension of Collineation Defined on Subsets of a Translation Plane. *Journal of Geom.*, 1(1971), 1-17.
42. On Semi-Symmetric Quasigroups. *Aequationes Math.*, 11(1974), 250-255.
43. Über die Bewertung der Vektorräume. Remark to the Problem of H.Hadwiger. *Reports of Meetings. Aequationes Math.* 11(1974), 299 and 309.
44. (With J.Acztél) Involutory Functions Induced by Abelian Groups. *Studia Univ.Babeş-Bolyai*, ser.Math.Mech., 1974, 32-40.
45. A Generalization of Valued Vector Spaces. Applications to Homomorphisms of Projective Spaces and to Mjelslev Geometries. Preprint, Cluj-Napoca, 1975, 29 pages.
46. Remark on a Functional Equation. *Aequationes Math.*, 14(1976), 228-230.
47. (With R.Miron) On the Definition of a Module and a Vector Space by independent Axioms. *Mathematica*, 18(41), (1976), 179-186.
48. (With B.Orbán) Nomograms with Minimal Global Error. *L'analyse numérique et la théorie de l'approximation*, 6(1977), 163-170.
49. A generalization of the valued vector space, *Mathematica*, 19(42), 203-209.
50. Affine Geometries and Affine Barbilian Structures. *Lucrările Col. Geom.şi top.*, 22-24 sept.1978, Cluj-Napoca, 27-45.
51. On the Definition of Skew Fields. *Archiv der Mathematik*, 32(1979), 441-444.
52. On the Characterization of Affine Isometries. *Resultate der Mathematik*, 2(1979), 171-185.
53. On the Characterization of Plane Affine Isometries. *Resultate der Math.*, 3(1980), 70-73.
54. Affine Barbilian Structures. *Journ.of Geom.*, 14(1980), 75-102.
55. Plane affine Barbilian (Affine Barbilian Planes). *Lucrările colocv. nat.de geom.şi top.*, Vaslui-Valea Hogeii-Bacău, 27-30 iunie 1980, 87-94.

56. Pre-Euclidean Planes over Pythagorean Fields and Orderable Fields. *Lucrările colocv.nat.de Geom.și top.*, București, 27-30 iunie 1981, București, 1983, 316-322.
57. A Characterization of the Semi-Isometries of a Minkowski Plane over a Field K . *Journ.of Geom.*, 21(1983), 164-183.
58. (With C.Tarția) A Characterization of the Isometries of the Rational Minkowski Plane. *Mathematica*, 26(1984), 183-187.
59. Teoreme de tip Beckman-Quarles in plane Minkowski generalizate peste un cîmp. (Beckman-Quarles Type Theorems in Generalized Minkowski Planes over Fields). *Lucrările colocv.nat.de geom.și top.*, Piatra Neamț, iunie 1983, Iași 1984, 15-24.
60. On the Characterization of the Semi-Isometries of a Galois Space. *Lucrările colocv.nat.de geom.și top.*, Timișoara, 2-7 iulie 1984, 237-240.
61. Mappings of the Galois Planes Preserving the Unit Euclidean Distance. *Aequationes math.*, 29(1985), 1-6.
62. (With I.Muntean) Un model al geometriei din manualul pentru clasa a IX-a. *Lucrările seminarului de "Didactica Matematicii"*, Cluj-Napoca, 1985, 67-82 (A model for the geometry presented in the textbook for grade IX).
63. On mappings of the Galois space. *Israel Journal of Math.*, 53(1986), 217-230.
64. On the Combinatorial Types of Polytopes over an Ordered Skew Field. *Research Seminars, Faculty of Mathematics, Cluj-Napoca*, no.10(1986), 3-8.
65. (With D.Andreescu and D.Vălcan) Mappings of E^n into E^m Preserving two Distances. *Research Seminars, Faculty of Math.*, Cluj-Napoca, 10(1986), 9-22.
66. On the Semi-Isometries of Metric Vector Spaces. *Lucrările colocv.nat.de geom.și top.*, Fîrgoviște, 12-14 aprilie 1986, București, 1988, 239-244.
67. (With J.A.Baker) Pexider's Equation and Aggregation of Allocations. *Aequationes Math.*, 32(1987), 227-239.
68. On the Euclidean Multifacility Location Problem. *Operations Res.*, 36 no.3(1988), 485-492.

B. Lecture Notes, Textbooks, Books

1. *Analitikus mértan (Analytic Geometry)* (hungarian), Lithographed, Cluj, Ed.1, 1954, Ed.2, 1958.
2. (With L.Bal) *Lecții de nomografie (Lectures in nomography)*, Ed. Tehnică, București, 1956.
3. (With V.Cseke and B.Kiss) *Feladatgyűjtemény középiskolai matematikai körök számára (Exercise book for mathematical circles in secondary school)*. Ed.Tehnică, București, vol.1; 1957, vol.2, 1959.
4. (With P.Szilágyi) *Analitikus mértan példatár (Exercise Book for Analytical Geometry)*, Litographed, Cluj, 1958.
5. *Curs de geometrie (A course in Geometry)*, Univ.Babeș-Bolyai, Cluj, 1972.
6. *Lecții de programare liniară (Lectures in Linear Programming)*. Univ.Babeș-Bolyai, Cluj-Napoca, 1975.
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THE MORSE-SMALE CHARACTERISTIC OF A COMPACT DIFFERENTIABLE MANIFOLD

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This paper deals with a general study of the Morse-Smale characteristic (Definition 2.1.) of a compact manifold M . Using some ideas which appear in the author's papers [1], [2], [3] one shows that the number $\chi(M)$ represents a differential invariant of M (Theorem 2.4.). The connection with the singular homology of the manifold is presented in Theorem 3.1. and Corollary 3.4. As applications of the general results the Morse-Smale characteristics of S^m , S_g^m , P^n and $P^n \times P^m$ are determined (Examples 3.5.) and a partial answer to an open problem concerning the existence of some special Morse functions on M is given (Theorem 4.1.).

1. Preliminaries. Let M^m be a compact m -dimensional smooth manifold with $\partial M = \emptyset$. For a smooth real mapping $f: M \rightarrow \mathbb{R}$ let us define by $C[f] = \{p \in M : (df)_p = 0\}$ the critical set of f and by $B[f] = f(C[f])$ the bifurcation set of f . An element $p \in C[f]$ represents a critical point